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VOLUME
I

Objective MATHEMATICS

~ Contents ~

1	Number System	2
2	Percentages	57
3	Simple & Compound Interest	78
4	Profit, Loss & Discount	98
5	Ratio & Proportion	121
6	Averages	150
7	Allegation, Mixtures	168
8	Time & Work, Pipe & Cisterns	190
9	Time, Speed & Distance	223
10	Algebra	272

Objective
MATHEMATICS
for RRB and other Exams

TOPRANKPREP

Number System

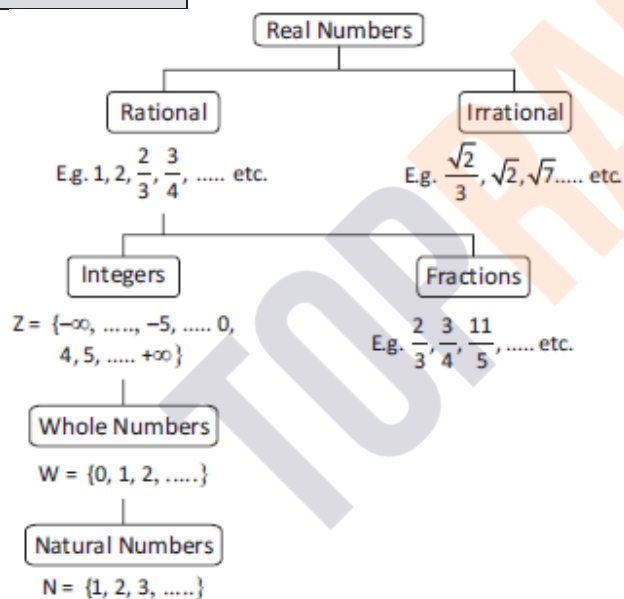
01

Number system is the fundamental building block of quantitative aptitude. Number system deals with various operations that can be performed on numbers. In this chapter, we will learn various concepts like types of numbers and fractions, finding unit's digit, remainders, factors, multiples, factorials, base system and calendars.

BASICS OF NUMBERS

First we will see various types of numbers that are there. We will also see their basic properties. Numbers in Mathematics can be related with a tree like structure given below:

Numbers Tree



Rational Numbers

A number which can be expressed in the form p/q (where p and q are integers and $q \neq 0$) is called a rational number. For example, 7, 5, -3, $2/7$, $-8/13$ etc are rational numbers. Rational numbers are either

integers or terminating decimals or recurring decimals.

Irrational Numbers

Numbers which are not rational but can be represented on the number line are called irrational numbers. These are decimals which are neither terminating nor recurring. For example, $\sqrt{2}$, $\sqrt{3}$, π , etc are irrational numbers.

Prime Numbers

All natural numbers, can be distinguished as prime numbers, composite numbers and 1.

All numbers which are only divisible by 1 and itself are known as Prime Numbers. In other words, we can say that all the numbers which have exactly two factors i.e. 1 and itself are known as Prime Numbers. 2 is the only even prime number.

There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

Prime Numbers till 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

There is no general formula to determine prime numbers.

General form of a prime number

All prime numbers greater than 3 can be written in the form of $6N - 1$ or $6N + 1$ (where N is a natural number) but each and every number of this form is not necessarily a prime number. It means that this is a necessary condition but not a sufficient condition for a number to be prime.

For example, $83 = 6 \times 14 - 1$, is a prime number, but $121 = 6 \times 20 + 1$, is not a prime number.

Composite Numbers

A number is composite if it has more than two factors. e.g. 4, 6, 8,

The smallest composite number is 4 and 1 is neither prime nor composite.

How to determine whether a number is prime or not?

For small numbers, we can determine by checking, whether that number is divisible by any other prime number till that number itself.

But for larger numbers like, 631, there is an alternative method.

Step 1: Find the approximate square root of the given number, i.e., 25.

Step 2: Check if any prime number from 2 to 25 divides 631.

The Prime numbers from 2 to 25 are 2, 3, 5, 7, 11, 13, 17, 19 and 23. Since, none of these numbers divide 631 exactly. Hence, 631 is a prime number.

Example 1: Is 1469 prime or not?

Solution:

Approximate square root of 1469 = 38

Prime numbers till 38 = 2, 3, 5, 7, 11, 13, 19, 23, 29, 31, 37

As 13 divides 1469 completely, hence 1469 is not a prime number.

Example 2: If $x + y = z$; x , y and z all are prime numbers and $y < x$. What is the value of y ?

Solution:

Here 'z' is given to be a prime number. The value of 'z' has to be odd as if it is even, then it has to be 2 and 2 is the smallest prime number, so it cannot be written as the sum of two other prime numbers. Now, as 'z' is odd; so one of 'x' or 'y' should be even and the other has to be odd. As $y < x$, so 'y' has to be 2, because 2 is the only even and smallest prime number. Hence, the answer is (b).

Example 3: Let x and y be positive integers such that x is prime and y is composite. Then which of the following is definitely true?

- I. $y - x$ cannot be an even integer.
- II. xy cannot be an even integer.
- III. $\frac{x+y}{y}$ cannot be an even integer.
- IV. None of these

Solution:

In case I, Let $y = 4$ and $x = 2$, then $y - x$ can be even.

In case II, Let $y = 4$ and $x = 2$, then xy can be even.

In case III, $\frac{x+y}{y} = \frac{x}{y} + 1$

For $(x/y) + 1$ to be an even integer, x/y has to be an odd integer, as x is prime, x/y can never be an integer.

Hence, $(x+y)/y$ cannot be an even integer.

Even and Odd numbers

Even numbers are those which are divisible by 2 and

Odd numbers are those which are not divisible by 2.

The general form of even numbers is $2n$ (where $n = 1, 2, 3, 4$) and the general form of odd numbers in $2n + 1$ (where $n = 0, 1, 2, 3, 4$).

▪ **Facts about Even and Odd numbers**

Even + Even = Even

Even - Even = Even

Even $\times$ Even = Even

Even $\div$ Even = Even or Odd

Odd + Odd = Even

Odd - Odd = Even

Odd $\times$ Odd = Odd

Odd $\div$ Odd = Odd

Even + Odd = Odd

Even - Odd = Odd

Even $\times$ Odd = Even

Even $\div$ Odd = Even

Odd + Even = Odd

Odd - Even = Odd

Odd $\times$ Even = Even

Odd $\div$ Even = (never divisible)

(Even)^{even/odd} = Even

(Odd)^{odd/even} = Odd

Example 4: If x and y are positive integers such that $x + y$ is always odd, then which of the following is always even?

I. $x - y$

II. $x^2 - y^2$

III. xy

IV. $x + y + xy$

Solution:

As $x + y$ is always odd, then one of the x or y is always odd and the other one is always even as only odd + even = odd

If one is odd and other is even then, $x - y = \text{odd}$

$$x^2 - y^2 = (x + y)(x - y)$$

$$= \text{odd} \times \text{odd}$$

$$= \text{odd}$$

$$xy = \text{even (as one of them is even)}$$

$$x + y + xy = \text{odd} + \text{even} = \text{odd}$$

Hence, only case III is even.

Example 5: Two of a , b , c and d are even and two are odd ($a + b > c + d$), not necessarily in the same order.

Which of the following is definitely even?

I. $a + b + c - 2d$

II. $a + 2b - c$

III. $2a + b + c - d$

IV. $a + b - c + d$

Solution:

Since we don't know which two are even and which two are odd, we will have to do a bit of hit-and-trial to solve this problem, with the help of options.

In case I, if a and b are even, and c and d are odd, then this will lead us to an odd number.

In case II, if a and b are even, and c is odd, then this will lead us to an odd number.

In case III, if a and b are odd, and c and d are even, then this will lead us to an odd number.

In case IV, whatever is the value of a , b , c and d , it is always going to be an even number.

Hence, the case IV is even.

FRACTIONS

A fraction is a part of an integer. The bottom part of the fraction is called the denominator. Think of it as the denomination – it tells you what size of fraction we are talking about: fourth, fifths, etc. The top of the fraction is called the numerator. It tells you how many fourths, fifths, etc.

The value of a fraction cannot be altered by multiplying or dividing the numerator and the denominator by the same number (other than 0).

Fractions are classified into the following categories:

I. Proper Fractions

A proper fraction is the one whose numerator is less than its denominator.

For example, $\frac{3}{7}$, $\frac{7}{13}$ are proper fractions.

II. Improper Fractions

An improper fraction is the one whose numerator is greater than its denominator.

For example, $\frac{7}{3}$, $\frac{11}{8}$ etc. are improper fractions.

III. Mixed Fractions

A mixed fraction consists of both an integral part and a fractional part. An improper fraction can be represented as a mixed fraction.

For example, $3\frac{1}{2}$, $7\frac{3}{5}$ are mixed fractions.

A mixed fraction can be written as a sum of integral part and the fractional part.

For example,

IV. Complex Fractions

Fractions whose numerators and denominators are fractions are known as complex fractions.

For example, $4\frac{2}{3} = 4 + \frac{2}{3} = \frac{14}{3}$ are complex fractions.

V. Equivalent Fractions

Two or more fractions are equivalent if they can be reduced to the same value.

For example, $\frac{3}{7}, \frac{5}{9}, \frac{4}{11}, \frac{3}{17}$ are all equivalent fractions.

Comparison of Fractions

Two fractions can be easily compared by cross multiplication method.

Let's see how to compare two fractions $\frac{5}{7}$ and $\frac{7}{9}$ by cross multiplication method.

$$\begin{array}{ccc} \frac{5}{7} & \nearrow \nwarrow & \frac{7}{9} \\ \Rightarrow & 5 \times 9 & 7 \times 7 \\ \Rightarrow & 45 & 49 \end{array}$$

As $45 < 49$, so the fraction $\frac{7}{9}$ is larger than $\frac{5}{7}$ i.e.

$$\frac{5}{7} < \frac{7}{9}$$

DECIMALS

Decimal is a way of writing numbers where each digit represents the different power of 10. It also needs a decimal point (dot) to separate the integral and fractional parts of the number.

A decimal in which a digit or a sequence of digits is repeated continually is called recurring decimal.

To represent a recurring decimal we put a bar on the repeating numerals.

$$\text{e.g. } 1/7 = 0.142857142857..... = 0.\overline{142857}$$

Conversion of recurring decimals to p/q form

A recurring decimal can be easily converted to a fraction. Let's try to understand the same by some examples.

Example 6: What is the p/q form of $0.33333 \dots$ (or $0.\overline{3}$)?

Solution:

$$\text{Let } x = 0.333\dots(\text{or } 0.\overline{3}) \quad \dots (1)$$

Multiply equation (1) by 10,

$$\therefore 10x = 3.333\dots(\text{or } 3.\overline{3}) \quad \dots (2)$$

Subtracting (1) from (2),

$$9x = 3$$

$$\text{So, } x = \frac{3}{9}$$

Example 7: What is the p/q form of $0.717171\dots$ (or $0.\overline{71}$)?

Solution:

$$\text{Let } x = 0.717171\dots(\text{or } 0.\overline{71}) \quad \dots (1)$$

Multiply equation (1) by 100,

$$\therefore 100x = 71.7171\dots(\text{or } 71.\overline{71}) \quad \dots (2)$$

Subtracting (1) from (2),

$$99x = 71$$

$$\text{So, } x = \frac{71}{99}$$

In the first example, we have multiplied the equation by 10 and in the second example by 100, as we need one more equations in which the recurring part is same, which is to be eliminated in the subtraction of the two equations.

Example 8: What is the $\frac{p}{q}$ form of $0.3\overline{1}$?

Solution:

$$\text{Let } x = 0.3\overline{1} \quad \dots (1)$$

Multiply equation (1) by 10,

$$\therefore 10x = 3.\overline{1} \quad \dots (2)$$

Now, the right hand side (RHS) has become a pure recurring decimal, so we can apply the same method that we had applied in pure recurring decimals.

Multiply equation (2) by 10,

$$100x = 31.\overline{1} \quad \dots (3)$$

Now, equation (2) and (3) have the same recurring part. So, Subtracting (2) from (3),

$$\therefore 90x = 31.\overline{1} - 3.\overline{1} = 28$$

$$\therefore x = \frac{28}{90} = \frac{14}{45}$$

After observing the above examples, we say that recurring decimals can be converted into the form of

$\frac{p}{q}$ as follows:

$\frac{p}{q}$

The form of a pure recurring decimal

$$= \frac{\text{The recurring part written once}}{\text{As many 9's as the number of digits in the recurring part}}$$

Thus, $0.\overline{71}$ can be straightaway written as $\frac{71}{99}$. $\frac{p}{q}$

The form of a mixed recurring decimal

$$= \frac{\left[\begin{array}{l} \text{(The non-recurring and the recurring part} \\ \text{written once)} - \text{(the non-recurring part)} \end{array} \right]}{\left[\begin{array}{l} \text{As many 9's as the number of digits in the} \\ \text{recurring part followed by as many 0's as} \\ \text{the number of digits in the non-recurring part} \end{array} \right]}$$

Thus, $0.\overline{31}$ can be written as $\frac{31-3}{90} = \frac{28}{90}$.**BODMAS**

BODMAS - Order of Simplification of an expression of numbers

B → Bracket

O → Of

D → Division

M → Multiplication

A → Addition

S → Subtraction

Of is solved by replacing by 'x' sign but is solved before division.

In a given expression of numbers, the above order of operations has to be strictly followed.

Example 9: What is the value of

$$\frac{3}{5} \text{ of } \frac{5}{9} \div \frac{1}{5} + \left(1 + \frac{1}{3} \right) - \left(\frac{29}{7} - \frac{8}{7} \right) ?$$

Solution:

$$= \frac{3}{5} \times \frac{5}{9} \div \frac{1}{5} + \frac{4}{3} - 3$$

$$= \frac{3}{5} \times \frac{5}{9} \div \frac{1}{5} + \frac{4}{3} - 3$$

$$= \frac{1}{3} \div \frac{1}{5} + \frac{4}{3} - 3$$

$$= \frac{1}{3} \times \frac{5}{1} + \frac{4}{3} - 3$$

$$= \frac{5}{3} + \frac{4}{3} - 3$$

$$= \frac{9}{3} - 3 = 3 - 3 = 0$$

SIMPLIFICATION FORMULAS

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- $(a + b)^2 - (a - b)^2 = 4ab$
- $a^2 - b^2 = (a - b)(a + b)$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Example 10: Simplify

$$\frac{527 \times 527 \times 527 + 183 \times 183 \times 183}{527 \times 527 - 527 \times 183 + 183 \times 183}$$

Solution:

The given expression is equivalent to

$$\begin{aligned} & \frac{(527)^3 + (183)^3}{(527)^2 - 527 \times 183 + (183)^2} \\ & \frac{(527 + 183)(527^2 - 527 \times 183 + 183^2)}{(527)^2 - 527 \times 183 + (183)^2} \\ & = 527 + 183 = 710 \end{aligned}$$

Example 11: Simplify $\left(\frac{(614 + 168)^2 - (614 - 168)^2}{614 \times 168} \right)$.**Solution:**

$$\left(\frac{(614 + 168)^2 - (614 - 168)^2}{614 \times 168} \right)$$

$$\frac{614^2 + 168^2 + 2 \times 614 \times 168 - 614^2 - 168^2 + 2 \times 614 \times 168}{614 \times 168} = \frac{4 \times 614 \times 168}{614 \times 168} = 4$$

Example 12: Simplify $\left(\frac{7^3 + 11^3 - 18^3}{693} \right)$.

Solution:

As $7 + 11 - 18 = 0$ ($a + b + c = 0$)

$$\therefore 7^3 + 11^3 + (-18)^3 = 3 \times 7 \times 11 \times (-18)$$

$$\Rightarrow -4158$$

$$\Rightarrow \frac{-4158}{693} = -6$$

COUNTING FORMULAS

- Sum of first n natural numbers $= \frac{n(n+1)}{2}$
- Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- Sum of the cubes of first n natural numbers $= \left[\frac{n(n+1)}{2} \right]^2$
- Sum of first n odd numbers $= n^2$
- Sum of first n even numbers $= n(n+1)$

Example 13: $N = 31 + 32 + 33 + \dots + 90$. What is the value of N ?

Solution:

$$N = (1 + 2 + 3 + \dots + 90) - (1 + 2 + 3 + \dots + 30)$$

$$= \frac{90 \times 91}{2} - \frac{30 \times 31}{2}$$

$$= 45 \times 91 - 31 \times 15 = 4095 - 465 = 3630$$

Example 14: $N = 11^2 + 12^2 + 13^2 + \dots + 20^2$. What is the value of N ?

Solution:

$$11^2 + 12^2 + 13^2 + \dots + 20^2$$

$$= (1^2 + 2^2 + 3^2 + \dots + 20^2) - (1^2 + 2^2 + 3^2 + \dots + 10^2)$$

$$= \frac{20 \times 21 \times 41}{6} - \frac{10 \times 11 \times 21}{6}$$

$$= \frac{10 \times 21}{6} (2 \times 41 - 11)$$

$$= \frac{10 \times 21}{6} \times (82 - 11)$$

$$= \frac{10 \times 21}{6} \times 71$$

$$= 5 \times 7 \times 71 = 2485$$

Example 15: $N = 1 + 3 + 5 + 7 + \dots + 39$. What is the value of N ?

Solution:

From 1 to 39, there are $\frac{39+1}{2} = 20$ odd numbers.

$$\therefore \text{Sum of first 20 odd numbers} = 20^2 = 400$$

Example 16: $N = 2^3 + 4^3 + 6^3 + 8^3 + \dots + 20^3$. What is the value of N ?

Solution:

$$N = (2 \times 1)^3 + (2 \times 2)^3 + (2 \times 3)^3 + (2 \times 4)^3 + \dots + (2 \times 10)^3$$

$$= 2^3 [1^3 + 2^3 + 3^3 + \dots + 10^3]$$

$$= 8 \times \left[\frac{10 \times 11}{2} \right]^2 = 24200$$

SURDS AND INDICES

Surds

Surds are those numbers which are written as the square root or cube root or any other root of the numbers. For example, $\sqrt{2}$, $\sqrt{3}$.

General form of the surd is $\sqrt[n]{a}$, where a is called the radical and it must be a positive rational number. n is called the order of the surd and it must be a natural number.

If $\sqrt[n]{a}$ is an integer, then it is not a surd.

Please note that $\sqrt{4}$ is not a surd. (As it is an integer)

Types of Surds

Simple Surd: A surd having a single term only is called simple surd. For example, $\sqrt{3}$, $5\sqrt{5}$, etc., are simple surds.

Compound Surd: The Algebraic sum of two or more simple surds or a simple surd and a rational

number is called a compound surd. For example, $\sqrt{2} + \sqrt{3}$, $5 + \sqrt{7}$, etc, are compound surds.

Properties of Surds

Addition: $\sqrt{x} + \sqrt{y}$ cannot be written as a single surd.

However, $\sqrt{x} + \sqrt{x} = 2\sqrt{x}$

For example, $\sqrt{5} + 3\sqrt{5} + 6\sqrt{5} = 10\sqrt{5}$

Multiplication: $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$

For example, $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$

Sometimes multiplication of surds can give rational numbers as $\sqrt{3}$ and $\sqrt{27}$ are surds, whereas 9 is a rational number.

- **Conjugate surd:** Recall that $\sqrt{3}$ is an example of a simple and $2 + \sqrt{3}$ is an example of a compound surd.

Observe that $(\sqrt{3})^2$ is rational, but $(2 + \sqrt{3})^2 = 4 + 3 + 2 \times 2 \times \sqrt{3}$ is still not rational (that is, it can't be expressed as a fraction).

The compound surds $(a + \sqrt{b})$ and $(a - \sqrt{b})$ are called conjugate surds. They are the roots of the equation, $x^2 - 2ax + a^2 - b = 0$.

Conjugate surds can be used to express a fraction which has a compound surd as its denominator with a rational denominator. You multiply the top and bottom of the fraction by the conjugate of the denominator. This is useful for simplifying expressions, as shown below.

$$\frac{5}{2 - \sqrt{3}} = \frac{5}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{5(2 + \sqrt{3})}{2^2 - (\sqrt{3})^2} = 5(2 + \sqrt{3})$$

Notice that the denominator will always become a difference of two squares, which must be rational.

$$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \times \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = \frac{3(5 + \sqrt{2})}{25 - 2} = \frac{3}{23}(5 + \sqrt{2})$$

Indices

When a quantity is multiplied by itself a certain number of times, the product thus obtained is called a power of that quantity.

Thus, a^m means 'a' is multiplied 'm' times consecutively and 'm' is called the exponent or the index and 'a' is called the base.

Rules of Indices:

- $a^m \times a^n = a^{m+n}$

- $\frac{a^m}{a^n} = a^{m-n}$

- $(a^m)^n = a^{mn}$

- $a^{-m} = \frac{1}{a^m}$

- $a^0 = 1$

- $(ab)^m = a^m b^m$

- $\sqrt[m]{a} = a^{\frac{1}{m}}$

- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

- $(x^a)^b \neq x^{a^b}$

For example, $(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$

and $2^{3^2} = 2^9 = 512$

Example 17: What is the value of

$$\frac{1}{(216)^{\frac{2}{3}}} + \frac{1}{(256)^{\frac{3}{4}}} + \frac{1}{(243)^{\frac{1}{5}}}$$

Solution:

$$\frac{1}{(216)^{\frac{2}{3}}} + \frac{1}{(256)^{\frac{3}{4}}} + \frac{1}{(243)^{\frac{1}{5}}}$$

$$(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + (243)^{\frac{1}{5}}$$

$$(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + (3^5)^{\frac{1}{5}}$$

$$= 6^2 + 4^3 + 3^1$$

$$= 36 + 64 + 3$$

$$= 103$$

Example 18: Find the value of $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$.

Solution:

$$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$$

$$\frac{2^n + \frac{2^n}{2}}{2^n \times 2^1 - 2^n} = \frac{2^n \left(1 + \frac{1}{2}\right)}{2^n (2 - 1)} = \frac{3}{2}$$

LOGARITHMS

The logarithmic function is the inverse of exponential function.

Definition: If any number N is expressed in the form of a^x , (where $a > 0$ and $a \neq 1$), then the index X is called the logarithm of the number N to the base 'a'.

Thus, if $N = a^x$, then $x = \log_a N$.

Properties of Logarithms

- $\log_a (XY) = \log_a X + \log_a Y$
- $\log_a \left(\frac{X}{Y}\right) = \log_a X - \log_a Y$
- $\log_a (X^k) = k \log_a X$
- $\log_a \sqrt[k]{X} = \frac{1}{k} \log_a X$
- $\log_{a^k} X = \frac{1}{k} \log_a X$
- $\log_{a^{1/k}} X = k \log_a X$
- $\log_a 1 = 0$ [$\because a^0 = 1$]
- $\log_x X = 1$
- $\log_a X = \frac{1}{\log_x a}$
- $\log_a X = \frac{\log_b X}{\log_b a}$
- $\log_{b^n} a^m = \frac{m}{n} \log_b a$
- $a^{(\log_a X)} = X$
- When base is not mentioned, it will be taken as 10.

Example 19: Find the value of $\log_9 16 \times \log_{32} 27$.

Solution:

$$\begin{aligned} \log_9 16 \times \log_{32} 27 \\ = \frac{\log 16}{\log 9} \times \frac{\log 27}{\log 32} \end{aligned}$$

$$\begin{aligned} &= \frac{\log 16}{\log 32} \times \frac{\log 27}{\log 9} = \frac{\log 2^4}{\log 2^5} \times \frac{\log 3^3}{\log 3^2} \\ &= \frac{4 \log 2}{5 \log 2} \times \frac{3 \log 3}{2 \log 3} \\ &= \frac{4}{5} \times \frac{3}{2} = \frac{6}{5} \end{aligned}$$

Example 20: Find the value of $\frac{8 \log_8 8}{2 \log_{\sqrt{8}} 8}$.

Solution:

$$= \frac{8 \log_8 8}{2 \log_{\sqrt{8}} 8} = \frac{8 \times 1}{2 \log_{\sqrt{8}} \sqrt{8}^2} = \frac{8}{4 \log_{\sqrt{8}} \sqrt{8}} = \frac{8}{4} = 2$$

Example 21: If $\log_5 [\log_3 (\log_2 x)] = 1$, then what is the value x?

Solution:

$$\begin{aligned} \log_5 [\log_3 (\log_2 x)] &= 1 \\ \Rightarrow \log_3 (\log_2 x) &= 5^1 \Rightarrow \log_3 (\log_2 x) = 5 \\ \Rightarrow \log_2 x &= 3^5 \Rightarrow \log_2 x = 243 \\ \text{So, } x &= 2^{243} \end{aligned}$$

Example 22: What is the value of $(\log_{1998} 2 + \log_{1998} 27 + \log_{1998} 37)^{1998}$?

Solution:

$$\begin{aligned} &(\log_{1998} 2 + \log_{1998} 27 + \log_{1998} 37)^{1998} \\ &= \left(\frac{\log 2}{\log 1998} + \frac{\log 27}{\log 1998} + \frac{\log 37}{\log 1998} \right)^{1998} \\ &= \left(\frac{\log 2 + \log 27 + \log 37}{\log 1998} \right)^{1998} \\ &= \left(\frac{\log 2 \times 27 \times 37}{\log 1998} \right)^{1998} = \left(\frac{\log 1998}{\log 1998} \right)^{1998} \\ &\Rightarrow 1^{1998} = 1 \end{aligned}$$

So, required value = 1

REMAINDERS

When we divide 57 by 9, then 9 is the divisor, 57 is the dividend, 6 is the quotient and 3 is the remainder

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Example 23: If dividend is 15968, quotient is 89 and the remainder is 37, then what is the divisor?

Solution:

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\begin{aligned}\text{Divisor} &= \left(\frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} \right) \\ &= \left(\frac{15968 - 37}{89} \right) = 179\end{aligned}$$

Note:

A number when divided by 'd' leaves remainder 'r' can be represented as $(d \times n) + r$, where 'n' is a natural number.

Basic facts about Remainder

- When any positive number A is divided by any other positive number B, and if $B > A$, then the remainder will be A itself.

$$\text{Remainder of } \frac{5}{12} = 5$$

$$\text{Remainder of } \frac{212}{678} = 212$$

- Remainder should always be calculated in its actual form. For example,

$$\text{Remainder of } \frac{1}{2} = 1$$

$$\text{Remainder of } \frac{2}{4} = 2$$

$$\text{Remainder of } \frac{3}{6} = 3$$

It can be observed that despite of all fractions being equal, remainders are different in each case. Therefore you cannot reduce the fraction to its lowest form to find the remainder.

- Product of any two or more natural numbers has the same remainder when divided by any natural number, as the product of their remainders and the same is true for addition and subtraction of numbers as well. Let's understand this through an example

Example 24: Find the remainder when 12×13 is divided by 7?

Solution:

$$\text{Remainder} = \frac{156}{7} = 2$$

Instead of multiplying the numbers we can find their individual remainders and then can multiply the respective remainders to get the answer.

So, first of all, we will find out the remainders of each number individually and then we will multiply these individual remainders to find out the final remainder.

$$\text{Remainder} = \frac{12}{7} = 5$$

$$\text{Remainder} = \frac{13}{7} = 6$$

$$\text{Remainder} = \text{Remainder} \frac{(5 \times 6)}{7}$$

$$\text{Remainder} = \frac{30}{7} = 2$$

Example 25: What is the remainder obtained when $(1421 \times 1423 \times 1425)$ is divided 12?

Solution:

$$\text{Remainder} = \frac{1421}{12} = 5$$

$$\text{Remainder} = \frac{1423}{12} = 7$$

$$\text{Remainder} = \frac{1425}{12} = 9$$

$$\text{Remainder} = \frac{(1421 \times 1423 \times 1425)}{12} = \frac{(5 \times 7 \times 9)}{12}$$

$$\text{Remainder} = \frac{(5 \times 63)}{12} = 3$$

Concept of Negative Remainder

Although remainders can't be negative, but for calculation purpose they have an existence in this topic.

Let's understand the concept of negative remainder by taking an example.

17 when divided by 6 leaves positive remainder of +5 as $17 = 6 \times 2 + 5$

Now, 17 can also be written as $17 = 6 \times 3 - 1$

So, we can say that 17 when divided by 6 gives negative remainder as -1.

Similarly, 16 when divided by 6 leaves positive remainder +4 and the negative remainder will be -2. One important point to note over here is that the absolute sum of the positive and the negative remainders is always equal to the divisor.

Important results about remainders

1. $\frac{(A+1)^N}{A}$ will always give remainder 1 for all natural values of A and N.

Let's find the remainder when 9^{100} is divided by 8?

For $A = 8$, it satisfies the above condition. So, remainder = 1

2. $\frac{(A-1)^N}{A}$ will always give remainder $(-1)^N$ for natural values of A and N. If N is even, remainder is 1 and if N is odd, remainder is -1.

Let's find the remainder when 5^{86} is divided by 6?

For $A = 6$, it satisfies the above condition. So, remainder = 1.

3. $\frac{(mA+1)^N}{A}$ will always give remainder 1 for all natural values of A and N.

$\frac{(mA-1)^N}{A}$ will always give remainder $(-1)^N$ for all natural values of A and N (where mA is a multiple of A).

Let's find the remainder when 9^{88} is divided by 5?

$9^{87} = (10-1)^{87}$, if $A = 5$, then 10 is of form mA.

Hence, remainder is 1.

Example 26: What is the remainder when 2^{1998} is divided by 7?

Solution:

Here, 2 is a number less than 7, we try to convert the product into product of numbers higher than 7, such that it leaves a remainder of +1 or -1 when it is divided by 7. Notice that $8 = 2 \times 2 \times 2$. The remainder when 8 is divided by 7 is 1.

Hence, $2^{1998} = (2^3)^{666} = 8^{666}$. When divided by 7 the remainder is 1.

Example 27: What is the remainder when 2^{2000} is divided by 7?

Solution:

This problem is like the previous one, except that 2000 is not an exact multiple of 3 so we cannot convert it completely into the form 8^x . We will write it in following manner: $2^{2000} = 2^2 \times 2^{1998} = 4 \times (2^3)^{666}$.

Now, 8^{666} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2000} is divided by 7 is the remainder when the product 4×1 is divided by 7. Therefore, remainder = 4

Example 28: What is the remainder when 25^{36} is divided by 9?

Solution:

Again $25^{36} = (18+7)^{36} = (18+7)(18+7)\dots 36 \text{ times} = 18K + 7^{36}$

Hence, remainder when 25^{36} is divided by 9 is same as the remainder when 7^{36} is divided by 9.

Now, $7^{36} = 7^3 \times 7^3 \times 7^3 \dots (12 \text{ times}) = 343 \times 343 \times 343 \dots (12 \text{ times})$.

When, 343 is divided by 9, the remainder is 1. Hence, the remainder when 25^{36} is divided by 9 is 1.

Some special cases

When both the Dividend and the Divisor have a Factor in Common

To find the remainder,

- I. Divide both dividend and divisor by the common factor (K) i.e. HCF of dividend and divisor.
- II. Divide the resulting dividend by the resulting divisor and find the remainder (R_1).
- III. The real remainder R is the remainder R_1 multiplied by the common factor (K).

Example 29: What is the remainder when 2^{96} is divided by 96?

Solution:

The common factor between 2^{96} and 96 is 32.

Divide both dividend and divisor by 32.

Now, we will get the resulting dividend and the divisor and we will find the remainder when 2^{91} is divided by 3.

- When, 2^{91} is divided by 3, we get the remainder as -1 or $+2$ i.e. $(3 - 1)$. Now, since we cancelled the common factor from dividend and divisor, the final remainder will be obtained after multiplying the remainder with cancelled factor, the final remainder will be $(+2) \times 32 = 64$.

Rules pertaining to $(a^n + b^n)$ and $(a^n - b^n)$

- $(a^n + b^n)$ is always divisible by $(a + b)$, if n is odd.
- $(a^n - b^n)$ is always divisible by $(a - b)$, if n is odd.
- $(a^n - b^n)$ is always divisible by $(a - b)$ and $(a + b)$, if n is even.

Example 30: What is the remainder when $(15^{23} + 23^{23})$ is divided by 19?

Solution:

It can be observed that $(15^{23} + 23^{23})$ is divisible by 38 i.e. $(15 + 23)$, so it will be divisible by 19 also. Hence, the remainder is 0.

Example 31: What is the remainder when $16^3 + 17^3 + 18^3 + 19^3$ is divided by 70?

Solution:

$$16^3 + 17^3 + 18^3 + 19^3 \\ = (16^3 + 19^3) + (17^3 + 18^3) = 35a + 35b$$

So, the expression is divisible by 35 and it is also an even number, So it must be divisible by 70. Hence the remainder is 0.

Concept of Successive Division

Suppose we say that **N** is any number which is divided successively by 3 and 5, then what we mean to say is first, we divide **N** by 3 and then the quotient obtained is divided by 5.

For example, let's see the case when 50 is divided by 7 and 3 successively.

50 divided by 7 gives 7 as the quotient and 1 as the remainder. Now, we will divide 7 by 3.

It gives a quotient of 2 and remainder of 1 i.e. $50 = 7(3 \times 2 + 1) + 1$

Example 32: When a number **N** is divided successively by 3 and 5, remainder obtained are 1 and 2 respectively. What is the remainder when **N** is divided by 15?

Solution:

It can be seen that we are required to calculate it from back-end.

Number which when divided by 5 give remainder 2 are of the form $= 5S + 2$

$$\text{So, } N = 3(5S + 2) + 1 = 15S + 7$$

Now, when **N** is divided by 15, remainder = 7.

UNIT DIGIT CONCEPT

To find the unit's digit of the sum or product of two or more numbers, we have to add or multiply the unit's digit of the numbers individually to find the unit's digit of the resultant number. For example, to find the unit's digit of the product of 728 and 234, we will multiply the unit's digit of the two numbers i.e. 8 and 4 which is 32. So, the unit's digit of the resultant number is 2.

Now, let's see, how to find the unit's digit of higher powers of any number.

Basically, the last digit of the powers of any number follow a cyclic pattern i.e. they repeat after a certain number of steps.

Let's look at the power of 2

Unit's digit of 2^1 is 2

Unit's digit of 2^2 is 4

Unit's digit of 2^3 is 8

Unit's digit of 2^4 is 6

Unit's digit of 2^5 is 2

Unit's digit of 2^6 is 4

Unit's digit of 2^7 is 8

Unit's digit of 2^8 is 6

Unit's digit of 2^9 is 2

Unit's digit of 2^{10} is 4

Unit's digit of 2^{11} is 8

Unit's digit of 2^{12} is 6

Here, The unit's digit of $2^1, 2^5, 2^9$ is 2

The unit's digit of $2^2, 2^6, 2^{10}$ is 4

The unit's digit of $2^3, 2^7, 2^{11}$ is 8

The unit's digit of $2^4, 2^8, 2^{12}$ is 6

As, after every four powers of 2, the unit's digit of the number starts repeating, thus, we can say that the unit's digit cyclicity of higher power of 2 is 4.

Similarly the unit's digit cyclicity of 3, 7 and 8 is also 4.

And, the unit's digit cyclicity of 4 and 9 is 2.

Note:

As we know cyclicity of 4 and 9 both is 2, that means their unit digit repeat after every two powers, so what we conclude from here is:

4^{odd} gives unit digit 4; 9^{odd} gives unit digit 9

4^{even} gives unit digit 6; 9^{even} gives unit digit 1

Also any powers of numbers whose unit's digit is 0, 1, 5, 6 always ends in 0, 1, 5 and 6 respectively. So, 0, 1, 5 and 6 have a cyclicity of 1.

For example $21^2 = 441$, $16^2 = 256$ etc.

So, collectively the cyclicity of the digits is as follows:

Digit	Cyclicity
2, 3, 7, 8	4
4, 9	2
0, 1, 5, 6	1

Example 33: Find the unit's digit of 213^{147} ?

Solution:

As the cyclicity of 3 is 4, we will divide 147 by 4.

147 when divided by 4 leaves remainder 3. So, the last digit of 213^{147} is same as the last digit of 3^3 i.e. = 7.

Note:

Please note that the number of digits in the base will not make a difference to the unit's digit of the number. It is only the unit's digit of the base which decides the unit digit of the number. So finding the unit's digit of 213^{147} is same as finding the unit's digit of 3^{147} .

Example 34: Find the unit's digit of $17^{43} \times 28^{21}$?

Solution:

As both 7 and 8 exhibit a cyclicity of 4.

The unit's digit of 17^{43} is same as the unit's digit of 7^3 i.e. 3.

The unit's digit of 28^{21} is same as the unit's digit of 8^1 i.e. 8.

So, the unit's digit of $17^{43} \times 28^{21}$ is same as the unit's digit of the product of 3 and 8 (24) i.e. 4.

DIVISIBILITY RULES

A divisibility rule is a shorthand way of determining whether a given number is divisible by a fixed divisor without performing the actual division, usually by examining its digits.

Divisibility by 2, 4, 8, 16, 32,

A number is divisible by 2, 4, 8, 16, 32, ..., 2^n when the number formed by the last one, two, three, four, five ... n digits is divisible by 2, 4, 8, 16, 32, ..., 2^n respectively.

It means that the divisibility rule of 2 is related with the unit digit, the divisibility rule of $4(2^2)$ is related with the last two digits, the divisibility rule of $8(2^3)$ is related with the last three digits and so on.

For example, 23784 is divisible by 8 because the number formed by the last three digits i.e. 784 is divisible by 8. Also the number 23784 is divisible by 4 because the number formed by the last two digits, 84 is divisible by 4.

Now, 48532 is divisible by 4 since the last two digits 32 are divisible by 4, but the number 48532 is not divisible by 8 since the last three digits 532 is not divisible by 8. Now, since the number is not divisible by 8 so, it will leave a non zero remainder when it is divided by 8. This remainder can be obtained by dividing the last three digits by 8. So, 532 when divided by 8 leaves remainder 4. Also the complete number i.e. 48532 when divided by 8 leaves remainder 4 as well.

Divisibility by 5, 25, 125, 625,

A number is divisible by 5, 25, 125, 625, ..., 5^n when the number formed by the last one, two, three, four, five ... n digits is divisible by 5, 25, 125, 625, ..., 5^n respectively.

It means that the divisibility rule of 5 is related with the unit digit, the divisibility rule of $25(5^2)$ is related with the last two digits, the divisibility rule of $125(5^3)$ is related with the last three digits and so on.

For example, 856375 is divisible by 125 because the number formed by the last three digits i.e. 375 is divisible by 125. Also the number 7875 is divisible by 25 because the number formed by the last two digits, 75 is divisible by 25.

The number 756350 is not divisible by 125 since the last three digits of 350 is not divisible by 125. Now, since the number is not divisible by 125 so, it will leave a non zero remainder when it is divided by 125. This remainder can be obtained by dividing the last three digits by 125. So, 350 when divided by 125 leaves remainder 100. Also the complete number i.e. 756350 when divided by 125 leaves remainder 100 as well.

Divisibility by 3 and 9

A number is divisible by 3 or 9 when the sum of the digits of the number is divisible by 3 or 9 respectively. For example, 58632 is divisible by 3 because the sum of the digits ($5 + 8 + 6 + 3 + 2 = 24$) is divisible by 3.

The number 953163 is divisible by 9 because the sum of the digits ($9 + 5 + 3 + 1 + 6 + 3 = 27$) is divisible by 9.

The number 364577 is not divisible by 9 as the sum of the digits ($3 + 6 + 4 + 5 + 7 + 7 = 32$) is not divisible by 9.

Now, since the number is not divisible by 9 so, it will leave a non zero remainder when it is divided by 9. This remainder can be obtained by dividing the sum of the digits by 9. So, 32 when divided by 9 the

remainder is 5. Also the complete number i.e. 364577 when divided by 9 leaves remainder 5 as well.

Similarly, the remainder from 3 can also be obtained from the sum of the digits.

Divisibility by 6, 12, 14, 15, 18,

If N can be written as a product of two number A and B

$$N = A \times B$$

A number N is divisible by the product $A \times B$ of two nonzero whole numbers A and B if and only if it is divisible by both A and B , and A and B have only 1 as a common whole number factor.

Whenever we have to check the divisibility of a number N by a composite number C , the number N should be divisible by all the prime factors (the highest power of every prime factor) present in C .

Divisibility by 6: the number should be divisible by both 2 and 3.

Divisibility by 12: the number should be divisible by both 3 and 4.

Divisibility by 18: the number should be divisible by both 2 and 9.

Divisibility by 24: the number should be divisible by both 3 and 8.

Divisibility by 72: the number should be divisible by both 8 and 9.

Divisibility by 88: the number should be divisible by both 8 and 11.

For example, 2568 is divisible by 6 as it is divisible by 2 as well as 3.

Divisibility by 11

A number is divisible by 11 if the difference between the sum of the digits at the odd and the sum of the digits at the even places is 0 or a multiple of 11.

For example, for the number 292215

$$\text{By rule, } (5 + 2 + 9) - (1 + 2 + 2) = 11$$

So, the number is divisible by 11

Now for the number 1760673

$$\text{By rule, } (3 + 6 + 6 + 1) - (7 + 0 + 7) = 2$$

So, the number is not divisible by 11, but it will leave a remainder of 2 when it is divided by 11.

One important point to note over here is if the difference is not 0 or a multiple of 11, the difference value will become the remainder when the given number is divided by 11.

Note:

Any six-digit, or twelve-digit or eighteen-digit or any such number with number of digits equal to a multiple of 6, is divisible by EACH of 7, 11 and 13 if all of its digits are same.

For example, 666666, 888888888888, etc. are all divisible by 7, 11, and 13.

Example 35: What least value should be assigned to X so that the number 451 X 603 is exactly divisible by 9?

Solution:

Sum of digits = $19 + X$ should be divisible by 9.

So, X must be 8.

Example 36: What least value should be assigned to X so that the number 63576X2 is divisible by 8?

Solution:

6X2 is divisible by 8.

So, X can be 3 or 7 but the least value of X is 3.

Example 37: If a number 968A96B is to be divisible by 72, the respective values of A and B will be?

Solution:

The number should be divisible by both 8 and 9.

Now, '96B' should be divisible by 8. So, B can be 0 or 8.

Now, sum of digits should be divisible by 9.

Sum of digits = $38 + A + B$

If B is '0' then $A = 7$ ($38 + 7 = 45$)

and If B = 8 then $A = 8$ ($38 + 8 + 8 = 54$)

Possible pairs are (7, 0) and (8, 8).

NUMBER OF FACTORS/DIVISORS

When we talk about the number of divisors of any number, we are basically talking about positive integral divisors of that number. Likewise it can be

observed that 20 has six divisors namely 1, 2, 4, 5, 10 and 20.

It is very essential to understand the fundamental of formation of divisors.

Since $20 = 2^2 \times 5^1$

We will take three power of 2 viz. 2^0 , 2^1 and 2^2 and two powers of 5 viz. 5^0 and 5^1 .

Divisors will come from all the possible combinations of powers of 2 and powers of 5.

$$2^0 \times 5^0 = 1$$

$$2^0 \times 5^1 = 5$$

$$2^1 \times 5^0 = 2$$

$$2^1 \times 5^1 = 10$$

$$2^2 \times 5^0 = 4$$

$$2^2 \times 5^1 = 20$$

Now, to find out number of divisors of any number, we can use the following formula:

If N is any number which can be factorized as $N = a^p \times b^q \times c^r \times \dots$, where a, b and c are prime numbers or the Prime Factors of 'N'

Then Number of divisors = $(p + 1)(q + 1)(r + 1) \dots$

Example 38: Find the number of divisors of 420?

Solution:

$$N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$$

So, Number of divisors = $(2 + 1)(1 + 1)(1 + 1)(1 + 1) = 24$.

Example 39: Find the number of even divisors and composite divisors of 420?

Solution:

$$N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$$

Odd divisors will come only if we take zero power of 2 (since any number multiplied by any power of 2 will give us an even number)

So, Odd divisors will come if we take $N_1 = 2^0 \times 3^1 \times 7^1 \times 5^1$

$$\text{So, Odd divisors} = (0 + 1)(1 + 1)(1 + 1)(1 + 1) = 8$$

So, Total number even divisors = Total number of divisors – Number of odd divisors

$$= 24 - 8 = 16$$

Alternatively, We can also find out the number of even divisors of $N = 420$ directly (or, in general for any number).

$$420 = 2^2 \times 3^1 \times 7^1 \times 5^1$$

To obtain the factors of 420 which are even, we will not consider 2^0 , since $2^0 = 1$

So, number of even divisors of $420 = (2)(1+1)(1+1)(1+1) = 16$.

(We are not adding 1 in the power of 2, since we are not taking 2^0 here i.e. we are not taking one power of 2.)

Prime divisor = 4 (namely 2, 3, 5 and 7 only)

Composite Divisors = Total Divisors – Prime Divisors – 1

$= 24 - 4 - 1 = 19$ (as 1 is neither prime nor composite).

HCF AND LCM

Meaning of HCF (Highest Common Factor)

Factors are those positive integral values of a number, which can divide that number. HCF is the highest value which can divide the given numbers. HCF is also known as Greatest Common Divisor (GCD). There are two methods of finding the HCF.

I. Division Method

II. Factor Method

I. **HCF by Division Method**

Example 40: Find the HCF of 210 and 495.

Solution:

$$\begin{array}{r}
 210 \overline{)495} (2 \\
 \underline{420} \\
 75 \overline{)210} (2 \\
 \underline{150} \\
 60 \overline{)75} (1 \\
 \underline{60} \\
 15 \overline{)60} (4 \\
 \underline{60} \\
 \hline
 \end{array}$$

Hence, the HCF is 15.

II. HCF by Factor Method

In this method first we break (or resolve) the numbers into prime factors then take the product of all the common factors. The resultant product obtained is known as the HCF of the given numbers.

Example 41: Find the HCF of 250, 5400 and 8100.

Solution:

$$250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$$

$$5400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 = 2^3 \times 3^3 \times 5^2$$

$$8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^4 \times 5^2$$

So, the product of common factors $= 2^1 \times 5^2 = 50$

Hence, the HCF is 50.

Meaning of LCM (Least Common Multiple)

A common multiple is a number that is a multiple of two or more than two numbers. The common multiples of 3 and 4 are 12, 24,

The least common multiple (LCM) of two numbers is the smallest positive number that is a multiple of both the numbers.

I. LCM by Division Method

First of all, write down all the given numbers in a line separated by a comma (,). Divide these numbers by the least common prime factors say 2, 3, 5, 7, 11, ... of the given numbers, then write the quotients just below the actual numbers separated by a comma (,). If any number is not divisible by such a prime factor then write that number as it is just below itself, then continue this process of division by considering higher prime factors, if the division is complete by lower prime factor, till the quotient in the last line is 1. Then take the numbers (or quotients) in different lines (or steps). This product will be the LCM of the given numbers.

Example 42: Find the LCM of 420, 9009 and 6270.

Solution:

2	420, 9009, 6270
---	-----------------

2	210, 9009, 3135
3	105, 9009, 3135
3	35, 3003, 1045
5	35, 1001, 1045
7	7, 1001, 209
11	1, 143, 209
13	1, 13, 19
19	1, 1, 19
	1, 1, 1

LCM of 420, 9009 and 6270 is $= 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 11 \times 13 \times 19 = 3423420$

II. LCM by Factor Method:

Resolve the given numbers into their prime factors and find product of all common prime factors of the number. Then take the product of all the prime factors of the first number with those prime factors of second number which are not common to the prime factors of the first number.

Now, this resultant product can be multiplied with those prime factors of the third number which are not common to the factors of the previous product and this process can be continued for further numbers if any. The product of common and uncommon factors will give the final LCM.

Example 43: Find the LCM of 48, 72 and 140.

Solution:

We can write the given numbers as:

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$140 = 2 \times 2 \times 5 \times 7$$

So, the LCM of 48, 72 and 140 is:

$$= 2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 5040$$

Standard formulas related with HCF and LCM

1. $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

This formula can be applied only in case of two numbers. However if all the pairs of numbers are relatively prime to each other (i.e. HCF of numbers = 1), then this formula can be applied for any number of numbers.

2. LCM of fractions

$$= \frac{\text{LCM of numerator of all the fractions}}{\text{HCF of denominator of all the fractions}}$$

3. HCF of fractions

$$= \frac{\text{HCF of numerator of all the fractions}}{\text{LCM of denominator of all the fractions}}$$

Note:

While finding the LCM and HCF of fractions, we first convert (if required) the fractions into their simplest form and then find their LCM or HCF.

Example 44: What is the LCM and HCF of $\frac{2}{3}$, $\frac{4}{9}$, $\frac{8}{6}$ and $\frac{7}{11}$?

Solution:

Here, $\frac{8}{6}$ will be taken as $\frac{4}{3}$.

$$\begin{aligned} \text{LCM} &= \frac{\text{LCM of numerators}}{\text{HCF of denominators}} \\ &= \frac{\text{LCM}(2, 4, 4 \text{ and } 7)}{\text{HCF}(3, 9, 3 \text{ and } 11)} = \frac{28}{1} = 28 \end{aligned}$$

$$\begin{aligned} \text{HCF} &= \frac{\text{HCF of numerators}}{\text{LCM of denominators}} \\ &= \frac{\text{HCF}(2, 4, 4 \text{ and } 7)}{\text{LCM}(3, 9, 3 \text{ and } 11)} = \frac{1}{99} \end{aligned}$$

Questions based on the application of LCM and HCF

Example 45: There are two bells in a temple. Both the bells toll at regular intervals of 20 seconds and 50 seconds respectively. After how much time will they toll together for the first time?

Solution:

Time taken by 1st bell to toll = 20 seconds

Time taken by 2nd bell to toll = 50 seconds

So, they will toll together after a time which is the lowest common multiple of 20 and 50 secs i.e. 100 seconds.

Example 46: Find the side of the largest possible square slabs which can be paved on the floor of a room 2.50 m long and 1.50 m broad. Also find the number of such slabs to pave the floor?

Solution:

The size of the largest possible square slab should be the HCF of the dimensions.

HCF (250 and 150) = 50 i.e. 0.5

So, slab size should be $0.5 \times 0.5 = 0.25 \text{ m}^2$

Number of slabs required = $\frac{2.5 \times 1.5}{0.5 \times 0.5} = 15$

Example 47: There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains same number of fruits of each type. What is the minimum number of rows required for this to happen?

Solution:

We can put one fruit in one row, and still in $(24 + 36 + 60)$ 120 rows, can we arrange all the fruits. Or, even we can put two fruits in one row and can arrange all the fruits in 60 rows. But for the rows to be minimum, number of fruits should be maximum in one row. i.e. HCF of 24, 36, 60 = 12, so, minimum number of fruits in each row so that each row has same number of fruits is 12.

Hence, number of rows = $\frac{24}{12} + \frac{36}{12} + \frac{60}{12}$

= $2 + 3 + 5 = 10$

LCM Type 1 Questions

In this type, a number N is divided by two or more divisors such that they give same remainders. Let a number N be divided by two divisors D_1 and D_2 such that remainder in each case is R then the number N must be in the form of $k(\text{LCM of } D_1, D_2) + R$, here k is any positive integer.

Here, $N = k(\text{LCM of } D_1, D_2) + R$

Example 48: Find the smallest three digit number that gives remainder 1 when divided by 2, 5 or 7.

Solution:

The number that satisfies the above condition must be in the form of $k(\text{LCM of } 2, 5 \text{ and } 7) + 1$, or the number must be in the form of $70k + 1$. Smallest 3 digit number will be 141.

Example 49: Find the smallest number greater than 6 and 8 which when divided by 6 or 8 leaves a remainder 3 in each case.

Solution:

Any number or group of numbers that satisfies the above condition can be given by $k(\text{LCM of } 6 \text{ and } 8) + 3$. Since, LCM of 6 and 8 = 24 so any number in the form of $24k + 3$ will satisfy the above condition and we will get smallest number when $k = 1$ i.e. smallest number is 27.

Example 50: Find the largest three digit number which when divided by 5 or 8 leaves remainder 4 in each case.

Solution:

Any number or group of numbers that satisfies the above condition can be given by $k(\text{LCM of } 5 \text{ and } 8) + 4$, since LCM of 5 and 8 = 40 so any number in the form of $40k + 4$ will satisfy above condition. Smallest number is when $k = 1$, i.e. 44. But here we need to find the largest three digit number that satisfies the above condition or is in the form of $40k + 4$. So we will first find largest 3 digit number that is divisible by 40. When we divide largest 3 digit number 999 by 40 then we will get a remainder 39. So, largest 3 digit number in the form of $40k$ is $999 - 39 = 960$. So, largest 3 digit number in the form of $40k + 4 = 964$.

LCM Type 2 Questions

In this type, a number N is divided by two or more divisors such that they give different remainders but the difference between the divisor and remainder is

same in each case. Let a number N be divided by two divisors D_1 and D_2 such that remainder are R_1 and R_2 respectively, and $D_1 - R_1 = D_2 - R_2 = d$ then the number N must be in the form of $k(\text{LCM of } D_1, D_2) - d$, where k is any +ve integer.

Example 51: Find the largest 3 digit number that gives remainder 3 and 6 when divided by 5 and 8 respectively.

Solution:

Here, in this case the difference between divisor and remainder i.e. $5 - 3 = 8 - 6 = 2$ is constant.

Now, the number must be of the form of $40k - 2$, since LCM of 5 and 8 is 40. The largest 3 digit number in the form of $40k - 2 = 998$.

LCM Type 3 Questions

In these type, of questions, neither the remainder nor the difference is same, so we have to use hit and trial method to find the smallest number that satisfies the above condition then any number in the form of any multiple of LCM of divisors added to the smallest number will satisfy the condition. Let us understand this with the help of examples.

Example 52: Find the largest 3 digit number that leaves a remainder 1 and 5 when divided by 5 and 8 respectively.

Solution:

In this case neither, the remainder is same (As in type 1) nor is the difference between divisor and remainder same (As in type 2). Here, we have to find the smallest number that satisfies above condition. Since N divided by 5 leaves remainder 1 hence N must be in the form of $(5x + 1)$, when this number is divided by 8 then it leaves remainder 5. Now we have to do hit and trial method to find the minimum value of x such that $5x + 1$ when divided by 8 gives a remainder 5.

For $x = 1$, $5x + 1 = 6$

For $x = 2$, $5x + 1 = 11$

For $x = 3$, $5x + 1 = 16$, it is divisible by 8 hence $x = 3$ will give us smallest value of N that satisfies these conditions. And that smallest value is $(16 + 5) = 21$. Now any number in the form of $k(\text{LCM of 5 and 8}) +$ smallest number or $40k + 21$ will satisfy this condition. Then largest 3 digit number that satisfy these conditions are $960 + 21 = 981$.

HCF Type 1 Questions

The largest number which divides the numbers a , b and c and gives the same remainder is given by HCF of $(a - b)$ and $(b - c)$. Here we are concerned with positive value of the difference. Point to note that $\text{HCF of } (a - b) \text{ and } (b - c) = \text{HCF of } (a - b) \text{ and } (a - c) = \text{HCF of } (b - c) \text{ and } (a - c)$.

Now, we will have a brief look at how it works. Let us assume common remainder as r and the largest such number as h then when a is divided by h it gives a remainder r , hence $(a - r)$ is divisible by h similarly $(b - r)$ and $(c - r)$ is divisible by h . We know that if numbers a and b are divisible by another divisor d then $(a - b)$ will also be divisible by d . So working on the same logic taking difference of the two in a pair we will get $(a - b)$ and $(b - c)$ both are divisible by h hence largest value of h is the HCF of $(a - b)$ and $(b - c)$ and so on.

Example 53: When 302, 752 and 1502 divided by N gives same remainder in each case then find the largest possible value of N .

Solution:

In this case since remainder is same hence required number is HCF of $(752 - 302)$ and $(1502 - 752)$ or HCF of 450 and 750 and it is 150, hence largest value of N is 150.

HCF Type 2 Questions

The largest number by which the numbers a , b and c are divided giving remainders as p , q and r respectively then the largest number is given by HCF of three number $(a - p)$, $(b - q)$ and $(c - r)$.

Example 54: Find the largest number by which 182, 228 and 275 are divided gives remainders 2, 3 and 5 respectively.

Solution:

Let the number be N hence $182 - 2 = 180$ is divisible by N and similarly $228 - 3 = 225$ and $275 - 5 = 270$ is divisible by N . So, largest value of N is given by HCF of 180, 225 and 270 and that is 45.

HIGHEST POWER IN A FACTORIAL

Exponent of any prime number p in $n!$

$$= \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots + \left[\frac{n}{p^x} \right],$$

where $[]$ denotes the greatest integer value i.e. we have to consider only the integral value.

Let us find out exponent of 5 in 1000!

$$= \left[\frac{1000}{5} \right] + \left[\frac{1000}{25} \right] + \left[\frac{1000}{125} \right] + \left[\frac{1000}{625} \right]$$

$$= 200 + 40 + 8 + 1 = 249$$

Process to find out the exponent of any composite number in $n!$

We have three different types of composite numbers:

1. Product of two or more than two prime numbers with unit power of all the prime numbers e.g. $15(5 \times 3)$, $30(2 \times 3 \times 5)$ etc.
2. Product of two or more than two prime numbers with power of any one prime number more than 1. e.g. $12(2^2 \times 3)$, $72(2^3 \times 3^2)$ etc.

Example 55: What is the highest power of 3 in 100!?

Solution:

$$\text{Highest power of } 3 = \left[\frac{100}{3} \right] + \left[\frac{100}{9} \right] + \left[\frac{100}{27} \right] + \left[\frac{100}{81} \right]$$

$$= 33 + 11 + 3 + 1 = 48$$

Example 56: Find the highest power of 15 in 100!?

Solution:

15 is the product of two distinct prime numbers 5 and 3. So, to find out the exponents of 5 and 3 individually.

$$\text{Highest power of } 5 = \left[\frac{100}{5} \right] + \left[\frac{100}{25} \right] = 20 + 4 = 24$$

$$\text{Highest power of } 3 = \left[\frac{100}{3} \right] + \left[\frac{100}{9} \right] + \left[\frac{100}{27} \right] + \left[\frac{100}{81} \right]$$

$$= 33 + 11 + 3 + 1 = 48$$

So, obviously the answer is 24.

Note:

No need to find powers of 3 as it would obviously be greater than the power of 5.

Example 57: What is the highest power of 25 in 100!?

Solution:

In this case, we first find the exponents of 5 and then divide it by 2 (the power) to find the exponents of 25.

$$\text{Highest power of } 5 = \left[\frac{100}{5} \right] + \left[\frac{100}{25} \right] = 20 + 4 = 24$$

So, highest power of 25 = $24/2 = 12$

CALENDAR

An ordinary year consists of 365 days (52 weeks and 1 odd day). An extra day is added once in every fourth year which is called as leap year, which has 366 days (52 weeks and 2 odd days). An year which is divisible by 4 is a leap year e.g. 1896, but if the year is a century year, then it should be divisible by 400, only then it would be a leap year. e.g. 1700 is not a leap year whereas 1600 is a leap year.

To find the day of any given date of the year, you need to understand the calendar calculations:

1. First thing to remember is that first January 1 AD was Monday therefore, we must count days from Sunday. This means the 0th day was Sunday, so the 7th day was Sunday again and so on and so forth.
2. The day gets repeated after every seventh day (concept of a week), if today is Monday, then 28th day from now will also be Monday, as 28 is a multiple of 7 i.e. $(28/7 = 4)$, so four weeks). Here, the 30 day will be calculated by $30/7$, which is 4 weeks and 2 days, these two days are called odd days. With starting day as Monday and two odd days, the 30th day will be Wednesday; this point is the most critical in calendars. The other way to

look at it is since the 28th day is Monday, so the 30th day will be Wednesday. But you have to understand and use the concept of odd days as the question may be about thousands of years. Each day gets repeated after 7 days i.e. one week, if today is Monday then the day after 7 days would be Monday and after 14, 28 days would be Monday as they are multiples of 7. Now a month having 30 days would have $30/7$ i.e. 4 weeks and 2 days. Hence, if today is Monday then after 30 days it would be Wednesday as after 4 weeks i.e. 28 days we would have a Monday but these 2 odd days would make it a Wednesday. This concept of Odd days is important while Calculating for thousands of year.

- In a normal year, there are 365 days, so 52 weeks and 1 odd day but in a leap year there are 366 days, so 52 weeks and 2 odd days.
- In 100 years, there are 24 leap years and 76 normal years(as 100 is an ordinary year because it is not divisible by 400), so the number of odd days are $24(2) + 76(1) = 124$, which is 17 weeks + 5 odd days, so 100 years have 5 odd days.
- In 200 years, the number of odd days is twice the number in 100 years which is 10, which is one week and 3 odd days, so 200 years have 3 odd days.
- In 300 years, the number of odd days is 15, which is two weeks and 1 odd day, so 300 years have one odd day.
- In 400 years, the number of odd days become $20 + 1$ (from the leap year), so total days are 21, which is three weeks and 0 odd days. In 400 years there are 0 odd days.

Example 58: What was the day on 25th January, 1975?

Solution:

Counting the years $1600 + 300 + 74$

In 1600 years, there are zero odd days.

In 300 years, there is one odd day

In 74 years, there are 18 leap years (74 divided by 4 quotient is 18) and 56 normal years, so the odd days are:

$18(2) + 56(1) = 36 + 56 = 92$, which is 13 weeks and 1 odd day

In 25 days of January, 1975, there are 3 weeks and 4 odd days.

Total odd days = $0 + 1 + 1 + 4 = 6$

This final value which is coming 6 in this question decides which day it was.

Final Value	Day
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday
0	Sunday

So, it was a Saturday.

Example 59: What day of the week was on 20th June 1837?

Solution:

Counting the years $1600 + 200 + 36$

In 1600 years, there are zero odd days.

In 200 years, there are three odd days.

In 36 years, there are 9 leap years and 27 normal years, so the odd days are:

$9 \times 2 + 27 \times 1 = 48$, which is 6 weeks and 6 odd days.

In 1837, the odd days are:

$3(\text{Jan}) + 0(\text{Feb}) + 3(\text{Mar}) + 2(\text{Apr}) + 3(\text{May}) + 6(\text{June}) = 17$, which is 2 weeks and 3 odd days.

Total odd days = $0 + 3 + 6 + 3 = 12$ odd days, which is 5 odd days.

So, it was a Friday.

Example 60: Today is 3rd November, this day of the week is Monday. This is a leap year. What will be the day of the week on this date after 3 years?

Solution:

This is a leap year and the date is after 29th Feb. So none of the next 3 years will be leap years. Each year will give one odd day so the day of the week will be 3 odd days beyond Monday i.e. it will be Thursday.

Example 61: The calendar of year 1982 is same as which year?

Solution:

We need to have 0 odd days, counting from 1982

Year	Odd days	Total
1982	1	1
1983	1	2
1984	2	4
1985	1	5
1986	1	6
1987	1	7
1988	2	9
1989	1	10
1990	1	11
1991	1	12
1992	2	14
1993	1	

Therefore 1988 could be the year with the same calendar as 1982, but it's a leap year and 1982 is not. Therefore next is 1993, where it fits, the calendar of 1982 is same as of 1993.

BASE SYSTEM

In our decimal system of writing the numbers, we use 10 digits (0 – 9). In this system, largest number of

single digit = 9, and the moment we have to form a number bigger than this number we are needed to take resort to two-digit numbers starting from 10.

Similarly, largest number of two digits = 99 and after this we have 100, a number of three digits.

Now, let us assume a system of writing where we can use only 6 digits (0 – 5). Largest single digit number in this system will be 5 and next to this will be 10. Similarly, largest two digit number will be 55 and next to this is 100.

Some Commonly Used Bases

System	Total Digits	Digits
Binary	2	0, 1
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexa-Decimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Let us see some comparative values for octal, decimal and hexa-decimal bases.

Octal	Decimal	Hexa-Decimal
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
10	8	8

11	9	9
12	10	A
13	11	B
14	12	C
15	13	D
16	14	E
17	15	F
20	16	10

From the above table, we can see that $(8)_{10} = (10)_8$ and $(16)_{10} = (10)_{16}$. So, we can deduce that $(b)_{10} = (10)_b$

Generally questions based on conversion of bases are asked in three different form viz:

1. Any other base to base 10
2. Base 10 to any other base
3. Base x to base y (x, y is not equal to 10)

Let us see each one of these in detail now.

Any other base to base 10

If any number is given in base 10 = $(89)_{10}$

It can be written as $8 \times 10^1 + 9 \times 10^0$, as it is base 10.

So, if it is any other base for e.g. $(89)_9$

$$\therefore (89)_9 = 8 \times 9^1 + 9 \times 9^0 = 72 + 9 = (81)_{10}$$

So, to convert any number of base b into base 10 we can use:

$$(xyz)_b = [(x \times b^2) + (y \times b^1) + (z \times b^0)]_{10}$$

Base 10 to any other base

Let us say we have to convert $(89)_{10}$ in base 6. We divide 89 by 6 and make note of quotient and remainder.

6	89	Remainder
6	14	5
	2	2

↑
Quotient

Now, the last quotient will be written first and first remainder will be written last $(89)_{10} = (225)_6$.

Base x to any other y

If we have to convert a number $(143)_5$ to base 6, then we have to take base 10 as an intermediate base.

$$(143)_5 = (P)_{10} = (Q)_6$$

First we do, $(143)_5 = (P)_{10}$

$$(143)_5 = (1 \times 5^2) + (4 \times 5^1) + (3 \times 5^0) \\ = 25 + 20 + 3 = 48$$

$$\text{So, } (143)_5 = (48)_{10}$$

Now, we will find $(48)_{10} = (Q)_6$

6	48
	8
	0

$$\text{So, } (48)_{10} = (80)_6$$

$$\text{Therefore } (143)_5 \rightarrow (80)_6$$

Number with a decimal

$$\text{I. } (18.26)_{10} = (P)_8$$

For such conversions, we first deal with part before decimal

$$(18)_{10} = (A)_8$$

8	18
	2
	2

$$\text{So, } (18)_{10} = (22)_8$$

$$\text{Now, we work with } (0.26)_{10} = (B)_8$$

$$0.26 \times 8 = 2.08 \text{ } \} \text{ take only integral part}$$

$$0.08 \times 8 = 0.64 \text{ } \} \text{ stop here as integral part is 0.}$$

$$\text{So, } (0.26)_{10} = (0.20)_8$$

$$\text{Hence, } (18.26)_{10} = (22.20)_8$$

$$\text{II. } (14.342)_6 = (P)_{10}$$

Again, working with part before decimal.

$$(14)_6 = (A)_{10}$$

$$(14)_6 = (1 \times 6^1) + (4 \times 6^0)$$

$$= 6 + 4 = 10$$

$$(14)_6 = (10)_{10}$$

The part after decimal

$$(0.342)_6 = (B)_{10}$$

$$(0.342)_6 = (3 \times 6^{-1}) + (4 \times 6^{-2}) + (2 \times 6^{-3})$$

$$= 0.5 + 0.111 + .009$$

$$(0.342)_6 = (0.620)_{10}$$

$$\text{Consolidating, } (14.342)_6 = (10.620)_{10}$$

Example 62: Convert $(ABCD)_{14}$ into decimal system?

Solution:

$$(ABCD)_{14} = A \times 14^3 + B \times 14^2 + C \times 14^1 + D \times 14^0$$

$$\Rightarrow 27440 + 2156 + 168 + 13$$

$$\Rightarrow (29777)_{10}$$

$$\therefore (ABCD)_{14} = (29777)_{10}$$

Example 63: Convert $(197)_{10}$ in base 3.

Solution:

3	197	
3	65	2
3	21	2
3	7	0
	2	1

$$\therefore (197)_{10} = (21022)_3$$

Addition and Subtraction in different bases

I. **Addition:**

$$(325)_7 + (456)_7$$

Start with the units place digit, $5 + 6 = 11$ which is $(14)_7$. So, unit digit is 4 and carry over is 1.

Next is tens place digit, $2 + 5 + 1$ (carry over) = 8 which is $(11)_7$. So, tens digit is 1 and carry over is again 1.

Next is $3 + 4 + 1$ (carry over) = 8 which is $(11)_7$.

$$(325)_7 + (456)_7 = (1114)_7$$

II. **Subtraction:**

$$(456)_8 - (367)_8$$

Starting with the units digit, since 6 is smaller than 7, we will borrow 1 from the tens place digit.

So, now it is 14 (when the base is 10, we get 10 but here base is 8, so will get 8.) and 7 subtracted from it = $14 - 7 = 7$, which is the units digit.

Next, tens digit is now 4 and we have to subtract 6 from it. We will again borrow 1 from hundred's place digit. So, now it is 12, and $12 - 6 = 6$, which is the tens place digit.

Now, hundred's place digit is $3(4 - 1)$, so $3 - 3 = 0$

$$\text{So, } (456)_8 - (367)_8 = (67)_8$$

Note:

Another method of doing these kind of calculations is to convert these values (in whatever base) into decimal system, then do the actual calculation in decimal system itself and finally converting the numbers into the required base system.

Example 64: What is the value of $(2345)_6 + (134)_6$?

Solution:

$$\begin{array}{r} 1 \quad 1 \\ 2 \quad 3 \quad 4 \quad 5 \\ + 1 \quad 3 \quad 4 \\ \hline 2 \quad 5 \quad 2 \quad 3 \end{array}$$

$$\therefore \text{Required value} = (2523)_6$$

Example 65: What is the value of $(4371)_8 - (1726)_8$ in hexadecimal system?

Solution:

$$\begin{array}{r} 4 \quad 3 \quad 7 \quad 1 \\ - 1 \quad 7 \quad 2 \quad 6 \\ \hline 2 \quad 4 \quad 4 \quad 3 \end{array}$$

$$\text{Now, } (2443)_8 = (1315)_{10} = (523)_{16}$$

$$\therefore \text{Required value} = (523)_{16}$$

Practice Exercise Level 1

- Which among 667, 559, 901 is prime?
(A) 667 (B) 559
(C) 901 (D) None of these
- A student was asked to find the sum of all the prime numbers between 10 and 40. He found the sum as 180. Which of the following statements is true?
(A) He missed one prime number between 10 and 20.
(B) He missed one prime number between 20 and 30.
(C) He added one extra non-prime number between 10 and 20.
(D) None of these
- Which of the following can't be written as the sum of two prime number?
(A) 78 (B) 88
(C) 51 (D) 60
- $N = A + B$, where A and B are two prime numbers. Which of the following statements is/are true?
A. Unit's digit of N can't be 8.
B. Unit's digit of N can't be 0.
C. N can't be a prime number.
(A) Only A (B) Only B
(C) Only C (D) None of these
- The sum of three consecutive even numbers is always divisible by:
(A) 12 (B) 6
(C) 18 (D) 24
- If $p = \frac{3}{5}$, $q = \frac{7}{9}$ and $r = \frac{5}{7}$, then
(A) $p < q < r$ (B) $q < r < p$
(C) $p < r < q$ (D) $r < q < p$
- The sum of a fraction and 3 times its reciprocal is $\frac{19}{4}$. What is the fraction?
(A) $\frac{3}{4}$ (B) $\frac{4}{3}$
(C) $\frac{5}{4}$ (D) $\frac{4}{5}$
- What is the value of $\frac{0.07 \times 0.07 \times 0.07 - 0.05 \times 0.05 \times 0.05}{0.07 \times 0.07 + 0.07 \times 0.05 + 0.05 \times 0.05}$?
(A) 0.002 (B) 0.02
(C) 0.2 (D) 0.0002
- What is the value of $\frac{(0.73)^3 + (0.27)^3}{(0.73)^2 + (0.27)^2 - 0.73 \times 0.27}$?
(A) 1 (B) 0.4087
(C) 0.73 (D) 0.46
- What is the value of $\frac{(27.5 - 19.3)^2 + (19.3 + 27.5)^2}{(19.3)^2 + (27.5)^2}$?
(A) 4 (B) 3
(C) 2 (D) 1
- What is the value of $\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 4.7 \times 3.8 - 1.5 \times 3.8}$?
(A) 0 (B) 1
(C) 10 (D) 30
- What is the value of $\left\{ 7\frac{1}{2} + \frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{4} - \frac{2}{5} \times 2\frac{1}{3} \div 1\frac{7}{8} \text{ of } \left(1\frac{2}{5} - 1\frac{1}{3} \right) \right\}$?
(A) $3\frac{1}{5}$ (B) $2\frac{1}{24}$
(C) $4\frac{1}{30}$ (D) None of these
- What is the value of $7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$?
(A) $\frac{2}{9}$ (B) 1
(C) $4\frac{1}{2}$ (D) $1\frac{77}{288}$

14. If $165^2 = 27225$, then what is the value of $\sqrt{272.25} + \sqrt{2.7225} + \sqrt{0.027225}$?
 (A) 12.25 (B) 34.65
 (C) 19.8 (D) 18.315
15. If the product of three consecutive integers is 120, then the sum of the integers is:
 (A) 9 (B) 12
 (C) 14 (D) 15
16. The denominator of a fraction is 1 more than its numerator. If 1 is deducted from both the numerator and the denominator, the fraction becomes equivalent to 0.5. The fraction is:
 (A) $\frac{3}{4}$ (B) $\frac{4}{5}$
 (C) $\frac{2}{3}$ (D) $\frac{7}{8}$
17. A two-digit number is three times the sum of its digits. If 45 is added to the number, its digits are interchanged. The sum of the digits of the number is:
 (A) 5 (B) 7
 (C) 9 (D) 11
18. P, Q and R are three consecutive odd numbers in ascending order. If the value of three times P is 3 less than two times R, find the value of R?
 (A) 5 (B) 7
 (C) 9 (D) 11
19. How many two-digit numbers are there which end in 7 and are divisible by 3?
 (A) 2 (B) 3
 (C) 4 (D) 5
20. On multiplying a number by 7, all the digits in the product appear as 3's. The smallest such number is:
 (A) 47619 (B) 47719
 (C) 48619 (D) 47649
21. How many digits are required for numbering the pages of a book containing 185 pages?
 (A) 435 (B) 185
 (C) 447 (D) 235
22. If n is an integer, which of the following must be even?
 (A) n^2 (B) $2n + 1$
 (C) $n + 2$ (D) $2n$
23. If a , b , and c are consecutive positive integers and $a < b < c$, which of the following must be true?
 I. $c - a = 2$
 II. abc is an even integer.
 III. $\frac{(a+b+c)}{3}$ is an integer.
 (A) I and II only (B) II and III only
 (C) I, II, and III only (D) I only
24. If p is an even integer and q is an odd integer, which of the following must be an odd integer?
 (A) $\frac{3p}{q}$ (B) pq
 (C) $2(p + q)$ (D) $2p + q$
25. What is the difference between the largest and the smallest four-digit numbers that can be formed using all the digits 0, 1, 2 and 8 (none of the digits being repeated)?
 (A) 7182 (B) 8128
 (C) 8082 (D) 198
26. If all the numbers between 11 and 100 are written on a piece of paper. How many times will the number 4 be used?
 (A) 20 (B) 19
 (C) 9 (D) 18
27. Shaurya writes first hundred whole numbers. Let, 'A' and 'B' be the number of times he writes '0' and '9' respectively. Find the value of $A + B$.
 (A) 31 (B) 30
 (C) 34 (D) 32
28. If we subtract 1 from a number, it becomes a perfect square. The original number cannot end in which of the following?
 (A) 1 or 2 (B) 2 or 3
 (C) 3 or 4 (D) 5 or 6

29. $23\bar{5}$ in $\frac{p}{q}$ form equals to:
- (A) $\frac{212}{990}$ (B) $\frac{53}{225}$
 (C) $\frac{106}{900}$ (D) $\frac{212}{999}$
30. Convert $0.234\bar{5}$ in $\frac{p}{q}$ form of rational number.
- (A) $\frac{129}{550}$ (B) $\frac{469}{1980}$
 (C) $\frac{2368}{9900}$ (D) None of these
31. If n^3 is odd, then which of the following must be true?
- I. n is odd II. $n \times n$ is odd
 III. $n \times n$ is even
- (A) (I) and (II) (B) (I) and (III)
 (C) (II) and (III) (D) (I), (II) and (III)
32. What is the value of $4 + 32 + 108 + \dots + 4000$?
- (A) 12000 (B) 12100
 (C) 122000 (D) 12400
33. How many numbers are there between 500 and 600 in which 9 occurs only once?
- (A) 19 (B) 18
 (C) 20 (D) 21
34. In all the numbers from 501 to 700 what is the total number of times the digit 6 appeared?
- (A) 138 (B) 139
 (C) 140 (D) 141
35. A two-digit number is such that the product of the digits is 14. When 45 is added to the number, then the digits interchange their places. Find the number.
- (A) 72 (B) 27
 (C) 37 (D) 14
36. The unit's digit of a two-digit number is one more than the digit at ten's place. If the number is more than five times of the sum of the digits of the number, then find the sum of all such possible numbers.
- (A) 246 (B) 275
 (C) 290 (D) 301
37. If $x = -0.5$, then which of the following has the smallest value?
- (A) $2^{1/x}$ (B) $\frac{1}{x}$
 (C) $\frac{1}{x^2}$ (D) 2^x
38. The sum of 10 consecutive natural numbers cannot be:
- (A) 785 (B) 755
 (C) 385 (D) None of these
39. $\sqrt{2}$, $\sqrt[3]{4}$ and $\sqrt[4]{6}$ in an ascending order are:
- (A) $\sqrt{2}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$ (B) $\sqrt[4]{6}$, $\sqrt{2}$, $\sqrt[3]{4}$
 (C) $\sqrt[4]{6}$, $\sqrt[3]{4}$, $\sqrt{2}$ (D) $\sqrt{2}$, $\sqrt[4]{6}$, $\sqrt[3]{4}$
40. What is the value of $\sqrt[4]{80} + \sqrt[3]{3125} - \sqrt[4]{6480}$?
- (A) $\sqrt[4]{80}$ (B) $\sqrt[4]{5}$
 (C) $-\sqrt[4]{5}$ (D) $-\sqrt[4]{80}$
41. What is the positive square root of $31 + 4\sqrt{57}$?
- (A) $\sqrt{19} + 4\sqrt{3}$ (B) $\sqrt{19} + 2\sqrt{3}$
 (C) $\sqrt{17} + \sqrt{14}$ (D) $\sqrt{17} + 3\sqrt{14}$
42. If $N = \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}}$, then what is the value of $N + \frac{1}{N}$?
- (A) $2\sqrt{2}$ (B) 5
 (C) 10 (D) 13
43. If $\frac{\sqrt{2+x} + \sqrt{2-x}}{\sqrt{2+x} - \sqrt{2-x}} = 2$, then what is value of x ?
- (A) $\frac{4}{8}$ (B) $\frac{3}{5}$
 (C) $\frac{8}{5}$ (D) $\frac{1}{5}$
44. What is the value of $\left(\frac{3^n + 3^{n-1}}{3^{n+1} - 3^n}\right)$?
- (A) $\frac{1}{2}$ (B) $\frac{3}{2}$
 (C) $\frac{2}{3}$ (D) $\frac{2}{1}$

45. What is the value of a , if $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$?
 (A) 4 (B) 2
 (C) 8 (D) 5
46. What is the value of $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25$?
 (A) 3 (B) 5
 (C) 2 (D) $\log\left(\frac{5}{2}\right)$
47. What is the value of x , if $0.01^x = 2$?
 (A) $\frac{\log 2}{2}$ (B) $\frac{2}{\log 2}$
 (C) $\frac{-2}{\log 2}$ (D) $\frac{-\log 2}{2}$
48. What is the value of $\log_{10} 10 + \log_{10} 10^2 + \dots + \log_{10} 10^n$?
 (A) $n^2 + 1$ (B) $n^2 - 1$
 (C) $\left(\frac{n^2 + n}{3}\right)$ (D) $\frac{n^2 + n}{2}$
49. What is the remainder when 6729 is divided by 35?
 (A) 11 (B) 7
 (C) 9 (D) 13
50. On dividing a certain number by 357, the remainder is 39. On dividing the same number by 17, what will be the remainder?
 (A) 0 (B) 3
 (C) 5 (D) 11
51. If a perfect square, not divisible by 6, be divided by 6, then what will be the remainder?
 (A) 1, 3 or 5 (B) 1, 2 or 5
 (C) 1, 3 or 4 (D) 1, 2 or 4
52. What is the remainder when 2^{93} is divided by 7?
 (A) 1 (B) 2
 (C) 4 (D) 6
53. What is the remainder when 4^{1000} is divided by 7?
 (A) 6 (B) 5
 (C) 4 (D) None of these
54. What is the remainder when 3^{40} is divided by 11?
 (A) 3 (B) 9
 (C) 5 (D) 1
55. What is the remainder when 2^{23} is divided by 10?
 (A) 2 (B) 3
 (C) 4 (D) 8
56. What is the remainder when 7^{84} is divided by 342?
 (A) 0 (B) 1
 (C) 49 (D) 341
57. What is the remainder when 5^{48} is divided by 124?
 (A) 123 (B) 1
 (C) 0 (D) 13
58. What is the remainder when 3^{65} is divided by 28?
 (A) 0 (B) 1
 (C) 9 (D) 19
59. What is the remainder when $4^{109} + 6^{109}$ is divided by 25?
 (A) 2 (B) 10
 (C) 15 (D) 5
60. When two numbers are divided by a certain divisor, the remainders are 9 and 10 respectively. When the sum of the numbers is divided by the same divisor, the remainder is 6. What is the divisor?
 (A) 12 (B) 13
 (C) 14 (D) 15
61. A woman collects just above 800 sticks for firewood and when she bundles them in groups of 4 she is left with 2 sticks and when she bundles them in groups of 5 she is left with 3 sticks. How many sticks are left if she bundles them in groups of 12?
 (A) 10 (B) 2
 (C) 8 (D) 4

62. A number when divided by a divisor which is 8 times the quotient, leaves 4 as the remainder. What is the number if the quotient is six times the remainder?
 (A) 4436 (B) 4254
 (C) 4612 (D) 4524
63. In a division sum, the remainder is 6 and the divisor is 5 times the quotient and is obtained by adding 2 to thrice the remainder. The dividend is:
 (A) 40 (B) 42
 (C) 80 (D) 86
64. What is the unit's place of 29^{136} ?
 (A) 1 (B) 3
 (C) 7 (D) 9
65. What is the unit's digit of $21^3 \times 2^{12} \times 34^7 \times 46^8$?
 (A) 4 (B) 8
 (C) 6 (D) 2
66. What is the last digit of the product $2^{22} \times 3^{33} \times 4^{44} \times 5^{55}$?
 (A) 2 (B) 4
 (C) 8 (D) 0
67. What is the unit's digit of $214^{870} + 149^{127}$?
 (A) 7 (B) 1
 (C) 3 (D) 5
68. What is the unit's digit of the number $1^{101} + 2^{101} + 3^{101} + 4^{101} + \dots + 9999^{101}$?
 (A) 5 (B) 2
 (C) 4 (D) 0
69. What is the unit's digit in the product $(4387)^{245} \times (621)^{72}$?
 (A) 1 (B) 2
 (C) 5 (D) 7
70. What is the unit's digit of the sum $(24)^{372} + (24)^{373}$?
 (A) 5 (B) 0
 (C) 2 (D) 4
71. If $517*324$ is divisible by 3, then what smallest digit is there in place of *?
 (A) 0 (B) 1
 (C) 2 (D) None of these
72. What least value must be given to * so that the number $97215*6$ is divisible by 11?
 (A) 3 (B) 2
 (C) 1 (D) 5
73. What least value must be given to * so that the number $91876*2$ is divisible by 8?
 (A) 1 (B) 2
 (C) 3 (D) 4
74. What least value must be given to * so that the number $42573*$ is exactly divisible by 72?
 (A) 4 (B) 5
 (C) 6 (D) 7
75. If the number $653xy$ is completely divisible by 80, then the value of $(x + y)$ is:
 (A) 2 (B) 3
 (C) 4 (D) 6
76. How many numbers between 200 and 600 are divisible by 4, 5 and 6?
 (A) 5 (B) 6
 (C) 7 (D) 8
77. From a group of first 100 natural numbers, all the numbers which are multiples of 3 and 4 but not 12 are removed. What is the number of the remaining numbers?
 (A) 32 (B) 68
 (C) 42 (D) 58
78. If $1826X73$ is divisible by 9, then what can be the value of 'X'?
 (A) 9 (B) 0
 (C) 3 (D) (A) or (B)
79. If $256X561$ is divisible by 11, then what can be the value of 'X'?
 (A) 3 (B) 0
 (C) 6 (D) 8
80. $4K56$ is divisible by 33. What is the value of 'K'?
 (A) 3 (B) 4
 (C) 5 (D) 6

81. What is the value of M and N respectively if M8458N is divisible by 88, where M and N are single digits?
(A) 5, 4 (B) 8, 6
(C) 6, 4 (D) 3, 2
82. How many factors does 1200 have?
(A) 10 (B) 15
(C) 30 (D) 45
83. What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?
(A) 7, 8 (B) 8, 6
(C) 6, 4 (D) 5, 4
84. The seven-digit number 2A5A756 is divisible by 3. What is the sum of the possible values of A?
(A) 8 (B) 12
(C) 16 (D) 18
85. For what value of the digit A, the number 56788A be divisible by 12?
(A) 4 (B) 8
(C) 6 (D) 0
86. For what value of N, 5N82N is divisible by 18?
(A) 2 (B) 4
(C) 6 (D) 8
87. The five-digit number 35N2N is divisible by 2, 3, 4, 6, 8, 9, and 12. What is the value of the digit N?
(A) 2 (B) 4
(C) 6 (D) None of these
88. The number 1X38X is divisible by 12 when $X = P$, and by 9 when $X = Q$. What is the value of $P + Q$?
(A) 2 (B) 3
(C) 6 (D) 9
89. The sum of two numbers is 45. Their difference is $\frac{1}{9}$ of their sum. What is their LCM?
(A) 100 (B) 150
(C) 200 (D) 250
90. If the HCF of two numbers (each greater than 13) be 13 and LCM be 273, then the sum of the numbers will be:
(A) 286 (B) 130
(C) 288 (D) 290
91. The HCF and LCM of two numbers are 12 and 72 respectively. If the sum of the two numbers is 60, then one of the two numbers will be:
(A) 12 (B) 24
(C) 60 (D) 72
92. The ratio between two numbers is 2 : 3. If their LCM is 150, then the numbers are:
(A) 30, 40 (B) 48, 64
(C) 50, 75 (D) None of these
93. The ratio between two numbers is 15 : 11. If their HCF is 13, then the numbers are:
(A) 75, 55 (B) 105, 77
(C) 15, 11 (D) 195, 143
94. Three different containers contain different quantities of mixtures of milk and water, whose measurements are 403 kg, 434 kg and 465 kg. What should be the highest weight which can measure all the different quantities exactly?
(A) 1 kg (B) 7 kg
(C) 31 kg (D) 41 kg
95. Three measuring rods are 64 cm, 80 cm and 96 cm in length. The least length of cloth that can be measured exact number of times, using any rod, is:
(A) 0.96 m (B) 9.60 m
(C) 19.20 m (D) 96 m
96. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 : 20 hours, then at what time will they change again simultaneously?
(A) 8 : 27 : 12 hrs. (B) 8 : 21 : 24 hrs.
(C) 8 : 27 : 36 hrs. (D) 8 : 27 : 48 hrs.
97. Three sets of English, Mathematics and Science Books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject-wise and

the height of each stack is the same. Total number of stacks will be:

- (A) 14 (B) 21
(C) 22 (D) 48

98. What is the greatest number of 3 digits which when divided by 6, 9, 12 leaves a remainder 3 in each case?

- (A) 903 (B) 939
(C) 975 (D) 996

99. How many two-digit numbers when successively divided by 6, 8 and 9 will leave remainders 3, 4 and 2 respectively?

- (A) 2 (B) 1
(C) 3 (D) None of these

100. What is the second largest five-digit number which when divided by 17 and 13 leaves remainders of 5 and 1 respectively?

- (A) 99948 (B) 99952
(C) 99880 (D) None of these

101. What is the greatest five-digit number which when divided by 2, 3, 4, 5 and 6 leaves a remainder of 1 in each case and when divided by 7, leaves no remainder?

- (A) 99939 (B) 99540
(C) 99960 (D) 99841

102. What is the largest number which when divides 44 and 66 leave remainders of 2 and 3 respectively?

- (A) 7 (B) 14
(C) 21 (D) 42

103. What is the largest number which divides 150, 180, 144 leaving the same remainder in each case?

- (A) 12 (B) 3
(C) 6 (D) 8

104. A certain number when successively divided by 15 and 5, leave remainders of 5 and 2 respectively. What is the least possible number in this case?

- (A) 83 (B) 35

- (C) 21 (D) 57

105. A military commander is grouping the soldiers into smaller battalions for operation Vijay. He finds that when he groups them into battalions each of 7, 8, 13 soldiers, he always finds 3 soldiers left. Which of the following could be the number of soldiers?

- (A) 1459 (B) 1452
(C) 1467 (D) 1472

106. Banti, the rice merchant has 149 kg rice of one quality and 177 kg rice of another quality. He packs and sells them in bags of equal quantity, which are of the largest possible size, and finds that 5 kg of first quality and 9 kg of the second quality are still remaining. Had he mixed both the qualities of rice and packed them in bags which are 1.25 times the weight of each of the present bags, then how many kilograms of rice remain?

- (A) 14 (B) 12
(C) 26 (D) 18

107. LCM of two distinct natural numbers is 191. What is their HCF?

- (A) 43 (B) 191
(C) 1 (D) 61

108. HCF and LCM of two numbers are 7 and 140 respectively. If the numbers are between 20 and 45 the sum of the numbers is:

- (A) 70 (B) 77
(C) 63 (D) 56

109. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. Minimum number of rows in which the trees may be planted is:

- (A) 20 (B) 17
(C) 15 (D) 3

110. The smallest positive number x , which leaves a remainder 1 when divided by 2, 3, 4 and 5 is:

- (A) greater than 75 (B) divisible by 7

- (C) a prime number (D) None of these
111. Two numbers are in the ratio 3 : 4. Their LCM is 84. The greater number is:
 (A) 21 (B) 24
 (C) 28 (D) 84
112. A chocolate dealer has to send chocolates of three brands to a shopkeeper. All the brands packed in boxes has to be sent in lots of 96 of brand A, 240 of brand B and 336 of brand C. These boxes are to be packed in cartons of same size containing equal number of boxes. Each carton should contain boxes of same brand of chocolates. What could be the minimum number of cartons that the dealer has to send?
 (A) 20 (B) 14
 (C) 22 (D) 24
113. What is the largest four-digit number which is divisible by 3, 4 and 6?
 (A) 9994 (B) 9996
 (C) 9998 (D) 9992
114. The least perfect square number which is divisible by 3, 4, 5, 6 and 8, is:
 (A) 900 (B) 1200
 (C) 2500 (D) 3600
115. What is the least number that must be added to 1856 to make it divisible by 7, 12 and 16?
 (A) 76 (B) 160
 (C) 167 (D) None of these
116. When Santa distributes some cookies among 40 kids, four cookies are left. If he distributes the same number of cookies to the 40 kids and the father, eight cookies are left. What is the minimum number of cookies the Santa has?
 (A) 1443 (B) 1476
 (C) 1488 (D) 1484
117. The HCF of two numbers is 6 and their sum is 60. How many pairs of such numbers exist?
 (A) 1 (B) 2
 (C) 3 (D) 4
118. What is the highest power of 5 contained in 60!?
 (A) 12 (B) 18
 (C) 14 (D) 25
119. What is the highest power of 21 contained in 342!?
 (A) 54 (B) 196
 (C) 48 (D) 146
120. What is the highest power of 8 contained in 80!?
 (A) 10 (B) 11
 (C) 26 (D) 9
121. Today is Friday. After 62 days it will be?
 (A) Friday (B) Thursday
 (C) Wednesday (D) Friday
122. The year after 1988 having the same calendar as that of 1988 is:
 (A) 1988 (B) 1992
 (C) 1993 (D) 1995
123. Calendar for 2000 will also serve for:
 (A) 2003 (B) 2006
 (C) 2007 (D) 2005
124. If March 1, 2006 was Wednesday, what was the day on March 1, 2002?
 (A) Wednesday (B) Thursday
 (C) Friday (D) Saturday
125. Convert decimal 99 to binary.
 (A) 1100101 (B) 1101001
 (C) 11100011 (D) 1100011
126. Convert 1153 from base 10 to base 15?
 (A) 51D (B) 61E
 (C) 51C (D) 61C
127. Convert 1234 from base 6 to base 10?
 (A) 3100 (B) 3010
 (C) 301 (D) 310
128. Convert 1556 from base 10 to base 16?
 (A) 641 (B) 64A
 (C) 6A4 (D) 614
129. Convert 413 from base 7 to base 10?
 (A) 613 (B) 362

(C) 206

(D) 216

Practice exercise Level - 2

1. Let x , y and z be three distinct odd positive integers. Which of the following statements can't be true?

(A) $x^3 y^3 z^2$ is odd (B) $(x^y + x)^x$ is even
(C) $(x + y)x^y$ is odd (D) $(x^2 + x)^y$ is even

2. If $\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}} = 2$, then what is the value of x ?

(A) $-\frac{1}{3}$ (B) -1
(C) $\frac{1}{3}$ (D) 1

3. What is the value of

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6}?$$

(A) $\frac{1}{30}$ (B) $\frac{7}{30}$
(C) $\frac{11}{30}$ (D) $\frac{13}{30}$

4. What is the value of

$$\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{99}\right)\left(1 - \frac{1}{100}\right)?$$

(A) $\frac{1}{25}$ (B) $\frac{1}{50}$
(C) $\frac{1}{100}$ (D) $\frac{2}{99}$

5. What is the value of

$$\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right)\left(1 + \frac{1}{x+3}\right)\left(1 + \frac{1}{x+4}\right)?$$

(A) $\frac{x+5}{x+1}$ (B) $\left(x + \frac{1}{x+5}\right)$
(C) $\frac{1}{(x+5)}$ (D) $\frac{x+6}{x+5}$

6. What is the value of

$$\left(\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16}\right)?$$

(A) $\frac{41}{7280}$ (B) $\frac{5}{16}$ (C) $\frac{3}{8}$ (D) $\frac{1}{3}$

7. What is the value of

$$\left\{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{n(n+1)}\right\}?$$

(A) $\frac{2(n-1)}{n}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{n}$

8. J, K, L and M are four integers. The product of the larger three integers is 4199, and that of the smaller three integers is 1547. What is their sum?

(A) 67 (B) 81
(C) 53 (D) 56

9. Out of the three-digit integers greater than 700, how many have two digits that are equal to each other and the remaining digit different from the other two?

(A) 82 (B) 36
(C) 80 (D) 90

10. X, Y, Z and W are integers. The expression $X - Y - Z$ is even and the expression $Y - Z - W$ is odd. If X is even, then what must be true?

(A) W must be odd (B) $Y - Z$ must be odd
(C) W must be even (D) Z must be odd

11. What is the value of $1^2 + 3^2 + 5^2 + \dots + 19^2$?

(A) 1330 (B) 2100
(C) 2485 (D) 2470

12. What is the value of $(2^2 + 4^2 + 6^2 + \dots + 20^2)$?

(A) 770 (B) 1155
(C) 1540 (D) 420

13. How many numbers from 287 to 803 will contain 2 as one of its digits?

(A) 108 (B) 107
(C) 109 (D) 112

14. If x and y are any odd numbers and $x > y$, then which of the following is an odd number?

- (A) $(x^y + y^x)(x - y)(x^y + x)$
 (B) $(x^y + y^x)(x + y)(x^y + x)$
 (C) $y^x(x^2 - y)(x^y - x)$
 (D) None of these

Directions (15-16) Read the passage below and solve the questions based on it.

There are 100 prisoners in 100 cells. Cells are numbered from 1 to 100 and every cell is occupied by one prisoner only. One day jailer decides to release some of the prisoners and for this he defines an algorithm of 100 steps which is as follows:

Step 1:	Reverse the position of all the cells which are divisible by 1.
Step 2:	Reverse the position of all the cells which are divisible by 2.
Step 3:	Reverse the position of all the cells which are divisible by 3.
Step 99:	Reverse the position of all the cells which are divisible by 99.
Step 100:	Reverse the position of all the cells which are divisible by 100.

Initially all the cells are closed. After executing all these steps, prisoners of all the cells which remain open are released.

15. How many prisoners are released?
 (A) 25 (B) 10
 (C) 90 (D) None of these
16. Which of the following cell number will be open at the end?
 (A) Cell number 56 (B) Cell number 64
 (C) Cell number 72 (D) Cell number 84

17. $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then what is the

value of $\frac{(a^2 + ab + b^2)}{(a^2 - ab + b^2)}$?

- (A) $\frac{3}{4}$ (B) $\frac{4}{3}$
 (C) $\frac{3}{5}$ (D) $\frac{5}{3}$

18. Which among the following is greatest
 $(\sqrt{7}-\sqrt{5})$, $(\sqrt{5}-\sqrt{3})$, $(\sqrt{9}-\sqrt{7})$, $(\sqrt{11}-\sqrt{9})$?

- (A) $(\sqrt{11}-\sqrt{9})$ (B) $(\sqrt{9}-\sqrt{7})$
 (C) $(\sqrt{7}-\sqrt{5})$ (D) $(\sqrt{5}-\sqrt{3})$

19. If $x = 7 - 4\sqrt{3}$, then what is the value of
 $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$?

- (A) 1 (B) 2
 (C) 3 (D) 4

20. If $x = \frac{2\sqrt{15}}{\sqrt{3}+\sqrt{5}}$, then what is the value of
 $\frac{x+\sqrt{5}}{x-\sqrt{5}} + \frac{x+\sqrt{3}}{x-\sqrt{3}}$?

- (A) $\sqrt{5}$ (B) $\sqrt{3}$
 (C) $\sqrt{15}$ (D) 2

21. What is the value of $\sqrt{37+20\sqrt{3}} - \sqrt{61+28\sqrt{3}}$?

- (A) -1 (B) 1
 (C) 2 (D) -2

22. $5^{x-1} + 5^x + 5^{x+1} = 775$, then the value of x, where x is a positive integer:

- (A) 1 (B) 3
 (C) 2 (D) 4

23. Which one among the following is greatest
 $\sqrt{7} + \sqrt{3}$, $\sqrt{6} + 2$, $\sqrt{5} + \sqrt{5}$, $\sqrt{2} + \sqrt{8}$?

- (A) $\sqrt{7} + \sqrt{3}$ (B) $\sqrt{6} + 2$
 (C) $\sqrt{5} + \sqrt{5}$ (D) $\sqrt{2} + \sqrt{8}$

24. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest?

- (A) $2^{1/2}$ (B) $3^{1/3}$

- (C) $4^{1/4}$ (D) $6^{1/6}$
25. What is the simplified value of $(2 + 1)(2^2 + 1)(2^4 + 1)(2^8 + 1)$?
 (A) $2^8 - 1$ (B) $2^{16} - 1$
 (C) $2^{32} - 1$ (D) $2^{64} - 1$
26. If $\log 3 = 0.4771$, then what is the value of $\log (0.81)^2 \times \log \left(\frac{27}{10}\right)^{\frac{2}{3}} \div \log 9$?
 (A) 2.689 (B) -0.0552
 (C) 2.2402 (D) 2.702
27. What is the value of $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$?
 (A) 4 (B) 5
 (C) 3 (D) 2
28. If $\log_y x = 8$ and $\log_{10y} 16x = 4$, then what is the value of y ?
 (A) 1 (B) 2
 (C) 3 (D) 5
29. In a question on division with zero remainder, a candidate took 12 as divisor, instead of 21. The quotient obtained by him was 35. What is the correct quotient?
 (A) 0 (B) 12
 (C) 13 (D) 20
30. When 7179 and 9699 are divided by another natural number N , remainder obtained is same. How many values of N will be ending with one or more than one zeroes?
 (A) 46 (B) 18
 (C) 169 (D) 24
31. What is the remainder when 47^{100} is divided by 100?
 (A) 1 (B) 51
 (C) 61 (D) 81
32. What is the remainder when 53^{1111} is divided by 51?
 (A) 25 (B) 9
- (C) 42 (D) 26
33. What is the remainder when $12^{13^{14}}$ is divided by 145?
 (A) 2 (B) 12
 (C) 133 (D) 132
34. What is the remainder when $(17^3 + 19^3 + 21^3 + 23^3)$ is divided by 80?
 (A) 10 (B) 20
 (C) 40 (D) 0
35. What is the remainder when $74^{13} - 41^{13} + 75^{13} - 42^{13}$ is divided by 66?
 (A) 2 (B) 64
 (C) 1 (D) 0
36. $(2^{51} + 2^{52} + 2^{53} + 2^{54} + 2^{55})$ is divisible by which of the following number?
 (A) 23 (B) 58
 (C) 124 (D) 127
37. If $1! + 2! + 3! + \dots + 50!$ is divided by 5, then what will be the remainder?
 (A) 3 (B) 33
 (C) 13 (D) 93
38. Two numbers, x and y , are such that when divided by 6, they leave remainders 4 and 5 respectively. What is the remainder when $(x^2 + y^2)$ is divided by 6?
 (A) 2 (B) 3
 (C) 4 (D) 5
39. What is the remainder if 2^{256} is divided by 17?
 (A) 1 (B) 16
 (C) 14 (D) None of these
40. What is the remainder if 30^{40} is divided by 17?
 (A) 1 (B) 2
 (C) 4 (D) 7
41. The expression $2^{7777} + 7^{2222}$ is divisible by:
 (A) 177 (B) 1001
 (C) 9999 (D) All of these
42. A young girl counted in the following manner on the fingers of her left hand. She started calling the thumb, 1, the index finger, 2, middle finger, 3, ring finger, 4, little finger, 5, then

reversed the direction, calling the ring finger, 6, middle finger, 7, index finger, 8, thumb 9 and then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her?

- (A) Thumb (B) Index finger
(C) Middle finger (D) Ring finger

43. It is given that $(2^{32} + 1)$ is exactly divisible by a certain number. Which of the following is also definitely divisible by the same number?

- (A) $(2^{16} + 1)$ (B) $(2^8 + 1)$
(C) $(2^{16} - 1)$ (D) $(2^{96} + 1)$

44. $\{(49)^{15} - 1\}$ is exactly divisible by?

- (A) 8 (B) 63
(C) 50 (D) 51

45. $N = 1234567$ and $M = 890$. Then which of the following statements is false?

- (A) $N \times M$ has ten digits.
(B) N^2 has 13 digits
(C) The unit's digit of $(N)^M$ is 9.
(D) $N \div M > (40)^2$

46. How many factors of 3600 are perfect squares?

- (A) 6 (B) 8
(C) 12 (D) 24

47. In how many ways 2500 can be written as a product of two different factors?

- (A) 7 (B) 6
(C) 15 (D) 16

48. Let $N = 55^3 + 17^3 - 72^3$. N is divisible by?

- (A) Both 7 and 13 (B) Both 3 and 13
(C) Both 17 and 7 (D) Both 3 and 17

49. Two 3-digit numbers have their HCF 29 and LCM 4147. What is the sum of the numbers?

- (A) 666 (B) 669
(C) 696 (D) 966

50. HCF of 3240, 3600 and a third number is 36 and their LCM is $(2^4 \times 3^5 \times 5^2 \times 7^2)$. What is the third number?

- (A) $2^5 \times 5^2 \times 7^2$ (B) $2^3 \times 3^5 \times 7^2$
(C) $2^2 \times 3^5 \times 7^2$ (D) $2^2 \times 5^3 \times 7^2$

51. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 seconds respectively. In 30 minutes, how many times do they toll together?

- (A) 4 (B) 10
(C) 15 (D) 16

52. What is the greatest number of 4 digits which is divisible by 12, 18, 21, 28?

- (A) 9848 (B) 9864
(C) 9828 (D) 9836

53. Which greatest number will divide 3026 and 5053 leaving remainders 11 and 13 respectively?

- (A) 15 (B) 30
(C) 45 (D) 60

54. What is the smallest four-digit number which when decreased by the largest two-digit number is exactly divisible by 15, 20 and 25?

- (A) 1299 (B) 1199
(C) 1599 (D) 1099

55. What is the smallest five-digit number which when divided by 4, 5, 6, 7 leaves remainders of 2, 3, 4, 5 respectively?

- (A) 96570 (B) 96600
(C) 10078 (D) 10080

56. During a flood relief programme, food packets were air dropped in lots of equal sizes from 3 different aircrafts containing 250, 490 and 850 packets. In each aircraft same number of damaged packets remained undropped. If the number of lots has to be minimum, and the number of damaged packets is less than the lot's size, then how many lots were dropped in all?

- (A) 3 (B) 7
(C) 11 (D) 10

57. When a number is successively divided by 7, 5, and 4, it leaves remainders 4, 2 and 3 respectively. What are the remainders when

- the smallest such number is successively divided by 8, 5 and 6?
- (A) 5, 0, 3 (B) 2, 2, 4
(C) 3, 0, 3 (D) 2, 4, 2
58. What is the greatest four-digit number which when divided by 5 leaves remainder 1 and when divided by 6 leave remainder 5?
- (A) 9941 (B) 9971
(C) 9911 (D) 8971
59. A gardener had a number of shrubs to plant in horizontal rows. At first, he tried to plant 5 shrubs in each row, then 6, then 8 and then 12, but had always 1 left. On trying 13, in each row he had none left. What is the smallest number of shrubs that he could have had?
- (A) 481 (B) 477
(C) 468 (D) 121
60. What is the power of 45 will exactly divide $123!$?
- (A) 28 (B) 30
(C) 31 (D) 59
61. $N = 2 \times 4 \times 6 \times 8 \times 10 \times \dots \times 100$. How many zeroes are there at the end of N ?
- (A) 24 (B) 13
(C) 12 (D) 15
62. What was the day of the week on 16th July, 1776?
- (A) Monday (B) Tuesday (C) Wednesday (D) None of these
63. Smt. Indira Gandhi died on 31st October, 1984. What was the day of the week?
- (A) Monday (B) Tuesday
(C) Wednesday (D) Friday
64. In an ordinary year, which months begin on the same day of the week.
- (A) February ; November
(B) January ; November
(C) February ; October
(D) January ; September
65. If the day after tomorrow is Sunday, what day was tomorrow's day before yesterday?
- (A) Friday (B) Thursday
(C) Monday (D) Tuesday
66. If the number 3402 is converted from base 10 to base x , it becomes 12630. What is the value of x ?
- (A) 6 (B) 8
(C) 9 (D) 7
67. For single digit numbers a , b and c $(abc)_7 = (cba)_9$. Then what is the value of $a + b + c$?
- (A) 11 (B) 8
(C) 16 (D) Can't say
68. $(152)_k = (86)_{10}$, then what is the value k ?
- (A) 6 (B) 7
(C) 8 (D) 9

Solution

Practice Exercise Level 1

- 1.(D) All are composite.
- 2.(D) His total is correct $(11 + 13 + 17 + 19 + 23 + 29 + 31 + 37) = 180$
- 3.(C) (a) $78 = 37 + 41$
(b) $88 = 41 + 47$
(d) $29 + 31 = 60$

Sum of two prime numbers is always even, except when one of the two prime numbers is 2. In choice (c) the sum of 51, an odd number. $51 = 2 + 49$ where 49 is not prime, so our choice is (c)

- 4.(D) (A) $37 + 41 = 78$

(B) $13 + 17 = 30$

(C) $2 + 3 = 5$

5.(B) $2n + 2n + 2 + 2n + 4$
 $= 6(n + 1)$

So, it is always divisible by 6.

6.(C) $p = \frac{3}{5} = 0.6, q = \frac{7}{9} = 0.777$

and $r = \frac{5}{7} = 0.714$

$0.6 < 0.714 < 0.777$

Hence, $p < r < q$.

7.(A) Let the fraction be x .

$\Rightarrow x + \frac{3}{x} = \frac{19}{4}$

$\Rightarrow 4x^2 - 19x + 12 = 0$

$\Rightarrow x = \frac{3}{4}$

8.(B) Given expression,

$$= \frac{(0.07)^3 - (0.05)^3}{(0.07)^2 + 0.07 \times 0.05 + (0.05)^2}$$

$$= \frac{(a^3 - b^3)}{(a^2 + ab + b^2)}, \text{ where } a = 0.07$$

and $b = 0.05$

So, $a - b = .07 - .05 = .02$

9.(A) Given expression,

$$= \frac{(a^3 + b^3)}{(a^2 + b^2 - ab)}, \text{ where } a = 0.73$$

and $b = 0.27$

$= (a + b) = (0.73 + 0.27) = 1$

10.(C) Given expression,

$$= \frac{(a-b)^2 + (a+b)^2}{(a^2 + b^2)}, \text{ where } a = 27.5$$

and $b = 19.3$

$$= \frac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$$

11.(C) Given expression,

$$= \frac{(a^3 + b^3 + c^3 - 3abc)}{(a^2 + b^2 + c^2 - ab - bc - ca)}$$

where $a = 1.5, b = 4.7, c = 3.8$

$= (a + b + c) = (1.5 + 4.7 + 3.8)$

$= 10$

12.(C) Given expression,

$$= \frac{15}{2} + \frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{4} - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8}$$

of $\left(\frac{7}{5} - \frac{4}{3}\right)$

$$= \frac{15}{2} + \frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{4} - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text{ of } \frac{1}{15}$$

$$= \frac{15}{2} + \frac{1}{2} \div \frac{1}{8} - \frac{2}{5} \times \frac{7}{3} \div \frac{1}{8}$$

$$= \frac{15}{2} + \frac{1}{2} \times \frac{8}{1} - \frac{2}{5} \times \frac{7}{3} \times \frac{8}{1}$$

$$= \frac{15}{2} + 4 - \frac{112}{15} = \frac{23}{2} - \frac{112}{15}$$

$$= \frac{345 - 224}{30} = \frac{121}{30} = 4 \frac{1}{30}$$

13.(C) Given expression,

$$= \frac{15}{2} - \left[\frac{9}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

$$= \frac{15}{2} - \left[\frac{9}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{9-2-1}{6} \right) \right\} \right]$$

$$= \frac{15}{2} - \left[\frac{9}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \right\} \right]$$

$$= \frac{15}{2} - \left[\frac{9}{4} \div \frac{3}{4} \right] = \frac{15}{2} - 3 = \frac{9}{2} = 4 \frac{1}{2}$$

14.(D) $16.5 + 1.65 + 0.165$

$= 18.315$

15.(D) By hit and trial, we have $4 \times 5 \times 6 = 120$

$\therefore$ Required sum $= (4 + 5 + 6) = 15$

16.(C) Let Numerator = x . Then,

Denominator = $(x + 1)$

$$\frac{x-1}{(x+1)-1} = 0.5 = \frac{1}{2}$$

$\Rightarrow 2(x - 1) = x \Rightarrow x = 2$

$\therefore$ Fraction $= \frac{2}{3}$

17.(C) Let ten's digit be x and unit digit be y . Then,

$10x + y = 3(x + y) \Rightarrow 7x - 2y = 0$

$(10x + y) + 45 = 10y + x$

$\Rightarrow -9x + 9y = 45$

$\Rightarrow -x + y = 5$

On solving (1) and (2), we get

$$x = 2 \text{ and } y = 7$$

The number = 27 and sum of its digits

$$= (2 + 7) = 9$$

- 18.(C)** As P, Q and R are consecutive odd numbers
 $Q = P + 2$ and $R = P + 4$. Now, $3P = 2(P + 4) - 3$. On solving this equation, we get $P = 5$.
 Therefore, $R = 5 + 4 = 9$.

- 19.(B)** 27, 57 and 87 are two digit numbers which ends in 7 and are divisible by 3.

- 20.(A)** By hit and trial, we find that smallest number consisting of entirely of 3's entirely and divisible by 7 is 333333.

$$\text{Now, } (333333 \div 7) = 47619$$

- 21.(C)** Number of digits required

$$1 - 9 \rightarrow 9 \text{ digits}$$

$$10 - 99 \rightarrow 2 \times [(99 - 10) + 1]$$

$$= 2 \times 90 = 180$$

$$100 - 185 \rightarrow 3 \times [(185 - 100) + 1]$$

$$= 3 \times 86 = 258$$

$$\text{Total number of digits} = 9 + 180 + 258 = 447 \text{ digits}$$

- 22.(D)** If n is an integer i.e. 3.

$$(1) n^2 = (3)^2 = 9$$

$$(2) 2n + 1 = 2 \times 3 + 1 = 7$$

$$(3) n + 2 = 3 + 2 = 5$$

$$(4) 2n = 2 \times 3 = 6$$

Hence, 2n is always be even.

- 23.(C)** Let a, b and c be 4, 5 and 6 respectively.

$$\text{I. } c - a = 2$$

$$6 - 4 = 2$$

$$\text{II. } abc = 4 \times 5 \times 6 = 120 = \text{even integer}$$

$$\text{III. } \frac{a+b+c}{3} = \frac{4+5+6}{3} = \frac{15}{3} = 5$$

which is an integer

Hence, I, II and III all follows.

- 24.(D)** Let p and q be 4 and 5 respectively

$$\text{I. } \frac{3p}{q} = \frac{3 \times 4}{5} = \text{Not an integer}$$

$$\text{II. } pq = 4 \times 5 = 20 = \text{even integer.}$$

$$\text{III. } 2(p + q) = 2(4 + 5) = 2 \times 9 = 18 = \text{even integer}$$

$$\text{IV. } 2p + q = 2 \times 4 + 5 = 8 + 5 = 13 = \text{odd integer}$$

- 25.(A)** Largest four digit number using the digits = 8210

$$\text{Smallest four digit number using the digits} = 1028$$

$$\text{Difference between them} = 7182$$

- 26.(B)** From 1 to 100, 4 comes 20 times. So, from 11 to 100, 4 will come 19 times.

- 27.(B)** First 100 whole numbers are 0 to 99. '0' will come 10 times (A) and '9' will come 20 times (B)

$$A + B = 10 + 20 = 30$$

- 28.(C)** A perfect square cannot end in 2, 3, 7 or 8. Therefore, before subtracting 1, we should not get a number that ends in 3, 4, 8 or 9.

$$\text{29.(B)} \quad 0.235 = \frac{235 - 23}{900}$$

$$= \frac{212}{900} = \frac{106}{450} = \frac{53}{225}$$

$$\text{30.(D)} \quad 0.2345 = \frac{2345 - 23}{9900}$$

$$= \frac{2322}{9900} = \frac{1161}{4950}$$

- 31.(A)** If n^3 is odd, so 'n' must be odd. Also, n^2 will be odd. So, (I) and (II) must be true.

- 32.(B)** $4 + 32 + 108 + \dots + 4000 = 4(1 + 8 + 27 + \dots + 1000)$

$$= 4(1^3 + 2^3 + 3^3 + \dots + 10^3)$$

$$= 4 \times \left(\frac{10 \times 11}{2} \right)^2$$

$$= 4 \times 55^2 = 12100$$

- 33.(B)** Required numbers are 509, 519, 529, 539, 549, 559, 569, 579, 589, 590, 591, 592, ..., 598. So, there are 18 numbers in all.

Note: In any set of 100 natural numbers digit from 1 to 9 will appear 20 times and will appear in 19 numbers at unit's and ten's place.

- 34.(C)** In every 100 numbers a digit appears 20 times at units and tens place. So, in 200 number the digit 6 appears $20 \times 2 = 40$ times at units and tens place and 100 times in the hundred place.

$$\text{Total} = 40 + 100 = 140 \text{ times}$$

- 35.(B)** Let the digits be a and b such that the number is $10a + b$.

$$\therefore ab = 14 \text{ and } 10a + b + 45 = 10b + a$$

$$\text{i.e., } 9a - 9b = -45$$

$$\text{i.e., } a - b = -5$$

$$(a + b)^2 = (a - b)^2 + 4ab = 25 + 4(14) = 81$$

$$\therefore a + b = 9$$

$$\therefore a = 2, b = 7$$

$$\therefore \text{The number is } 27.$$

Use options & check.

- 36.(C)** Let digit at ten's place be x.

$$\text{Digit at unit's place} = x + 1$$

$$5 \text{ times sum of digit} = 5(x + x + 1)$$

$$= 10x + 5$$

The number has to be greater than $(10x + 5)$.

Also x can take values from 1 to 8 only.

$$x = 1, \text{ no.} = 12$$

$$x = 2, \text{ no.} = 23$$

$$x = 3, \text{ no.} = 34$$

$$x = 4, \text{ no.} = 45$$

For all these values of 'x', number is not more than 5 times the sum of digits.

$$x = 5, \text{ no.} = 56$$

$$x = 6, \text{ no.} = 67$$

$$x = 7, \text{ no.} = 78$$

$$x = 8, \text{ no.} = 89$$

For all these numbers the numbers are greater than 5 times the sum of digits.

$$\text{So, sum of numbers} = 56 + 67 + 78 + 89 = 290$$

- 37.(B)** $x = \frac{-1}{2}$

$$\text{Option (a): } 2^x = 2^{\frac{-1}{2}} = 2^{-2} = \frac{1}{4}$$

$$\text{Option (b): } \frac{1}{x} = \frac{1}{\frac{-1}{2}} = -2$$

$$\text{Option (c): } \frac{1}{x^2} = \frac{1}{\left(\frac{-1}{2}\right)^2} = 4$$

$$\text{Option (d): } 2^x = 2^{\frac{-1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

$$\text{So, smallest is } \frac{1}{x}.$$

By looking at the options, we can find that the value given in option (b) will be negative and remaining options will give us a positive value.

So, the smallest value will be $\frac{1}{x}$.

- 38.(D)** The sum of first ten natural numbers is 55.

Now, the sum of next set of ten natural numbers is 65 (from 2 to 11)

Now, the sum of next set of ten natural numbers is 75 (from 3 to 12)

So, possible values are 55, 65, 75, 85,

All values with unit digit 5 are possible starting from 55.

- 39.(D)** Given surds are $2^{1/2}$, $4^{1/3}$ and $6^{1/4}$.

LCM of 2, 3 and 4 is 12.

$$2^{1/2} = (2^6)^{1/12} = (64)^{1/12}, 4^{1/3} = (4^4)^{1/12}$$

$$256^{1/12} \text{ and } 6^{1/4} = (6^3)^{1/12} = (216)^{1/12}$$

- 40.(B)** $\sqrt[4]{80} + \sqrt[4]{3125} - \sqrt[4]{6480}$

$$= \sqrt[4]{2^4 \times 5} + \sqrt[4]{5^4 \times 5} - \sqrt[4]{2^4 \times 3^4 \times 5^1}$$

$$= 2\sqrt[4]{5} + 5\sqrt[4]{5} - 6\sqrt[4]{5} = \sqrt[4]{5}$$

- 41.(B)** $31 + 4\sqrt{57}$

$$= (\sqrt{19})^2 + (2\sqrt{3})^2 + 2 \times \sqrt{19} \times 2\sqrt{3}$$

$$= (\sqrt{19} + 2\sqrt{3})^2$$

$$\text{Square root} = \sqrt{19} + 2\sqrt{3}$$

We can use options to get the answer.

Square the values and check which comes equal to $31+4\sqrt{57}$.

$$\begin{aligned}
 42.(B) \quad N + \frac{1}{N} &= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}} + \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} \\
 &= \frac{(\sqrt{7}-\sqrt{3})^2 + (\sqrt{7}+\sqrt{3})^2}{(\sqrt{7})^2 - (\sqrt{3})^2} \\
 &= \frac{7+3-2\sqrt{21}+7+3+2\sqrt{21}}{4} \\
 &= \frac{20}{4} = 5
 \end{aligned}$$

$$\begin{aligned}
 43.(C) \quad \frac{\sqrt{2+x} + \sqrt{2-x}}{\sqrt{2+x} - \sqrt{2-x}} &= 2 \\
 \sqrt{2+x} + \sqrt{2-x} &= 2 \left[(\sqrt{2+x}) - \sqrt{2-x} \right] \\
 \sqrt{2+x} + \sqrt{2-x} &= 2\sqrt{2+x} - 2\sqrt{2-x} \\
 3\sqrt{2-x} &= \sqrt{2+x} \\
 \text{Squaring both the sides,} \\
 18 - 9x &= 2 + x \\
 16 &= 10x \\
 x &= \frac{16}{10} = \frac{8}{5}
 \end{aligned}$$

$$\begin{aligned}
 44.(C) \quad \frac{3^n + 3^{n-1}}{3^{n+1} - 3^n} &= \frac{3^n + \frac{3^n}{3}}{3^n \times 3^1 - 3^n} \\
 &= \frac{3^n \left(1 + \frac{1}{3} \right)}{3^n (3-1)} \\
 &= \frac{4}{2} = \frac{4}{2} = \frac{2}{1}
 \end{aligned}$$

Put values of n as 0, 1, -1, to get the answer.

For $n = 1$;

$$\frac{3^n + 3^{n-1}}{3^{n+1} - 3^n} = \frac{3^1 + 3^0}{3^2 - 3^1} = \frac{4}{6} = \frac{2}{3}$$

$$\begin{aligned}
 45.(A) \quad \text{The given expression is:} \\
 \log_a (4 \times 16 \times 64 \times 256) &= 10 \\
 \text{i.e. } \log_a 4^{10} &= 10 \Rightarrow 10 \log_a 4 = 10 \\
 \Rightarrow \log_a 4 &= 1 \\
 \text{Thus, } a &= 4
 \end{aligned}$$

$$46.(C) \quad \log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25$$

$$\begin{aligned}
 &= \left(\frac{\log 10}{\log 5} \right) \times \left(\frac{\log 15}{\log 10} \right) \times \left(\frac{\log 20}{\log 15} \right) \\
 &\times \left(\frac{\log 25}{\log 20} \right) \\
 &= \frac{\log 25}{\log 5} = \frac{2 \log 5}{\log 5} = 2
 \end{aligned}$$

$$47.(D) \quad 0.01^x = 2$$

Take logarithm on both sides,

$$x \log \frac{1}{100} = \log 2$$

$$x \log 10^{-2} = \log 2$$

$$-2x \log 10 = \log 2$$

$$-2x = \log 2 \quad (\log 10 = 1)$$

$$x = \frac{-\log 2}{2}$$

$$\begin{aligned}
 48.(D) \quad \log_{10} 10 + \log_{10} 10^2 + \dots + \log_{10} 10^n \\
 = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}
 \end{aligned}$$

$$49.(C) \quad 6729 = 6720 + 9 = 35 \times 192 + 9$$

Hence the remainder is 9.

$$\begin{aligned}
 50.(C) \quad \text{Let the given number be} \\
 (357K + 39).
 \end{aligned}$$

$$(357K + 39) = (17 \times 21K) + (17 \times 2) + 5 = 17 \times (21K + 2) + 5$$

$$\therefore \text{Remainder} = 5$$

$$51.(C) \quad \text{Perfect squares that are not divisible by 6 are 9, 16, 25 etc.}$$

When they are divided by 6, the remainders are 3, 4 and 1.

$$\begin{aligned}
 52.(A) \quad 2^{93} &= [(2)^3]^{31} = (8)^{31} \\
 &= (7+1)^{31}
 \end{aligned}$$

If we expand it, each term except last one (1^{31}) will have 7 as a factor, so the required remainder is 1.

$$53.(C) \quad \text{Remainder} = \frac{4^{1000}}{7}$$

$$\begin{aligned}
 &= \frac{[4^3]^{333} \times 4}{7} \\
 &= \frac{(64)^{333} \times 4}{7}
 \end{aligned}$$

$$= \frac{(63+1)^{333} \times 4}{7}$$

$$= \frac{(1)^{333} \times 4}{7} = \text{Remainder is 4.}$$

$$54.(D) \quad \frac{3^{40}}{11} = \frac{(3^5)^8}{11} = \frac{(243)^8}{11}$$

$$= \frac{(242+1)^8}{11} = \text{Remainder is 1}$$

55.(D) Remainder with 10 is finding the unit's digit. The digits at unit's place in powers of 2 repeats itself after power 4.

On dividing 23 by 4, we get 3 as remainder. Therefore, unit's digit will be the unit digit of $2^3 = 8$

Hence, remainder on division by 10 = 8.

$$\frac{2^{23}}{10} = \frac{2 \times 2^{22}}{2 \times 5} = \frac{(2^2)^{11}}{5} = \frac{4^{11}}{5}$$

$$\text{This is of the form } \frac{(A-1)^n}{A}.$$

$\therefore$ Remainder = -1 as n is odd

$\therefore$ Remainder $5 - 1 = 4$

$\therefore$ Actual remainder = $4 \times 2 = 8$

56.(B) $7^3 = 343$, when divided by 342, leaves a remainder of 1.

$$7^{84} = (7^3)^{28}$$

$$= \frac{(343)^{28}}{342} = \frac{(342+1)^{28}}{342}$$

$$\Rightarrow (1)^{28} \Rightarrow 1$$

$\therefore$ Remainder = 1

$7^4 = 2104$, when divided by 342, leaves a remainder of 7.

$7^5 = 16807$, when divided by 342, leaves a remainder of 49.

$7^6 = 117649$, when divided by 342, leaves a remainder of 1 and so on.

$\therefore 7^{84}$, when divided by 342, will leave a remainder of 1.

$$57.(B) \quad \frac{5^{48}}{124} = \frac{(5^3)^{16}}{124} = \frac{125^{16}}{124}$$

$$= \frac{(124+1)^{16}}{124} = 1^{16} = 1$$

$$58.(D) \quad \frac{3^{65}}{28} = \frac{3^2(3^3)^{21}}{28} = \frac{9(27)^{21}}{28}$$

$$= \frac{9 \times (28-1)^{21}}{28} = \frac{9 \times (-1)^{21}}{28} = -9$$

Now, Remainder = $-9 + 28 = 19$

59.(D) We see that $4^2 + 6^2 = 52$ when divided by 25, remainder is 2.

$4^3 + 6^3 = 280$, when divided by 25, leaves remainder 5.

$4^4 + 6^4 = 1552$, when divided by 25, leaves remainder 2.

When power is odd, the remainder is 5.

When power is even, the remainder is 2.

Hence, remainder = 5

60.(B) Let n be that divisor. So, first number will be $(na + 9)$ and second number will be $(nb + 10)$.

Sum of numbers = $n(a + b) + 19$.

Remainder of $\frac{n(a+b)+19}{n}$ is 6.

i.e. $19/n$ leaves remainder 6.

i.e. $n = 13$

61.(B) The number of sticks are 818 as 818

$$= 4 \times x + 2$$

$$818 = 5 \times y + 3$$

Now, when she bundles them in groups of 12, she is left with 2 sticks.

62.(C) Dividend = Divisor $\times$ Quotient + Remainder

$$\text{Remainder} = 4$$

$$\text{Quotient} = x$$

$$\text{Divisor} = 8x$$

$$\text{Now, quotient} = 6(4) = 24$$

$$\text{and divisor} = 8(24) = 192$$

$$\text{Number} = 192 \times 24 + 4$$

$$= 4612$$

63.(D) Remainder = 6

$$\text{Divisor} = 3(6) + 2 = 20$$

$$\text{Quotient} = \frac{20}{5} = 4$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$= 20 \times 4 + 6 = 86$$

- 64.(A)** $9^{\text{even}} \rightarrow$ unit's place is 1
- 65.(A)** Unit digit of $21^3 \times 2^{12} \times 34^7 \times 46^8$ is $1 \times 6 \times 4 \times 6$ i.e. 4.
- 66.(D)** Unit digit of $2^{22} \times 3^{33} \times 4^{44} \times 5^{55}$ is 0 ($4 \times 3 \times 6 \times 5$) = 360.
- 67.(D)** Unit's digit of $(214)^{870} + (149)^{127}$
 Since power of (214) is even it gives unit's digit "6", while (149) with odd power gives unit's digit 9.
 So, unit digit will be $= 6 + 9 = 5$
- 68.(D)** All the indices are equal to 101.
 $N = 1^{101} + 2^{101} + \dots + (9999)^{101}$
 In general, a^k and a^{k+4} have the same units digits.
 Dividing the common index by 4, we get the remainder 1.
 $\therefore$ Last digit of N is same as last digits of $M = 1 + 2 + 3 + 4 + \dots + 9999$
 Taking the sum of first 10 numbers
 $= \frac{10 \times 11}{2} = 55$
 Last digit of first set of 10 numbers is 5.
 There are $\frac{9999}{10} = 999$ such sets of 10 numbers.
 Then, sum of all these numbers end in 5 and the sum of the remaining numbers also end in 5.
 $\therefore$ The sum M ends in 0.
 For odd values of n, $a^n + b^n$ is divisible by $a + b$.
 $\therefore 1^{101} + 9999^{101}$ is divisible 10000
 Similarly, $2^{101} + 9998^{101}$ divisible by 10000 and so on.
 $4999^{101} + 5001^{101}$ is divisible by 10000.
 $\therefore 1^{101} + 9999^{101}$ is divisible 10000
 $\therefore$ N is divisible by 10000.
 $\therefore$ Not just the unit's digits, but the last 4 digits (atleast) are zero.
- 69.(D)** Since on dividing 245 by 4, we get 1 as remainder.
- Therefore, unit's digit in the product of $(4387)^{245}$ and $(621)^{72}$ = unit's digit in the product of $(7)^{245} \times (1)^{72} = 7^1 \times 1^0 = 7$
- 70.(B)** Unit's digit of $(24)^{372}$ is 6 and unit's digit of $(24)^{373}$ is 4.
 So, the unit's digit of sum is 0 i.e. $(6 + 4 = 10)$.
- 71.(C)** $(5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x)$ must be divisible by 3.
 $\therefore x = 2$
- 72.(A)** $(9 + 2 + 5 + 6) - (7 + 1 + x)$ must be equal to 0 or a multiple of 11.
 $\therefore x = 3$
- 73.(C)** As we know that the last three digits must be divisible by 8 and 632 is divisible by 8.
 $\therefore *$ is to be replaced by 3.
- 74.(C)** $72 = (9 \times 8)$. So, the given number 42573 * must be divisible by both 9 and 8.
 For divisibility by 8, the number 73* must be divisible by 8.
 $\therefore *$ must be replaced by 6.
 Also $(4 + 2 + 5 + 7 + 3 + 6) = 27$, which is divisible by 9.
 $\therefore 425736$ is divisible by (8×9) , i.e. 72.
- 75.(D)** The given number has to be divisible by 16 and 5.
 As the number must be divisible by 16 and 5. So, $y = 0$.
 Now, $53x0$ must be divisible by 16. So, $x = 6$
 (As $16 = 2^4$ $\therefore$ last four digits has to be divisible by 16)
- 76.(B)** Each such number will be divisible by LCM of (4, 5, 6), i.e. 60 i.e. multiples of 60 between 200 and 600 are 240, 300, 360, 420, 480, 540. Total such numbers are 6.
- 77.(D)** Number which are divisible by 3 = $\frac{100}{3} = 33$
 Number which are divisible by 4
 $= \frac{100}{4} = 25$
 Number which are divisible by 12

$$= \frac{100}{12} = 8$$

Number which are divisible by 3 and 4 but not 12

$$= (33 - 8) + (25 - 8) = 42$$

$$\text{Remaining numbers} = 100 - 42$$

$$= 58$$

78.(D) 27 + 'X' should be divisible by 9, so 'X' can be 0 or 9.

79.(A) $(1 + 5 + 6 + 2) - (6 + X + 5)$

$$= 3 - X \text{ must be 0 or divisible by 11}$$

So, X must be 3.

80.(A) 4K56 must be divisible by both 11 and 3.

$$\text{Sum of digits is} = 15 + K$$

So, 'K' can be '0', '3', '6' or '9'.

Now it is divisible by 11.

$$(6 + K) - (5 + 4)$$

$$K - 3 = 0$$

$$\text{So, } K = 3$$

Use options to get the answer.

81.(A) Divisible by 88 means it should be divisible by both 11 and 8.

$$58N \text{ should be divisible by 8.}$$

$$\text{So, } N = 4$$

$$\text{Now, } M84584 \text{ is divisible by 11.}$$

$$(4 + 5 + 8) - (8 + 4 + M)$$

$$= 5 - M$$

$$\text{So, } M \text{ is } 5.$$

82.(C) $1200 = 2^4 \times 3^1 \times 5^2$

$$(4 + 1) \times (1 + 1) \times 5(2 + 1) = 5 \times 2 \times 3 = 30$$

83.(C) A number is divisible by 8 if the number formed by the last three digits is divisible by 8, i.e., 58N is divisible by 8.

$$\text{Hence, } N = 4$$

Again, a number is divisible by 11 if the difference between the sum of digits at even place and sum of digits at the odd places is either 0 or divisible by 11.

$$\text{i.e., } (M + 9 + 4 + 4 + 8) - (3 + 0 + 8 + 5 + N)$$

$$= M + 25 - (16 + N) = M - N + 9$$

$$(M - N) + 9 \text{ must be zero or divisible by 11}$$

$$\text{i.e., } M - N = 2$$

$$\Rightarrow M = 2 + 4 = 6$$

$$\text{So, } M = 6, N = 4$$

84.(B) In order for 2A5A756 to be divisible by 3, the sum of the digits must be divisible by 3. The sum of the digits is $2A + 25$ and that needs to be a multiple of 3. The smallest possible value of $2A + 25$ is $2(0) + 25 = 25$ and the largest possible value is $2(9) + 25 = 43$. The multiples of 3 between 25 and 43 are 27, 30, 33, 36, 39, and 42. We can solve the equation $2A + 25 = 27$ or 30 or 33 or 36 or 39 or 42 to find A. Subtracting 25 gives us $2A = 2, 5, 8, 11, 14$, or 17. We then divide by 2, but ignore any decimals since A is a digit. The possible values of A then are: 1, 4 or 7 and they have a sum of 12.

85.(B) In order to be divisible by 12, a number should be divisible by both 3 and 4. The sum of digits of 56788A is $(34 + A)$, so A can be 2, 5 or 8 in order to make it divisible by 3. However, only a value of 8 will make it divisible by 4.

86.(C) For a five-digit number 5N82N to be divisible by 18, it must be divisible by both 2 and 9. Now, N has to be even. Also, the sum of the digits must be a multiple of 9. Sum of the digits is $5 + N + 8 + 2 + N = 15 + 2N$. For $N = 0$, we get $15 + 2 \times 0 = 15$, which is not a multiple of 9. For $N = 2$, we get $15 + 2 \times 2 = 19$. For $N = 4$, we get $15 + 2 \times 4 = 23$. For $N = 6$, we get $15 + 2 \times 6 = 27$, which is a multiple of 9. So, $N = 6$.

Use options and check.

87.(B) Clearly, N must be 0, 2, 4, 6 and 8. For 35N2N to be divisible by 3, the sum of the digits must be a multiple of 3. So, far we have $3 + 5 + N + 2 + N = 10 + 2N$. Let's use the possible values of N to evaluate this

expression. If $N = 0$, then $10 + 2N = 10 + 0 = 10$, which is not divisible by 3. If $N = 2$, then $10 + 2N = 10 + 4 = 14$, which is not divisible by 3. If $N = 4$, then $10 + 2N = 10 + 8 = 18$, which is divisible by 3. Neither $N = 6$ nor $N = 8$ gives a multiple of 3 so 4 is our only candidate. Let's make sure that the number 35424 is divisible by all the other numbers listed. It is divisible by 4 since the last two digits 24 are divisible by 4. It is divisible by 6 since it is divisible by 2 and 3. It is divisible by 8 since the last three digits 424 are divisible by 8. It is divisible by 9 since the sum of the digits (18) is divisible by 9. Finally, it is divisible by 12 since it passes the test for 3 and 4. So, N is 4.

- 88.(B)** 1P38P has to be divisible by 12 so both by 3 and 4. Divisibility by 3 means that the sum of digits ($2P + 12$) is divisible, which means P can be one of the digits [0, 3, 6, 9]. For divisibility by 4, P can be one of the digits [0, 4, 8]. The only digit that fits both criteria is $P = 0$.

1Q38Q is divisible by 9, so its sum of digits has to be divisible by 9. That means ($2Q + 12$) is a multiple of 9, and the only possible value for the digit Q is 3. Therefore, $P + Q = 0 + 3 = 3$.

- 89.(A)** Let the numbers be x and y . Then,

$$\left\{ \begin{array}{l} x + y = 45, x - y = \left(\frac{1}{9} \times 45 \right) = 5 \end{array} \right\}$$

$$\Rightarrow x = 25, y = 20$$

$$\text{LCM of 25 and 20} = (5 \times 5 \times 4)$$

$$= 100$$

- 90.(B)** Let the numbers be $13a$ and $13b$, where a and b are co-primes.

$$\text{Then, } 13a \times 13b = (13 \times 273)$$

$$\Rightarrow ab = 21$$

Two co-primes with product 21 are 3 and 7.

$\therefore$ Numbers are $(13 \times 3, 13 \times 7)$, i.e., 39 and 91.

$$\text{Their sum} = (39 + 91) = 130$$

- 91.(B)** Let the numbers be $12a$ and $12b$, where a and b are co-primes.

$$\text{Then, } 12a \times 12b = 12 \times 72$$

$$\Rightarrow ab = 6$$

Co-primes with product 6 are 2 and 3.

$$a = 2, b = 3$$

$\therefore$ Numbers are $(12 \times 2, 12 \times 3)$, i.e., 24 and 36.

- 92.(C)** Let the numbers be $2x$ and $3x$. Then, the LCM is $6x$, and $6x = 150$

$$\Rightarrow x = 25$$

So, the numbers are 50 and 75.

- 93.(D)** Let the numbers be $15x$ and $11x$. Then, their HCF is x .

$\therefore x = 13$ and so the numbers are $(15 \times 13, 11 \times 13)$ i.e., 195, 143.

- 94.(C)** HCF of 403 kg and 434 kg is 31 kg.

Required measure = HCF of 403 kg, 434 kg and 465 kg.

$$= \text{HCF of } 31 \text{ kg and } 465 \text{ kg} = 31 \text{ kg}$$

- 95.(B)**

16	64, 80, 96
2	4, 5, 6
	2, 5, 3

$$\text{LCM} = (16 \times 2 \times 2 \times 5 \times 3)$$

$$960 \text{ cm}$$

$$\text{Required length} = 9.6 \text{ m}$$

- 96.(A)** Interval of change = LCM of 48 sec., 72 sec., 108 sec.

$$= 432 \text{ Seconds} = 7 \text{ min. } 12 \text{ sec.}$$

Thus, the light will change simultaneously after every 7 min. 12 sec.

Time of next change

$$= 8 : 27 : 12 \text{ hrs}$$

- 97.(A)** $336 = 2^4 \times 3 \times 7$

$$240 = 2^4 \times 3 \times 5$$

$$96 = 2^5 \times 3$$

$$\text{HCF} = (2^4 \times 3) = 48$$

So, we make stacks of 48 books each.

Number of stacks

$$= \left(\frac{336}{48} + \frac{240}{48} + \frac{96}{48} \right)$$

$$= (7 + 5 + 2) = 14$$

98.(C) LCM of 6, 9 and 12

$$= (3 \times 2 \times 3 \times 2) = 36$$

Greatest number of 3 digits

$$= 999$$

On dividing 999 by 36, Remainder = 27

$$\text{Required number} = (999 - 27 + 3) = 975$$

99.(D) $N = 6 \{8(9a + 2) + 4\} + 3$

$$= 6(72a + 20) + 3$$

$$= 432a + 123$$

Smallest number is 123 which is a three-digit number.

100.(C) Use options.

101.(D) Numbers which when divided by 2, 3, 4, 5, and 6 leaving a remainder of 1 in each case are of the form = LCM (2, 3, 4, 5, 6) $K + 1 = 120K + 1$

Now, by using options,

$$\text{Greatest such number possible} = 99841$$

102.(C) The number should exactly divide $(44 - 2)$ and $(66 - 3)$ i.e. 42 and 63. HCF of (42, 63) is 21.

103.(C) The number should be a factor of $(180 - 150)$, $(180 - 144)$ and $(150 - 144)$ i.e. 30, 36, and 6.

By hit & trail we get 6.

104.(B) General form of that number is $= 15 \times (5x + 2) + 5$

$$= 75x + 35$$

$$\text{Smallest such number} = 35$$

105.(A) LCM of 7, 8 and 13 is 728.

Number of soldiers all in the form of $728k + 3$.

Using the options, we get the value of the number of soldiers.

106.(C) Quantity of rice Banti had 149 kg and 177 kg.

Rice packed = $(149 - 5)$ and

$$(177 - 9) = 144 \text{ and } 168$$

HCF of (144 and 168) weight of each bag i.e. 24 kg.

On mixing both the quantities, bag weighs $149 + 177 = 326$ kg

$$\text{Weight of new bag} = 1.25 \times 24$$

$$= 30 \text{ kg}$$

$$\text{Remaining quantity of rice} = 326 - 30 \times 10 = 326 - 300 = 26 \text{ kg}$$

107.(C) 191 is a prime number whose factors are 1 and 191. Therefore, two distinct numbers are 191 and 1. So, there HCF is 1.

108.(C) Since the HCF is 7 therefore, possible numbers could be 21, 28, 35 and 42. LCM of the numbers does not have the factor of 3. But 21 and 42 are the numbers which have 3 as a factor. So, 21 and 42 will not be the numbers. If 21 and 42 are not the numbers, then the numbers are 28 and 35.

$$\text{Sum of the numbers} = 28 + 35 = 63$$

109.(B) HCF of 21, 42, 56 = 7

$$\text{So, number of rows} = \frac{119}{7} = 17$$

110.(C) LCM of 2, 3, 4 and 5 = 60

So, Smallest number will be $60 + 1 = 61$ and 61 is a prime number.

111.(C) Let the numbers be $3x$ and $4x$.

Their HCF is x and LCM is 84.

$$\text{So, } x = \frac{84}{12} = 7$$

$$\therefore \text{Greater number} = 4 \times 7 = 28$$

112.(B) For minimum number of cartons, there should be maximum number of chocolates in a carton that is HCF of 96, 240 and 336, which is 48.

$\therefore$ Minimum number of cartons

$$= \frac{96 + 240 + 336}{48} = \frac{672}{48} = 14$$

113.(B) By using options, we find that 9996 is divisible by 3, 4, and 6.

114.(D) LCM of (3, 4, 5, 6 and 8) = 120

But it is not a perfect square

$$120 = 2^3 \times 3^1 \times 5^1$$

So, it should be multiplied by $(2 \times 3 \times 5)$ to become a perfect square

So, required value = $120 \times 30 = 3600$

Use options and check which value is divisible by the given numbers and is a perfect square.

115.(B) LCM of (7, 12 and 16) is 336.

Now, 1856 when divided by 336, leaves remainder 176.

So, $336 - 176$ i.e. 160 must be added to 1856 to make it divisible by 336.

116.(D) If the cookies are distributed amongst 40 kids then 4 are left. So, the number of cookies is $40K_1 + 4$, where K_1 is a natural number. The same number of cookies are distributed amongst 41 people, then 8 are left, so the number of cookies is $41K_2 + 8$, when K_2 is a natural number.

$$\text{Now, } 40K_1 + 4 = 41K_2 + 8$$

$$K_1 = \frac{41K_2 + 4}{40}$$

The minimum value of K_2 for K_1 to be a natural number is 36.

Hence, the minimum number of cookies = $41 \times 36 + 8 = 1484$

117.(B) Let the numbers be $6x$ and $6y$ where x and y are co-prime to each other.

$$6x + 6y = 60$$

$$x + y = 10$$

Now, we have to find the pairs of the values of x and y which are co-prime to each other. Possible pairs are (1, 9) and (3, 7).

118.(C) Highest power of 5 is 60 ! is

$$\left[\frac{60}{5} \right] + \left[\frac{60}{5^2} \right] = 12 + 2 = 14$$

119.(A) Highest power of 21 in $342!$ is equal to the highest power of 7 in $342!$ which is

$$\left[\frac{342}{7} \right] + \left[\frac{342}{7^2} \right] = 48 + 6 = 54$$

120.(C) Highest power of 2 is 80 ! is

$$\left[\frac{80}{2} \right] + \left[\frac{80}{4} \right] + \left[\frac{80}{8} \right] + \left[\frac{80}{16} \right] + \left[\frac{80}{32} \right] + \left[\frac{80}{64} \right]$$

$$= 40 + 20 + 10 + 5 + 2 + 1$$

$$= 78$$

So, highest power of 8 is $\frac{78}{3} = 26$

121.(B) Each day of the week is repeated after 7 days.

After 63 days, it would be Friday.

So, after 62 days, it would be Thursday.

122.(C) Starting with 1988, we go on counting the number of odd days till the sum is divisible by 7.

Years	Odd days
1988	2
1989	1
1990	1
1991	1
1992	2

= 7 i.e. 0 odd day

∴ Calendar of 1993 is the same as that of 1988.

123.(D) Starting with 2000, count for number of odd days in successive years till the sum is divisible by 7.

2000	2
2001	+1
2002	+1
2003	+1
2004	+2
Total	7

So, Number of odd days up to 2004 = 0

So, Calendar of 2000 will serve for 2005 also.

- 124.(C)** Since in between 2002 to 2006, there is one leap year.

So, Total odd days

$$= (2006-2002) + 1 = 4 + 1$$

$$= 5 \text{ odd days}$$

So, required day = Wednesday - 5 = Friday

- 125.(D)**

2	99	
2	49	1
2	24	1
2	12	0
2	6	0
2	3	0
	1	1

Hence, 99 in binary system

$$= 1100011$$

- 126.(A)** We can see that the options are identical except for the last digit. So, we calculate only last digit. But for understanding we are demonstrating the sake of the entire procedure.

We divide 1153 by 15 and write the remainders and quotients at appropriate places.

15	1153	
15	76	13
	5	1

So, we write last quotient first and first remainder last. Also, $(13)_{10} = (D)_{15}$

$\therefore$ The number is 51D.

- 127.(D)** To convert from base 6 to base 10, multiply each digit to get number in base 6.

$$(1234)_6 = (4 \times 6^0 + 3 \times 6^1 + 2 \times 6^2 + 1 \times 6^3)_{10} = 4 + 18 + 72 + 216 = (310)_{10}$$

- 128.(D)** $(1556)_{10} = (?)_{16}$

16	1556	
16	97	4

	6	1
--	---	---

So, we write last quotient first and first remainder last. Also, $(1556)_{10} = (614)_{16}$

- 129.(C)** $(413)_7 = (3 \times 7^0 + 1 \times 7^1 + 4 \times 7^2)_{10} = (3 + 7 + 196) = (206)_{10}$

Practice Exercise Level 2

- 1.(C)** In option (c), as x and y are odd.

$(x + y)$ is always even and if a number is multiplied by an even number, the answer will always be even.

$$\begin{aligned} \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} &= 2 \Rightarrow \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = 2 \\ \Rightarrow \frac{(x+1)}{(2x+1)} &= 2 \end{aligned}$$

$$4x + 2 = x + 1$$

$$3x = -1$$

$$x = \frac{-1}{3}$$

- 3.(B)** Given expression, can be reconstructed as

$$\begin{aligned} \frac{4 \times 5 \times 6 + 5 \times 6 + 6 \times 2 + 3 \times 2}{1 \times 2 \times 3 \times 4 \times 5 \times 6} \\ = \frac{120 + 30 + 12 + 6}{1 \times 2 \times 3 \times 4 \times 5 \times 6} \\ = \frac{168}{2 \times 3 \times 4 \times 5 \times 6} = \frac{7}{30} \end{aligned}$$

- 4.(B)** Given expression, can be written as

$$\begin{aligned} \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{98}{99} \times \frac{99}{100} &= \frac{2}{100} \\ &= \frac{1}{50} \end{aligned}$$

- 5.(A)** Given expression, can be reconstructed as

$$\begin{aligned} \frac{(x+2)}{(x+1)} \times \frac{(x+3)}{(x+2)} \times \frac{(x+4)}{(x+3)} \times \frac{(x+5)}{(x+4)} \\ = \frac{(x+5)}{(x+1)} \end{aligned}$$

- 6.(B)** Given expression, can be reconstructed as

$$= \frac{1}{3} \left[\left(1 - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{10} \right) + \left(\frac{1}{10} - \frac{1}{13} \right) + \left(\frac{1}{13} - \frac{1}{16} \right) \right]$$

$$= \frac{1}{3} \left(1 - \frac{1}{16} \right) = \frac{1}{3} \times \frac{15}{16} = \frac{5}{16}$$

7.(C) Given expression, can be reconstructed as

$$= \left[\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \dots + \left(\frac{1}{n} - \frac{1}{(n+1)} \right) \right]$$

$$= \left(1 - \frac{1}{n+1} \right) = \frac{(n+1-1)}{(n+1)} = \frac{n}{(n+1)}$$

8.(D) According to the question,

$$KLM = 4199 \quad \dots (1)$$

$$JKL = 1547 \quad \dots (2)$$

On dividing equation (1) by (2), we get

$$\frac{KLM}{JKL} = \frac{4199}{1547}$$

$$\frac{M}{J} = \frac{19}{7}$$

$$\text{and } KL = 221$$

$$K \times L = 221 = 17 \times 13$$

$$\text{Therefore, } J = 7, K = 13, L = 17 \text{ and } M = 19$$

$$J + K + L + M = 7 + 13 + 17 + 19 = 56$$

9.(C) Three-digit number greater than 700 whose two digits are equal are

707, 711, 722, 727, 733, 737, 744, 747, 755, 757, 766, 767, 770, 771, 772, 773, 774, 775, 776, 778, 778, 779, 787, 788, 797, 799, 800, 808, 811, 818, 822, 828, 833, 838, 844, 848, 855, 858, 886, 868, 877, 878, 880, 881, 882, 883, 884, 885, 886, 887, 889, 898, 899, 900, 909, 911, 919, 922, 929, 933, 939, 944, 949, 955, 959, 966, 969, 977, 979, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998

$$7 _ (11, _ 99) = 8$$

(not including 77)

$$7 _ 7 (0 _ 9) = 9$$

(not including 7)

$$7 _ 7 (0 _ 9) = 9$$

(not including 7)

$$\text{Total } 8 + 9 + 9 = 26$$

Similarly, for 8 and 9 at hundredth place, numbers possible = $26 \times 2 = 52$

Total numbers possible

$$= 26 + 52 + 2 \text{ i.e. including 800 and 900}$$

$$= 80$$

10.(A) If X is even and $X - (Y + Z)$ is even,

So, $Y + Z$ must be even, also $Y - Z$ will be even

Now, $Y - Z - W$ is odd

So, W must be odd.

11.(A) $1^2 + 2^2 + \dots + 19^2$

$$= \frac{19(19+1)(38+1)}{6} = 2470$$

$$\text{and } 2^2 + 4^2 + \dots + 18^2$$

$$= 2^2 (1^2 + 2^2 + \dots + 9^2)$$

$$= \frac{4 \times 9(9+1)(18+1)}{6} = 1140$$

$$\therefore 1^2 + 3^2 + 5^2 + \dots + 19^2$$

$$= 2470 - 1140 = 1330$$

12.(C) $2^2 + 4^2 + 6^2 + \dots + 20^2$

$$= (2 \times 1)^2 + (2 \times 2)^2 + \dots + (2 \times 10)^2$$

$$= 2^2 (1^2 + 2^2 + \dots + 10^2)$$

$$= 4 \times \frac{10 \times 11 \times 21}{6} = 1540$$

13.(C)

Range	Numbers
287 – 299	13
300 – 799	95 (19 × 5)
800 – 803	1
Total	109

14.(D) x and y are odd numbers.

We know that,

When, we multiply an even number to any natural number the resultant number is even.

All the first three options will be even.

15.(B) The position of any cell can be changed by the factors of that particular cell number.

Let us discuss the fate of any particular cell number as per the algorithm given:

Cell Number 45

Factors of 45 are (1, 3, 5, 9, 15, 45)

Initially – Closed

After Step – 1	Open
After Step – 3	Close
After Step – 5	Open
After Step – 9	Close
After Step – 15	Open
After Step – 45	Close

It can be seen that for cell number 45, only those step numbers which are factors of 45 will have any impact on the position of cell number 45. These are going to be – Step 1, Step 3, Step 5, Step 9, Step 15, Step 45.

Beyond step 45, none of the steps will have any impact on cell number 45.

It can be concluded that the moment 1st factor is obtained (in the form of Step 1), cell will be opened.

And so on:

1 st Factor	Open
2 nd Factor	Close
3 rd Factor	Open
4 th Factor	Close
5 th Factor	Open
6 th Factor	Close
and so on	

We can see that when 1st or 3rd or 5th factor or any odd number of factor is obtained, cell gets opened.

However, when 2nd or 4th or 6th or any even number of factor is obtained, cell will get closed.

It is the case where when a number is having only odd number factors, cell remains open. This is possible only if the cell number is perfect square.

Hence, cell numbers which will remain open = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. These ten cells will remain open. And ten prisoners will be released.

16.(B) In the given options, only 64 is a perfect square number.

17.(B)
$$(a+b) = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$= \frac{(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)^2}$$

$$= \frac{2(5+1)}{(5-1)} = \frac{12}{4} = 3$$

$$(a-b) = \frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$= \frac{(\sqrt{5}+1)^2 - (\sqrt{5}-1)^2}{(\sqrt{5}-1)}$$

$$= \frac{4 \times \sqrt{5} \times 1}{4} = \sqrt{5} \text{ and } ab = 1$$

$$\therefore \frac{(a^2 + ab + b^2)}{(a^2 - ab + b^2)} = \frac{(a+b)^2 - ab}{(a-b)^2 + ab}$$

$$= \frac{(3^2 - 1)}{(\sqrt{5})^2 + 1} = \frac{8}{6} = \frac{4}{3}$$

18.(D) As $5 - 3 = 7 - 5 = 9 - 7$
then, $\sqrt{5} - \sqrt{3} > \sqrt{7} - \sqrt{5} >$
 $\sqrt{9} - \sqrt{7} > \sqrt{11} - \sqrt{9}$

Hence, greatest among the given is $\sqrt{5} - \sqrt{3}$.

19.(D) $x = 7 - 4\sqrt{3}$
 $x = (2)^2 + (\sqrt{3})^2 - 2\sqrt{3} \times 2$
 $= (2 - \sqrt{3})^2$
 $\frac{1}{x} = \frac{1}{7 - 4\sqrt{3}} \times \frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}}$
 $= \frac{7 + 4\sqrt{3}}{49 - 48} = 7 + 4\sqrt{3}$
 $\frac{1}{x} = (2)^2 + (\sqrt{3})^2 + 2 \times \sqrt{3} \times 2$
 $= (2 + \sqrt{3})^2$

Now, $\sqrt{x} + \frac{1}{\sqrt{x}}$

$$= \sqrt{(2-\sqrt{3})^2} + \sqrt{(2+\sqrt{3})^2}$$

$$= 2 + 2 = 4$$

20.(D)

$$x = \frac{2\sqrt{15}}{\sqrt{3} + \sqrt{5}}$$

$$\Rightarrow \frac{x}{\sqrt{3}} = \frac{2\sqrt{5}}{\sqrt{3} + \sqrt{5}}$$

$$\text{and } \frac{x}{\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{3} + \sqrt{5}}$$

Applying componendo and dividendo, we get

$$\frac{x + \sqrt{3}}{x - \sqrt{3}} = 9 + 2\sqrt{15}$$

$$\text{and } \frac{x + \sqrt{5}}{x - \sqrt{5}} = -(7 + 2\sqrt{15})$$

$$\Rightarrow \frac{x + \sqrt{5}}{x - \sqrt{5}} + \frac{x + \sqrt{3}}{x - \sqrt{3}} = 2$$

21.(D)

$$\sqrt{37 + 20\sqrt{3}} - \sqrt{61 + 28\sqrt{3}}$$

$$= \sqrt{25 + 12 + 20\sqrt{3}}$$

$$- \sqrt{49 + 12 + 28\sqrt{3}}$$

$$= \sqrt{(5)^2 + (2\sqrt{3})^2 + 2 \times 5 \times 2\sqrt{3}}$$

$$- \sqrt{(7)^2 + (2\sqrt{3})^2 + 2 \times 7 \times 2\sqrt{3}}$$

$$= \sqrt{(5 + 2\sqrt{3})^2} - \sqrt{(7 + 2\sqrt{3})^2}$$

$$(5 + 2\sqrt{3}) - (7 + 2\sqrt{3}) = -2$$

22.(B)

$$5^{x-1} + 5^x + 5^{x+1} = 775$$

$$\Rightarrow 5^x (5^{-1} + 1 + 5^1) = 775$$

$$\Rightarrow \frac{31}{5} \times 5^x = 775$$

$$\Rightarrow 5^x = 125 = 5^3 = 5^3$$

$$x = 3$$

Put values from the options and check.

23.(C)

Square all the given values

$$(\sqrt{7} + \sqrt{3})^2 = 7 + 3 + 2\sqrt{21}$$

$$= 10 + 2\sqrt{21}$$

$$(\sqrt{6} + \sqrt{4})^2 = 6 + 4 + 2\sqrt{24}$$

$$= 10 + 2\sqrt{24}$$

$$(\sqrt{5} + \sqrt{5})^2 = 5 + 5 + 2\sqrt{25}$$

$$= 10 + 2\sqrt{25}$$

$$(\sqrt{2} + \sqrt{8})^2 = 2 + 8 + 2\sqrt{16}$$

$$= 10 + 2\sqrt{16}$$

So, $\sqrt{5} + \sqrt{5}$ is greatest.

24.(B)

LCM of 2, 3, 4, 6 and 12 = 12

$$2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = \sqrt[12]{64}$$

$$3^{\frac{1}{3}} = 3^{\frac{1}{3} \times \frac{4}{4}} = (3^4)^{\frac{1}{12}} = \sqrt[12]{81}$$

$$4^{\frac{1}{4}} = 4^{\frac{1}{4} \times \frac{3}{3}} = (4^3)^{\frac{1}{12}} = \sqrt[12]{64}$$

$$6^{\frac{1}{6}} = 6^{\frac{1}{6} \times \frac{2}{2}} = (6^2)^{\frac{1}{12}} = \sqrt[12]{36}$$

$$12^{\frac{1}{12}} = \sqrt[12]{12}$$

So, $3^{\frac{1}{3}}$ is the largest.

25.(B)

$$(2^{16} - 1) = (2^8 + 1)(2^8 - 1)$$

$$= (2^8 + 1)(2^4 + 1)(2^2 + 1)(2 + 1)(2 - 1)$$

26.(B)

The given expression is:

$$2 \log \left(\frac{81}{100} \right) \times \frac{2}{3} \log \left(\frac{27}{10} \right) \div \log 9$$

$$= 2[\log 3^4 - \log 100] \times \frac{2}{3} [(\log 3^3 - \log 10)] \div$$

$$2 \log 3$$

$$= 2[4 \log 3 - 2] \times \frac{2}{3} [(3 \log 3 - 1)] \div 2 \log 3$$

Substitute $\log 3 = 0.4771$

$$\Rightarrow -0.0552$$

27.(D)

Given expression

$$= \frac{1}{\log xyz} + \frac{1}{\log xyz} + \frac{1}{\log xyz}$$

$$= \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log zx}{\log xyz}$$

$$= \frac{\log xy + \log yz + \log zx}{\log xyz}$$

$$= \frac{\log x^2 y^2 z^2}{\log xyz} = \frac{2 \log xyz}{\log xyz} = 2$$

28.(D)

$$\log_y x = 8$$

$$\Rightarrow x = y^8 \quad \dots (1)$$

$$\log_{10y} 16x = 4$$

$$\Rightarrow 16x = (10y)^4 \quad \dots (2)$$

Dividing (2) by (1),

$$10^4 y^{-4} = 16 \Rightarrow y = 5$$

29.(D) Number = $(35 \times 12) = 420$

Correct quotient = $420 \div 21 = 20$

30.(B) Difference = $9699 - 7179$
 $= 2520$

Factors of 2520 $\Rightarrow 2^3 \times 3^2 \times 5^1 \times 7^1$

For one or more zeros

Set of 2 and 5 is required.

i.e. $2^2 \times 3^2 \times 7^1 \times 10$

Total possible factors $(3 \times 3 \times 2) = 18$

31.(A) $[47]^{100} = [47^2]^{50} = [2209]^{50}$

$$\Rightarrow \frac{(09)^{50}}{100} \Rightarrow \frac{(-91)^{50}}{100} = 01$$

32.(D) $\frac{[(53)]^{1111}}{51} = \frac{[(2)]^{1111}}{51}$

$$= \frac{[(2^8)^{138} \times (2)^7]}{51}$$

$$= \frac{[(256)^{138} \times (2)^7]}{51} = \frac{(1)^{38} \times (2)^7}{51}$$

$$= \frac{[128]}{51} = 26$$

33.(B)

$$\frac{[(12)^{13^{14}}]}{145}$$

Now, we know that $(12)^2 = 144$

which will yield a remainder of (-1) . Hence, if we convert the power $(13)^{14}$ in power of '2'

$$\frac{13^{14}}{2} = (1)^{14}$$

Hence, $(13)^{14}$ can be written as $2k + 1$.

Also, $(13)^{14}$ will be an odd number, hence we need to check whether values of k would be even or odd.

$$(13)^{14} = 2k + 1$$

$$(13)^{14} - 1 = 2k$$

In order to check whether k is even or not we divide $(13)^{14} - 1$ by 4.

$$\frac{(13^{14} - 1)}{4} = (1)^{14} - 1 = 0$$

Hence, 'k' is even.

$$\text{So, } \frac{(12)^{13^{14}}}{145} = \frac{(12)^{2k+1}}{145}$$

$$= \frac{(12)^{2k} \times 12}{145} = \frac{[(12)^2]^k \times 12}{145}$$

$$\frac{(144)^k \times 12}{145} = \frac{(-1)^k \times 12}{145}$$

Now, as k is even.

$$\text{Hence, } \frac{1 \times 12}{145} = 12 \text{ is the remainder.}$$

34.(D) $(17)^3 + (19)^3 + (21)^3 + (23)^3$ is divided by 80.

$$\Rightarrow (17)^3 + (23)^3 + (19)^3 + (21)^3$$

$$= (17 + 23)(17^2 + 13^2 - 17 \times 23) + (19 + 21)$$

$$(19^2 + 21^2 - 19 \times 21)$$

$$= 40 \times (\text{Odd number}) +$$

$$40 \times (\text{Odd number})$$

$$= \frac{80}{80}$$

$$= \frac{40 \times (\text{Even number})}{80}$$

$$= \frac{80 \times k}{80} = \text{Remainder} = 0$$

Since, 17, 19, 21, 23 are in A.P. and power is odd.

$\therefore 17^3 + 19^3 + 21^3 + 23^3$ is divisible by $(17 + 19 + 21 + 23)$ expression becomes $\frac{80k}{80} \Rightarrow$

Remainder = 0.

35.(D) $(74)^{13} - (41)^{13} + (75)^{13} - (42)^{13}$

$$\Rightarrow (74 - 41) \times (74^2 + 74 \times 41 + 41^2) + (75 - 42) (75^2 + 75 \times 42 + 42^2)$$

$$\Rightarrow 33 \times (\text{Odd number}) + 33 \times (\text{Odd number})$$

$$\Rightarrow 33 \times (\text{Even number}) \Rightarrow 66k$$

$$\Rightarrow \frac{66k}{66} = \text{Remainder} = 0$$

36.(C) $2^{51} + 2^{52} + 2^{53} + 2^{54} + 2^{55}$

$$= 2^{51} [1 + 2 + 2^2 + 2^3 + 2^4]$$

$$= 2^{51} [1 + 2 + 4 + 8 + 16]$$

$$= 2^{51} \times 31$$

$$= 2^{49} \times 124$$

Hence, the expression is divisible by 124.

37.(A) $1! + 2! + 3! + \dots + 50!$ divided by 5

$$\begin{aligned} & \frac{1!}{5} + \frac{2!}{5} + \frac{3!}{5} + \frac{4!}{5} + \frac{5!}{5} + \frac{6 \times 5!}{5} + \dots \\ & + \frac{50 \times 49!}{5} \\ & = \frac{1 + 2 + 6 + 24 + 0 + 0 + \dots + 0}{5} \\ & = \frac{33}{5} = 3 \end{aligned}$$

38.(D) $x = 6q_1 + 4$

$$y = 6q_2 + 5$$

$$x^2 + y^2 = (6q_1 + 4)^2 + (6q_2 + 5)^2$$

$$\text{Remainder} = 4^2 + 5^2 = 16 + 25 = 41$$

Now, 41 when divided by 6, leaves remainder 5.

39.(A) $\frac{2^{256}}{17} = \frac{(2^4)^{64}}{17} = \frac{(16)^{64}}{17}$

$$= \frac{(17-1)^{64}}{17} = (-1)^{64} = 1$$

40.(A) $\frac{30^{40}}{17} = \frac{(34-4)^{40}}{17}$

$$= \frac{(-4)^{40}}{17} = \frac{4^{40}}{17}$$

$$= (4^2)^{20} = 16^{20}$$

$$= (17-1)^{20} = (-1)^{20} = 1$$

So, the required remainder is 1.

41.(B) $2^{7777} + 7^{2222}$

$$(2^7)^{1111} + (7^2)^{1111}$$

$$(128)^{1111} + (49)^{1111}$$

So, it must be divisible by 177 i.e. $(128 + 49)$.

42.(B)

Thumb	1	9		17		25
Index	2	8	10	16	18	24
Middle	3	7	11	15	19	23
Ring	4	6	12	14	20	22
Little	5		13		21	

Values coming at the thumb are 1, 9, 17, 25,

These values when divided by 8 the remainder is 1.

Now, 1993 when divided by 8, the remainder is 1.

So, 1994 will come at the index finger.

43.(D) $(2^{96} + 1) = (2^{32})^3 + (1)^3$

$$= (2^{32} + 1)(\dots)$$

The given expression is divisible by $(2^{32} + 1)$ so it must also be divisible by any factor of $2^{32} + 1$.

44.(A) $(x^n - 1)$ is divisible by $(x + 1)$, when n is even.
 $[(49)^{15} - 1] = [(7^2)^{15} - 1] = (7^{30} - 1)$, which is divisible by $(7 + 1)$, i.e. 8.

45.(D) (a) Is true: see last two cross-products $12 \times 89 = 105$
 $\Rightarrow N \times M$ should have ten $(7 + 3)$ digits.

If we get last two cross-products as two digits then $N \times M$ should have $9 (= 7 + 2)$ digits.

(b) N^2 has 13 digits as last two cross-products is a single digit no.

(c) $[(...7)^4]^{222} + (7)^2 = (...1) \times (...9) = 9$

OR

N^M , $M = 890$ and N has 7 at unit's place.

Cyclicity of 7 is 4.

$$\therefore \frac{890}{4} \Rightarrow \text{Remainder} = 2$$

$$\therefore \text{Unit's digit of } N = 7^2 = 9$$

(d) $N \div M > 1600$ is false.

46.(C) $3600 = 2^4 \times 3^2 \times 5^2$

Number of factors which are perfect square
 $= 3 \times 2 \times 2 = 12$

47.(A) $2500 = 2^2 \times 5^4$

Number of factors $= 3 \times 5 = 15$

$$\text{Number of ways} = \frac{15-1}{2} = 7$$

48.(D) $55^3 + 17^3 - 72^3$

$$= 55^3 + 17^3 + (-72)^3$$

Now, When $a + b + c = 0$

$$\text{Then, } a^3 + b^3 + c^3 = 3abc$$

So, $55^3 + 17^3 + (-72)^3$

$$= 3 \times 55 \times 17 \times (-72)$$

$$[\text{As } 55 + 17 - 72 = 0]$$

Now, it is divisible by both 3 and 17.

- 49.(C)** Let the numbers be $29a$ and $29b$, where a and b are co-primes.

$$\text{Then, } 29a \times 29b = 29 \times 4147$$

$$\Rightarrow ab = \frac{29 \times 4147}{29 \times 29} = 143$$

Co-primes with product 143 are 11 and 13.

$\therefore$ Numbers are 29×11 and 29×13 i.e., 319 and 377.

$$\text{Their sum} = (319 + 377) = 696$$

- 50.(C)** $3240 = 2^3 \times 3^4 \times 5$;

$$3600 = 2^4 \times 3^2 \times 5^2$$

$$\text{HCF} = 36 = 2^2 \times 3^2 ;$$

$$\text{LCM} = 2^4 \times 3^5 \times 5^2 \times 7^2$$

$\therefore 3^{\text{rd}}$ number must have $2^2, 3^5, 7^2$ as its factor.

- 51.(D)** LCM of 2, 4, 6, 8, 10 and 12

$$= (2 \times 2 \times 3 \times 2 \times 5) = 120$$

After every 2 minutes they toll together.

Number of times they toll in 30 minutes

$$= \left(\frac{30}{2} + 1 \right) \text{ times} = 16 \text{ times}$$

- 52.(C)** Required number is divisible by LCM of 12, 18, 21, 28 i.e. it is divisible by $(3 \times 2 \times 2 \times 7 \times 3)$

$$= 252$$

Greatest number of 4 digits

$$= 9999$$

$$\text{On dividing } 9999 \text{ by } 252, \text{ remainder} = 171$$

$$\text{Required number} = (9999 - 171)$$

$$= 9828$$

- 53.(C)** Required number = HCF of $(3026 - 11)$ and $(5053 - 13)$.

$$= \text{HCF of } 3015 \text{ and } 5040 = 45$$

- 54.(A)** LCM of (15, 20 and 25)

$$= 300$$

So, smallest four-digit number divisible by 15, 20 & 25 is 1200.

$$\text{So, required number} = 1200 + 99 = 1299$$

- 55.(C)** Use options.

- 56.(D)** Let x is the number of damaged packets So, $(250 - x)$, $(490 - x)$ and $(850 - x)$ are dropped respectively.

HCF of $(250 - x)$, $(490 - x)$ and $(850 - x)$ or $(490 - x) - (250 - x)$, $(850 - x) - (490 - x)$ and $(850 - x) - (250 - x)$ or 240, 360 and 600.

$$\text{HCF of } 240, 360, 600 = 120$$

$$\text{Required value} = \frac{240}{120} + \frac{360}{120} + \frac{600}{120}$$

$$= 2 + 3 + 5 = 10$$

- 57.(C)** General form of that number is $= 7 (5 (4x + 3) + 2) + 4$

$$= 140x + 123$$

$$\text{Smallest such number} = 123$$

and remainders with 8, 5 and 6

$$\text{are } 3, 0, 3$$

- 58.(B)** When a number is divided by 5, leaves remainder 1.

$$\text{i.e. number is } 5a + 1.$$

When the number is divided 6 leaves remainder 5 i.e. number is $6b + 5$

As the number is same,

$$5a + 1 = 6b + 5$$

$$a = \frac{6b + 4}{5}$$

Minimum value of "b" that gives integral values of "a" is $b = 1, a = 2$

Minimum such number is "11".

General form of that number is $11 + 30x$.

To obtain greatest four-digit number, x has to be 332.

$$\therefore \text{Highest possible number is } 9971.$$

- 59.(A)** Number can be written in the form of $120k + 1$ [(LCM of 5, 6, 8 and 12) + 1]

Number can also be written as $13k$.

By equating both,

$$k' = \frac{120k+1}{13}$$

Minimum value of k for which k' is an positive integer is +4.

$$k = 4 \Rightarrow k' = 37$$

Hence, the smallest number of shrubs is 481.

- 60.(A)** $45 = 3 \times 3 \times 5$. Here, 5 is the largest prime factor of 45 but for every single power of 5, 2 powers of 3 is required. So, we have

$$5: 24 + 4 = 28$$

$$3: 41 + 13 + 4 + 1 = 59$$

As $59 > 56$ (2×28), highest power of 45 in $123!$ is 28.

- 61.(C)** $N = 2 \times 4 \times 6 \times 8 \times 10 \times \dots \times 100$
 $= 2^{100} \times (1 \times 2 \times 3 \times 4 \times \dots \times 50)$
 $= 2^{100} \times 50!$

So, highest power of 5 = Number of zeros =

$$\frac{50}{5} + \frac{50}{25}$$

$$= 10 + 2 = 12$$

- 62.(B)** (i) 16^{th} July, 1776 means
 (1775 years + 6 months + 16 days)
 Now, 1600 years have 0 odd days.
 100 years have 5 odd days.
 75 years contain 18 leap and 57 ordinary years and therefore (36 + 57) or 93 or 2 odd days.

$\therefore$ 1775 years give $0 + 5 + 2 = 7$ odd days or 0 odd day. Also number of odd days from 1^{st} Jan, 1776 to 16^{th} July, 1776

Jan + Feb + March + April + May + June + July

$$3 + 1 + 3 + 2 + 3 + 2 + 2 = 16 \text{ odd days or } 2 \text{ odd days.}$$

$\therefore$ Total number of odd days

$$= 0 + 2 = 2$$

Hence, the day on 16^{th} July, 1776 was 'Tuesday'.

- 63.(C)** 1600 years have 0 odd days. 100 years have 5 odd days.

300 years have 15 or 1 odd day.

i.e. 1900 years have 1 odd day.

83 years contain 20 leap years and 63 ordinary years and therefore (40 + 0) odd days i.e. 5 odd days.

1983 years contain (0 + 1 + 5) i.e., 6 odd days. Number of odd days from Jan, 1984 to 31^{st} Oct 1984.

$$= (3 + 1 + 3 + 2 + 3 + 2 + 3 + 3 + 2 + 3) = 25 \text{ days} = 4 \text{ odd days}$$

$\therefore$ Total number of odd days = $6 + 4 = 10$ i.e. 3 odd days.

So, 31^{st} Oct, 1984 was Wednesday.

- 64.(A)** In an ordinary year, February has no odd day

So, February and March begin on same day of week.

Also, we know that, November and March begin on same day of the week.

- 65.(B)** The day after tomorrow is Sunday. Therefore, today is Friday.

Hence, the day on tomorrow's day before yesterday is given by:

$$= \text{Friday} - 1 = \text{Thursday}$$

- 66.(D)**

$$(12630)_x = (3402)_{10}$$

$$\Rightarrow (3402)_{10} = (0 \times x^0 + 3 \times x^1 + 6 \times x^2 + 2 \times x^3 + 1 \times x^4)_{10}$$

$$= (3x + 6x^2 + 2x^3 + x^4)_x$$

Now, we have 3 choices for x . We substitute from the given answer choices and check for last digit. We see that 9 gets eliminated. If we put $x = 7$, it satisfy the condition and hence correct option will be (d).

- 67.(A)**

$$(abc)_7 = (cba)_9$$

$$a \times 7^2 + b \times 7^1 + c \times 7^0 = c \times 9^2 + b \times 9^1 + c \times 9^0$$

$$49a + 7b + c = 81c + 9b + a$$

$$48a - 2b - 80c = 0$$

Only possible value of a , b and c that satisfy the equation are 2, 8 and 1 respectively.

So, $a + b + c = 11$.

68.(B)

$$(152)_k = (86)_{10}$$

$$2 \times k^0 + 5 \times k^1 + 1 \times k^2 = 86$$

$$k^2 + 5k - 84 = 0$$

$$(k - 7)(k + 12) = 0$$

$$k = 7 \text{ (ignoring negative value)}$$

Percentages

02

The word "Percentage" in the simplest sense means "per hundred" or "for every hundred". Therefore when we calculate something as a part of 100, that part is numerically termed as percentage.

CONVERSION OF A FRACTION / DECIMAL INTO A PERCENT

To convert a fraction/decimal into a percentage, simply multiply the value by 100.

For example, $\frac{1}{3}$ can be expressed in percentage

terms as $\frac{1}{3} \times 100 = 33.33\%$ and the decimal 0.2 can be expressed as a percentage as $0.2 \times 100 = 20\%$

CONVERSION OF A PERCENTAGE INTO A FRACTION

A percentage when divided by 100 is converted into a fraction/decimal. For example 20% as a fraction is

$$\frac{20}{100} = \frac{1}{5}$$

The % sign is dropped when we divide the percentage by 100. So, % = $\frac{1}{100}$

PERCENT OF A NUMBER

To find the percent of a number, convert the percent into fraction and multiply the resultant fraction with the number.

For example, P% of a number N is $= \frac{P}{100} \times N$

Example 1: What is 45% of 500?

Solution:

$$\text{Required value} = \frac{45}{100} \times 500 = 225$$

Example 2: What is 20% of 50% of 300?

Solution:

$$\text{Required value} = \frac{20}{100} \times \left(\frac{50}{100} \times 300 \right) = 30$$

Note:

To find what percentage of 'a' is 'b', use the formula
Percentage = $(b/a) \times 100$

In short, we should ask ourself "of what are we calculating percentage? We will get the answer as 'a'. Therefore, 'a' will form the whole and 'b' will form the part of the whole.

Example 3: What percentage of 270 is 90?

Solution:

$$\text{Percentage} = \frac{90}{270} \times 100 = 33.3\%$$

Example 4: 80 is what percentage of 500?

Solution:

$$\text{Percentage} = \frac{80}{500} \times 100 = 16\%$$

Example 5: If A = 200 and B = 150, then B is what percentage of A?

Solution:

$$\text{Percentage} = \frac{150}{200} \times 100 = 75\%$$

PERCENTAGE EQUIVALENTS OF FRACTIONS

Knowing conversion of common fraction into percentage helps you convert many fractions into percentage immediately. For example, knowing that $\frac{1}{8} = 12\%$ will help you to find 12.5% of any number. If we have to find 12.5% of 8400, then it is $\frac{1}{8} \times 8400$ i.e. 1050. Similarly, as $\frac{1}{6} = 16.66\%$. So, 16.66% of 612 is $\frac{1}{6} \times 612$ i.e. 102.

The following is the list of percentage values of some of the important fraction values.

Fraction	Percentage	Fraction	Percentage
$\frac{1}{2}$	50%	$\frac{1}{11}$	9.09%
$\frac{1}{3}$	33.33%	$\frac{1}{12}$	8.33%
$\frac{1}{4}$	25%	$\frac{1}{13}$	7.69%
$\frac{1}{5}$	20%	$\frac{1}{14}$	7.14%
$\frac{1}{6}$	16.66%	$\frac{1}{15}$	6.66%
$\frac{1}{7}$	14.28%	$\frac{1}{16}$	6.25%
$\frac{1}{8}$	12.5%	$\frac{1}{17}$	5.88%
$\frac{1}{9}$	11.11%	$\frac{1}{18}$	5.55%
$\frac{1}{10}$	10%	$\frac{1}{19}$	5.26%

Example 6: What is 14.28% of 392?

Solution:

$$\text{As, } 14.28\% = \frac{1}{7}$$

$$\text{So, } 14.28\% \text{ of } 392 = \frac{1}{7} \times 392 = 56$$

Example 7: What is 58.33% of 4896?

Solution:

58.33% can be written as 50% + 8.33%.

Now 50% of a number is $\frac{1}{2}$ of the number and

8.33% of a number is $\frac{1}{12}$ of the number.

$$\text{So, } 50\% \text{ of } 4896 = \frac{1}{2} \times 4896 = 2448 \text{ and}$$

$$8.33\% \text{ of } 4896 = \frac{1}{12} \times 4896 = 408$$

$$\text{So, the required value} = 2448 + 408 = 2856$$

PERCENTAGE INCREASE / DECREASE

Percentage increase/decrease of a quantity is the ratio of the actual increase or decrease of the quantity to the original amount of the quantity.

$$\text{Percentage Increase} = \frac{\text{Actual Increase}}{\text{Original quantity}} \times 100$$

$$\text{Percentage Decrease} = \frac{\text{Actual Decrease}}{\text{Original quantity}} \times 100$$

Example 8: If the sales of the number of airplanes went up from 220 in 2002 to 245 in 2003, then what is the percentage increase in the sales of airplanes from 2002 to 2003?

Solution:

$$\text{Actual increase in sales} = 245 - 220 = 25$$

$$\text{Original sales} = 220$$

$$\text{Percentage increase} = \frac{\text{Actual Increase}}{\text{Original quantity}} \times 100$$

$$= \frac{25}{220} \times 100 = 11\frac{4}{11}\%$$

Example 9: Due to poor performance, the salary of Johny has decreased from ₹ 400 per hour to ₹ 320 per hour. What is the percentage decrease in the salary of Johny?

Solution:

$$\text{Actual decrease in salary} = 400 - 320 = 80$$

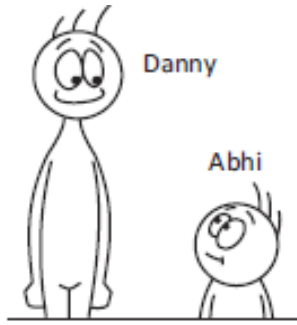
$$\text{Original salary} = 400$$

$$\text{Percentage Decrease} = \frac{\text{Actual Decrease}}{\text{Original quantity}} \times 100$$

$$= \frac{80}{400} \times 100 = 20\%$$

PERCENTAGE LESS THAN / GREATER THAN

We can see from the picture below that Danny is taller than Abhi. What will be the answers of the questions below?



- a) By what percentage Danny is taller than Abhi?
As we are looking at Danny from the viewpoint of Abhi, therefore in the denominator we have Abhi's height. Therefore, the percentage by which Danny is taller than Abhi is

$$= \frac{\text{Danny's height} - \text{Abhi's height}}{\text{Abhi's height}} \times 100$$

- b) By what percentage Abhi is shorter than Danny?
As we are looking at Abhi from the viewpoint of Danny, therefore in the denominator we have Danny's height. Therefore, the percentage by which Abhi is shorter than Danny is

$$= \frac{\text{Danny's height} - \text{Abhi's height}}{\text{Danny's height}} \times 100$$

Example 10: Income of Johny is 25% more than the income of Vicky. By what percentage is Vicky's income less than that of Johny's income?

Solution:

Let Vicky's income = 100

So, Johny's income = 125

Now, to find out by what percentage Vicky's income is less than that of Johny's, we will look at Vicky's income from Johny's side.

So, required percentage

$$= \frac{\text{Johny's income} - \text{Vicky's income}}{\text{Johny's income}} \times 100$$

$$= \frac{125 - 100}{125} \times 100$$

$$= \frac{25}{125} \times 100 = 20\%$$

So, Vicky's income is 20% less than the income of Johny.

Example 11: The price of TV is 20% less than the price of a DVD. By what percentage is the price of DVD more than the price of the TV?

Solution:

Let the price of DVD = 100

So, price of TV = 80

Now, to find by what percent the price of DVD is more than the price of the TV, we will look at the price of DVD from the viewpoint of the price of the TV.

$$\text{So, required percentage} = \frac{100 - 80}{80} \times 100$$

$$= \frac{20}{80} \times 100 = 25\%$$

Example 12: Salary of Vikrant is 9.09% less than the salary of Raman. By what percentage is the salary of Raman more than the salary of Vikrant?

Solution:

As by assuming the salary of Raman to be 100, solution will be a bit calculative, so let's solve it using fractions.

$$\text{As, } 9.09\% = \frac{1}{11}$$

So, in this fraction, 1 is 9.09% of 11.

So, salary of Raman = 11

Hence, salary of Vikrant = 10

$$\text{Therefore, required percentage} = \frac{11 - 10}{10} \times 100$$

$$= \frac{1}{10} \times 100 = 10\%$$

PRODUCT CONSISTENCY METHOD

Consider a equation where expenditure is calculated as product of price and quantity.

Expenditure (E) = Price (P) × Quantity (Q)

If E is constant, then $P \propto \frac{1}{Q}$

For example, if price gets doubled then quantity should be half to keep the expenditure constant.

Hence, we can say if product of two quantities is constant, then change in one is compensated by another quantity.

Example 13: If the price of mustard oil increases by 25%, then by what percentage a household must reduce their consumption so that the expenditure remains the same?

Solution:

Expenditure = Price $\times$ Consumption

Here, 25% = $\frac{1}{4}$, means 1 is 25% of 4.

So, let us see it through a table.

	Price	Quantity	Expenditure
Initial	4	5	$4 \times 5 = 20$
Final	5	4	$5 \times 4 = 20$

So, according to this table, if price becomes 5 from 4, the quantity has to be 4 from 5 to keep the expenditure same.

So, change in quantity = $\frac{1}{5} \times 100 = 20\%$

Alternate Method:

$$P \times Q = E$$

$$\text{If price} = 1.25P = \frac{5}{4}P$$

$$\text{then, quantity} = \frac{4}{5}Q$$

$$\text{Reduced in quantity} = Q - \frac{4}{5}Q$$

$$= \frac{1}{5}Q \text{ or } 20\%$$

Example 14: A 30% reduction in the price of wheat enables Mahesh to buy 72 kg extra wheat for Rs. 1200. What is the new price (per kg) of wheat?

Solution:

In the question, expenditure in both cases i.e. initial and final is 1200.

As, Expenditure = Price $\times$ Quantity

$$\text{Here, } 30\% = \frac{3}{10}$$

If initial value of price was 10, then the final value will become 7 i.e. $(10 - 3)$.

	Price	Quantity	Expenditure
Initial	10	7	$10 \times 7 = 70$
Final	7	10	$7 \times 10 = 70$

$$\text{Change in quantity} = \frac{10-7}{7} = \frac{3}{7}$$

So, $\frac{3}{7}$ of initial quantity is 72 kg.

$$\text{Initial quantity} = \frac{7}{3} \times 72 = 168 \text{ kg}$$

$$\therefore \text{New quantity} = 168 + 72 = 240 \text{ kg}$$

$$\text{Hence, new price per kg} = \frac{1200}{240} = ₹ 5/\text{kg}$$

Alternate Method: 1

Total expenditure each time = ₹ 1200

Let initial price be x /kg.

$$\text{and new price be } \left(x \times \left(1 - \frac{30}{100} \right) \right)$$

$$= \left(x \times \frac{70}{100} \right) / \text{kg} = 0.7x$$

Difference between the initial and new quantity is 72 kg.

$$\text{So, } \frac{1200}{0.7x} - \frac{1200}{x} = 72$$

By solving we get,

New price = ₹ 5/kg

Alternate Method: 2

$$P \times Q = 1200$$

$$\text{Price} = 0.7P = \frac{7}{10}P$$

$$\text{Quantity} = \frac{10}{7}Q$$

$$\text{Increase in quantity} = \frac{10}{7}Q - Q = \frac{3}{7}Q$$

$$\frac{3}{7}Q = 72$$

$$Q = 168 \text{ kg}$$

New quantity = $168 + 72 = 240$ kg

New price = $\frac{1200}{240} = ₹ 5/\text{kg}$

MULTIPLYING FACTOR FOR PERCENTAGE CHANGE

When a quantity 'a' increases/decreases by some percentage to become another quantity 'b', then

New quantity $b = a \left(1 + \frac{\text{Percentage increase}}{100} \right)$

OR $b = a \left(1 - \frac{\text{Percentage decrease}}{100} \right)$

Let us say that a quantity has increased by 20%.

So, new quantity = Old quantity $\times \left[1 + \frac{20}{100} \right]$

= Old quantity $\times 1.2$

Therefore, to find the final quantity, after an increase of old quantity by 20%, we directly multiply the old quantity by the multiplying factor of 1.2 and get the new quantity.

Similarly, for a 30% increase, we will multiply the old quantity by 1.3 and get the new quantity.

Now, let us say the quantity has decreased by 30%.

So, new quantity = Old quantity $\left(1 - \frac{30}{100} \right)$

= Old quantity $\times 0.7$

Therefore, to find the final quantity after a 30% decrease, we can directly multiply the old quantity by the multiplying factor of 0.7 and get the new quantity. Similarly, for a 20% decrease, we will multiply the old quantity by 0.8 and get the new quantity.

Note:

Multiplying factors, which are more than 1, always increase the value, and the multiplying factors, which are less than 1, always decrease the value.

Below is a table showing multiplying factors for various percentage changes:

Percentage Increase	Multiplying Factor	Percentage Decrease	Multiplying Factor
5%	1.05	5%	0.95
10%	1.1	10%	0.9
15%	1.15	15%	0.85
20%	1.2	20%	0.8
25%	1.25	25%	0.75
30%	1.3	30%	0.7
40%	1.4	40%	0.6
50%	1.5	50%	0.5
60%	1.6	60%	0.4

MULTIPLYING FACTOR FOR PERCENTAGE CHANGE USING FRACTIONS

As we have seen above, the multiplying factors in terms of decimals, now we will see these multiplying factors in terms of fractions as it will be easier than decimals.

Let us understand it through an example, Salary of Rajesh got increased by 20%, here the percentage value is 20%.

$20\% = \frac{1}{5}$; In this fraction 1 is the 20% of 5

Let initial salary be x.

New salary = $x + \frac{x}{5}$

= $x \times \frac{6}{5}$

So, to increase the salary by 20%, we multiplied the initial salary by $\frac{6}{5}$, so we can say that $\frac{6}{5}$ is the multiplying factor for 20% increase.

Similarly, for 30% increase,

$30\% = \frac{3}{10}$; In this fraction 3 is 30% of 10

$\therefore$ Multiplying factor = $\frac{10+3}{10} = \frac{13}{10}$

Now, we see the multiplying factor for decrease.

Salary of Rajesh has decreased by 20%.

20% = $\frac{1}{5}$; In this fraction 1 is the 20% of 5

Let initial salary be x .

$$\text{Final salary} = x - \frac{x}{5}$$

$$= x \times \frac{4}{5}$$

Hence, multiplying factor for 20% decrease is $\frac{4}{5}$.

Multiplying factor for 30% decrease.

30% = $\frac{3}{10}$; In this fraction 3 is the 30% of 10

$$\therefore \text{Multiplying factor} = \frac{10-3}{10} = \frac{7}{10}$$

Note:

Multiplying factors having numerator greater than denominator will always increase the value.

Multiplying factors having numerator smaller than denominator will always decrease the value.

Percentage Increase	Multiplying Factor	Percentage Decrease	Multiplying Factor
$5\% = \frac{1}{20}$	$\frac{21}{20}$	$5\% = \frac{1}{20}$	$\frac{19}{20}$
$10\% = \frac{1}{10}$	$\frac{11}{10}$	$10\% = \frac{1}{10}$	$\frac{9}{10}$
$20\% = \frac{1}{5}$	$\frac{6}{5}$	$20\% = \frac{1}{5}$	$\frac{4}{5}$
$30\% = \frac{3}{10}$	$\frac{13}{10}$	$30\% = \frac{3}{10}$	$\frac{7}{10}$
$33.33\% = \frac{1}{3}$	$\frac{4}{3}$	$33.33\% = \frac{1}{3}$	$\frac{2}{3}$

SUCCESSIVE PERCENTAGE CHANGES

Let us say in the first year, A's salary increases by 20% and in the second year, the salary increases by 10%.

Now, would the net increase over the two years be $20 + 10 = 30\%$?

Let us check:

If A's salary was 100 at the start, then after first year it would be $\text{Rs. } 100 \times 1.2 = \text{Rs. } 120$

After second year it would be $\text{₹ } 120 \times 1.1 = \text{₹ } 132$.

Thus, the net increase is 32% and not 30%.

This computation of the final salary can also be done by multiplying factor as

$$\begin{aligned} \text{Final salary} &= (\text{Initial Salary} \times 1.2) \times 1.1 \\ &= \text{Initial Salary} \times 1.32 \end{aligned}$$

So, the final salary is 32% more than the initial salary.

In the above problem, the 20% increase and 10% increase are called successive and the second percentage is affected on the new base i.e. on the salary after the first year and not on the initial salary.

Consider the same example with two successive percentages increase $a\%$ and $b\%$.

$$\begin{aligned} \text{Final value} &= \left(1 + \frac{a}{100}\right) \left(1 + \frac{b}{100}\right) \times \text{Initial value} \\ &= \left(1 + \frac{b}{100} + \frac{a}{100} + \frac{ab}{100 \times 100}\right) \\ &= \left[1 + \frac{\left(a + b + \frac{ab}{100}\right)}{100}\right] \end{aligned}$$

So, the new percentage change of $a\%$ and

$$b\% \text{ is an effective change } = \left(a + b + \frac{ab}{100}\right)\%$$

The last example, where in the first year, the salary increases by 20% and in the second year, the salary increases by 10%. The net change is:

$$= 20 + 10 + \frac{20 \times 10}{100} = 32\%$$

The given formula $\left(a + b + \frac{ab}{100}\right)$ also works out for two successive decreases or one increase and one decrease (for percentage decrease the sign would be negative).

For example, if the salary decreases by 20% in the first year and in the next year the salary again

decreases by 10%. Then the net percentage change in the salary is

$$= -20 - 10 + \frac{(-20) \times (-10)}{100} = -30 + 2 = -28\%$$

So, the net percentage change in the salary is a decrease of 28%.

Note:

The negative sign indicates a net decrease in the salary.

So, to find the net change for two successive changes, we have two approaches:

Either we multiply the two corresponding multiplying factors to get the net multiplying factor.

OR

We use the formula $[a+b+\frac{ab}{100}]$ where a and b are the change in percentages.

It is advisable to get used to both these methods, as for different situations, one might be more time consuming than the other.

The same concept is also applicable to any relation of the type $N = A \times B$

If A and B change by a% and b% respectively, then the net percentage change in **N** is $[a+b+\frac{ab}{100}]$ %

Note:

We cannot use the formula for more than two successive changes. If there are 3 successive changes of a%, b% and c%, then first calculate net of a% and b% (let us say it k%). Then, the final change is given by the net successive change of k% and c%.

Example 15: In a rectangle, if the length increases by 30% and the breadth decreases by 10%, then what is the percentage change in area?

Solution:

Area = Length $\times$ Breadth

$$\text{Percentage change in area} = 30 - 10 + \frac{(30) \times (-10)}{100}$$

$$= 20 - 3 = 17\%$$

So, the area will increase by 17%.

Example 16: If the price of a movie ticket increases by 20% and the number of viewers decreases 30%, then what is the net change in the revenue collection?

Solution:

Revenue = Price of ticket $\times$ Number of tickets sold

Percentage change in revenue

$$= 20 - 30 + \frac{(20) \times (-30)}{100} = -16\%$$

So, the revenue will decrease by 16%.

Example 17: The length and breadth of a rectangle are 30 cm and 20 cm respectively. The length decreases by 20% and the breadth increases by 5%. What is the new area of the rectangle?

Solution:

$$\text{Percentage change in area} = -20 + 5 + \frac{(-20 \times 5)}{100}$$

$$= -15 - 1 = -16\%$$

So, the area will decrease by 16%.

Original area = $30 \times 20 = 600 \text{ cm}^2$

New area = $0.84 \times 600 = 504 \text{ cm}^2$ (As there is a decrease of 16%, the MF = 0.84)

Example 18: The radius of a circle is increased by 20%. By what percentage the circumference and the area of the circle will change?

Solution:

Circumference of the circle = $2\pi r$

As 2π is constant and r increases by 20%. So, the circumference will also increase by 20%.

Area of circle = $\pi r^2 = \pi \times r \times r$

As there are two factors of radius in the expression of area, so the area will increase by $[20+20+\frac{20 \times 20}{100}]$ i.e. 44%.

APPLICATION OF PERCENTAGE

The concept of percentages is also applied in various situations like questions based on voting, based on exams etc.

The following examples will help us to understand application based questions on percentages.

Example 19: A candidate scores 25% marks and fails by 30 marks. Another candidate scores 50% marks and gets 20 marks more than the minimum marks required to pass the examinations. What are the maximum marks for the examination?

Solution:

Let maximum marks = x

Passing marks from 1st candidate = 25% of $x + 30$

Passing marks from 2nd candidate = 50% of $x - 20$

Equating the passing marks:

$$25\% \text{ of } x + 30 = 50\% \text{ of } x - 20$$

$$25\% \text{ of } x = 50 \quad \text{So, } x = 200$$

Example 20: In an election between two candidates, 75% of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. What was the total number of voters enrolled in that election?

Solution:

Let total votes in the voter's list = x

Votes polled = $0.75x$

Valid Votes = $0.98 (0.75x)$

Now, 75% of valid votes = 9261

$$0.75 \times (0.98 \times (0.75x)) = 9261$$

By solving, we get

$$x = 16800$$

$$\therefore \text{Total Voters} = 16800$$

Alternate Method:

Let total number of voters enrolled be x .

Now, according to problem,

Total voters who casted their votes = $x \times 3/4$

$$\text{Valid voters} = x \times \frac{3}{4} \times \frac{49}{50}$$

$$\text{Winning candidate votes} = x \times \frac{3}{4} \times \frac{49}{50} \times \frac{3}{4}$$

$$\text{Hence, } x \times \frac{3}{4} \times \frac{49}{50} \times \frac{3}{4} = 9261$$

$$x = 9261 \times \frac{4}{3} \times \frac{4}{3} \times \frac{50}{49} \quad x = 16800$$

Example 21: Out of the total employees of an MNC, 50% of the employees are male. 20% of the females are married and 10% of them have children. What percentage of the females are without children?

Solution:

Let total employees = 100

So, males = 50 and females = 50

Married females = 20% of 50 = 10

Females having children = 10% of 10 = 1

So, females without children = $50 - 1 = 49$

$$\therefore \text{Required percentage} = \frac{49}{50} \times 100 = 98\%$$

Example 22: Fresh grapes contain 80% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 50 kg of fresh grapes?

Solution:

When fresh grapes become dry grapes, some of the water content evaporates but the quantity of pulp remains the same.



In dry grapes water is 20%, so the percentage of pulp will be $(100 - 20)\% = 80\%$.

Now, 80% of dry grapes = 10 kg

$$\text{Dry grapes} = 10 \times \frac{100}{80} = 12.5 \text{ kg}$$

Practice exercise Level 1

1. What is 20% of 30% of 300?
(A) 20 (B) 25
(C) 18 (D) None of these
2. By what percentage is 60% of 1750 more than 45% of the same number?
(A) 15% (B) 33.33%
(C) 25% (D) None of these
3. If 40% of $x = y$, then $y\%$ of 40 is same as ____ of x .
(A) 16% (B) 4%
(C) 8% (D) 160%
4. If 60% of the students in a school are boys and number of girls is 812, how many boys are there in the school?
(A) 1128 (B) 1218
(C) 1821 (D) 1281
5. The population of two cities A and B is in the ratio 5 : 6. What is the percentage of the population of city A to the population of city B?
(A) 120% (B) 83.33%
(C) 16.66% (D) 20%
6. If 30% of C is same as 45% of D, then which of the following is true?
(A) $C = 2D$ (B) $D = 2C$
(C) $C = 1.5D$ (D) $D = 1.5C$
7. When a number is increased by 120, it becomes 130% of itself. What is the number?
(A) 400 (B) 520
(C) 460 (D) 580
8. A is 20% more than B and B is 30% less than C. If A is 210, then what is the value of C?
(A) 242 (B) 250
(C) 220 (D) 235
9. Three candidates in an election received 1136, 7636 and 11628 votes respectively. What percentage of the total votes did the winning candidate get?
(A) 11.6% (B) 60%
(C) 76.4% (D) None of these
10. If volume of water increases by 20% on freezing and decreases $x\%$ when melted to water, then what is the value of x ?
(A) 20% (B) 16.66%
(C) 37.5% (D) 25%
11. A report consists of 20 sheets each of 55 lines and each line consists of 65 characters. This report is reduced on to sheets each of 65 lines such that each line consists of 70 characters. What is the percentage reduction in number of sheets (approximately)?
(A) 20 (B) 5
(C) 30 (D) 35
12. If the numerator of a fraction is increased by 20% and the denominator is diminished by 10%, then the value of the new fraction is $16/21$. What is the original fraction?
(A) $3/5$ (B) $4/7$
(C) $2/3$ (D) $5/7$
13. Mayur weighs twice as much as Shweta. Shweta's weight is 60% of Deepika's weight. Rakesh weighs 50% of Vikas's weight. Vikas weighs 190% of Mayur's weight. Which of these people weighs the least?
(A) Mayur (B) Deepika
(C) Shweta (D) Rakesh
14. If I have to pay 4% sales tax in addition to the price of a certain article, then what is the price of the article if I have to pay Rs. 260 in all?
(A) Rs. 300 (B) Rs. 350
(C) Rs. 250 (D) Rs. 200
15. A man spends 80% of his income and saves the rest. When the cost of living increased, expenses increased by 37.5% and his income increased by 16.66%. What percentage of his income does he now save?
(A) 6.8% (B) 7.3%

- (C) 8.4% (D) 5.7%
16. In a family of four members, the first one spends 42% of the family income, the second 35% of the remaining, the third 21% of what remains and the fourth spends 21% of the whole income of the family. What percentage is saved?
(A) 9.6% (B) 8.7%
(C) 7.2% (D) 6.9%
17. The annual population of a country increases every year by 20%. What will be population in 2002, if the population in 2000 was 1,00,000?
(A) 1,44,000 (B) 1,20,000
(C) 1,40,000 (D) None of these
18. A person is addicted to surfing the Internet. If cost of surfing per hour is increased by 40%, then by how much percent should the person decrease his daily surfing hours so that his total expenditure on surfing remains same?
(A) 28.56% decrease (B) 30% decrease
(C) 50% decrease (D) 60% decrease
19. Because of scarcity of rainfall, the price of a land decreases by 12% and its production also decreases by 4%. What is the total effect on revenue?
(A) Loss of 16% (B) Loss of 15.48%
(C) Gain of 15.48% (D) Loss of 15.52%
20. If each edge of a cube is increased by 30%, then what is the percentage increase in the volume of cube?
(A) 100% (B) 119.7%
(C) 90% (D) None of these
21. If length, breadth and height of a cube is increased by 10%, 20% and 30% respectively, then what is the percentage increase in volume?
(A) 71.6% (B) 70%
(C) 80% (D) 90%
22. The length of a rectangle is increased by 60%. By what percent should the width be decreased to maintain the same area?
(A) $37\frac{1}{2}\%$ (B) 60%
(C) 75% (D) None of these
23. If length of a rectangle is increased by 10% and breadth is increased by 15%, then what will be the percentage increase in the area of rectangle?
(A) 25.5 (B) 25
(C) 28.4 (D) 26.5
24. If the base of triangle is increased by 10% and height is decreased by 20%, then what will be the percentage change in the area of a triangle?
(A) 30 (B) 20
(C) 22 (D) 12
25. If length and breadth of a rectangle are increased by 15% and 20% respectively, then what will be the percentage increase in area?
(A) 37 (B) 40
(C) 35 (D) 38
26. Income of Suman is 1st increased by 7% and then it is decreased by 7%. What is the net change in his income?
(A) 0.49% increase (B) 0.39% decrease
(C) 0.39% increase (D) 0.49% decrease
27. Wheat is now being sold at Rs. 27 per kg. During last month its cost was Rs. 24 per kg. By how much percent a family reduces its consumption so as to keep the expenditure fixed?
(A) 10.2% (B) 12.1%
(C) 12.3% (D) 11.1%
28. In a medical certificate, by mistake a candidate gave his height as 25% more than actual. In the interview panel, he clarified that his height was 5 ft. What should the percentage correction

made by the candidate from his stated height to his actual height?

- (A) 28.56 (B) 20
(C) 25 (D) 16.66

29. A student got 45% marks in an exam and failed by 10 marks. If the maximum marks in the exam is 200, then what is the minimum percentage marks required for passing the exam?

- (A) 48.5% (B) 50%
(C) 55% (D) 45%

30. An MBA student gets a monthly fellowship from which he spends 70% on personal expenses and 20% on books, fees, etc. The remaining amount is saved and it amounts to Rs. 4800 in a year. What is the value of the monthly fellowship?

- (A) 3000 (B) 3500
(C) 5000 (D) 4000

31. In a municipal election, there were two candidates. One who got 30% of the votes polled was defeated by 16000 votes. What is the total number of votes polled?

- (A) 24000 (B) 28000
(C) 30000 (D) 40000

32. At a school, 20% of the students are seniors. If all of the seniors attended the school play, and 60% of all the students attended the play, then what percent of the non-seniors attended the play?

- (A) 20% (B) 40%
(C) 50% (D) 100%

33. One litre of water is evaporated from 6 litres of a solution containing 5% salt. What is the percentage of salt in the remaining solution?

- (A) 8% (B) 10%
(C) 6% (D) 4%

34. In a co-educational school there are 15 more girls than boys. If the number of girls is increased by 10% and the number of boys is

also increased by 16%, there would be nine more girls than boys. What is the number of students in the school?

- (A) 140 (B) 125
(C) 265 (D) 255

35. The price of table depreciates every year by 20%. If the value of the table after 2 years will be Rs. 32000, then what is the present price of the table?

- (A) Rs. 48000 (B) Rs. 44000
(C) Rs. 50000 (D) Rs. 51000

Practice exercise Level 2

1. When 1 kg is kept on an electronic weighing scale, the reading is 950. What is the percentage of error in the measurement?

- (A) 5.26% (B) 3%
(C) 4.8% (D) 5%

2. If 25% of a number is less than 18% of 650 by 19, then what is the number?

- (A) 380.8 (B) 392
(C) 450 (D) 544

3. If the price of pen decreases by 20%, then a man can buy 10 more pens for Rs. 100. What is the new price of each pen?

- (A) Rs. 1 (B) Rs. 2
(C) Rs. 4 (D) Rs. 5

4. A's salary is 80% of B's salary. If A's salary is increased by 20% and B's salary is decreased by 30%, then A's salary is how much percentage of B salary?

- (A) 120% (B) 135%
(C) 137.14% (D) None of these

5. There is 26% iron in a mixture then how much mixture is needed to find 91 kg iron?

- (A) 450 kg (B) 240 kg
(C) 250 kg (D) 350 kg

6. If a bucket is 80% full, then it contains 2 litres more water than when it is $66\frac{2}{3}\%$ full. What is the capacity of the bucket?
 (A) 10 l (B) 15 l
 (C) $16\frac{2}{3}$ l (D) 20 l
7. A shopkeeper has certain number of eggs of which 5% are found to be broken. He sells 93% of the remainder and still has 266 eggs left. How many eggs did he originally have?
 (A) 3800 (B) 4000
 (C) 4200 (D) None of these
8. Three persons Mohit, Jayesh and Nandu whose salary together amounts to Rs. 54000 spend 60%, 70% and 50% of their salaries respectively. Their savings are in the ratio of 8 : 15 : 10 respectively. What is the salary of Jayash?
 (A) Rs. 20000 (B) Rs. 28000
 (C) Rs. 12000 (D) Rs. 30000
9. What is the percentage change in the curved surface area of a cylinder when the radius is increased by 20% and the height becomes three-fourth of its original value?
 (A) 5% decrease (B) 25% decrease
 (C) 10% decrease (D) No change
10. Area of a square is increased by 96% when side of square is increased by x%. What is the value of x?
 (A) 40% (B) 50%
 (C) 60% (D) 70%
11. If price of mangoes is increased by 20%, a person now can buy 5 mangoes less for Rs. 60. What is the original price of a mango?
 (A) Rs. 4 (B) Rs. 3
 (C) Rs. 2 (D) Rs. 2.4
12. When the price of sugar was increased by 32%, a family reduced its consumption in such a way that the expenditure on sugar was only 10% more than before. If 30 kg per month were consumed before, then what is the new monthly consumption?
 (A) 42 kg (B) 35 kg
 (C) 25 kg (D) 16 kg
13. If the population of a town at the beginning of a year was 15,30,000, and the birth rate was 53.2, while the death rate was 32.2 per 1000 of the population, then what is the net increase in the population at the end of the year?
 (A) 33130 (B) 32310
 (C) 32230 (D) 32130
14. "A" made 3 mistakes in an examination (no negative marking) where all questions carry equal marks, and obtained 72%. If he had attempted 6 more questions and made 4 wrongs, he would have had 82%. How many questions were there in the exam?
 (A) 36 (B) 48
 (C) 50 (D) Can't say
15. The rate of increase of the price of sugar is observed to be two percent more than the inflation rate expressed in percentage. The price of sugar, on January 1, 1994 is Rs. 20 per kg. The inflation rates of the years 1994 and 1995 are expected to be 8% each. What would be the expected price of sugar on January 1, 1996?
 (A) Rs. 23.60 (B) Rs. 24.00
 (C) Rs. 24.20 (D) Rs. 24.60
16. Forty percent of the employees of a certain company are men and 75% of the men earn more than Rs. 25,000 per year. If 45% of the company's employees earn more than Rs. 25,000 per year, then what fraction of the women employed by the company earn Rs. 25,000 or less per year?
 (A) $\frac{2}{11}$ (B) $\frac{1}{4}$
 (C) $\frac{1}{3}$ (D) $\frac{3}{4}$

17. In an election between two candidates, the candidate who got 60% of valid votes won by a majority of 200 votes. If out of total votes polled, 80% votes are valid, then what is the total number of votes polled?
- (A) 1000 (B) 1250
(C) 1500 (D) 1300

Directions (18-19) Refer to the following information to answer the questions that follow.

In an examination, a candidate who secures 25% of the maximum marks fails by 60 marks, but another candidate who secures 42% of the maximum marks gets 8 marks more than that necessary to pass.

18. What is the maximum marks?
- (A) 400 (B) 600
(C) 1000 (D) 420
19. What is The pass marks?
- (A) 660 (B) 160
(C) 400 (D) 260
20. There are two candidates Bhiku and Mhatre for an election. Bhiku gets 65% of the total valid votes. If the total votes were 6000, then what is the number of valid votes that the other candidate Mhatre gets if 25% of the total votes were declared invalid?
- (A) 1575 (B) 1625
(C) 1675 (D) 1525
21. 8% of the voters in an election did not cast their votes. In the election, there were only two candidates. The winner, by obtaining 48% of the total votes in the voters list defeated his contestant by 1100 votes. What was the total number of voters in the election?
- (A) 21000 (B) 22000
(C) 23500 (D) 27500
22. Sohan spends 23% of an amount of money on an insurance policy, 33% on food; 19% on children's education and 16% on recreation. He

deposits the remaining amount of Rs. 504 in bank. How much total amount does he spend on food and insurance policy together?

- (A) 3200 (B) 3126
(C) 3136 (D) 3048
23. A clothing supplier stores 800 coats in a warehouse, of which 15 percent are full-length-coats. If 500 of the shorter length coats are removed from the warehouse, then what percent of the remaining coats are full-length?
- (A) 5.62% (B) 34%
(C) 40% (D) 48%
24. A person spends 40% of his salary on his educational expenses. He spends 60% of it in purchasing books and one-half of the remaining in purchasing stationary items. If he saves Rs. 160 every month, which is one-fourth of the balance after spending over books and stationary items, then what is his monthly salary?
- (A) Rs. 8000 (B) Rs. 4800
(C) Rs. 9600 (D) Data inadequate
25. In a certain shop, there are stocks of four types of caps, there are one-third as many red caps as blue and half as many green caps as red caps. There are equal numbers of green caps and yellow caps. If there are 42 blue caps, then what percent of the total caps in the shop are blue?
- (A) 70% (B) 28%
(C) 60% (D) 14%
26. In a company, there are 75% skilled workers and remaining ones are unskilled. 80% of skilled workers and 20% of unskilled workers are permanent. If number of temporary workers is 126, then what is the number of total workers?
- (A) 480 (B) 510
(C) 360 (D) 377

27. At the end of year 1998, Shepherd bought nine dozen goats. Hence forth, every year he added $p\%$ of the goats at the beginning of the year and sold $q\%$ of the goats at the end of the year where $p > 0$ and $q > 0$. If Shepherd had nine dozen goats at the end of year 2002, after making the sales for that year, which of the following is true?
- (A) $p = q$ (B) $p < q$
 (C) $p > q$ (D) $p = \frac{q}{2}$
28. A company received two shipments of ball bearings. In the first shipment, 1% of the ball bearings were defective. In the second shipment, which was twice as large as the first, 4.5% of the ball bearings were defective. If the company received a total of 100 defective ball bearings, then how many ball bearings were in the first shipment?
- (A) 2000 (B) 1000
 (C) 900 (D) 3000
29. In an examination, 40% marks are required to pass. A obtains 10% less than the number of marks required to pass. B obtains $11\frac{1}{9}\%$ less than A and C obtained $41\frac{3}{17}\%$ less than the number of marks obtained by A and B together. What percent marks did C get?
- (A) 50 (B) 40
 (C) 35 (D) 45
30. The number of votes not cast for the Praja Party increased by 25% in the National General Elections over those not cast for it in the previous Assembly polls, and the Praja party lost by a majority twice as large as that by which it had won the Assembly Polls. If a total of 2,60,000 people voted each time, then how many voted for the Praja Party in the previous Assembly Polls?
- (A) 110000 (B) 150000
 (C) 140000 (D) 120000
31. A man buys a house for Rs. 10000 and rents it. He puts 12.5% of each month's rent aside for upkeep and repairs, pays Rs. 325 per year as taxes and realizes 5.5% on his investment. What is the monthly rent?
- (A) 50 (B) 150
 (C) 83.33 (D) 100.33
32. One-fourth of the subscribers to a certain fund gave Rs. 20 each, one-sixth of the remainder gave Rs. 10 each, and the rest gave Rs. 1 each. If the three sets of subscribers raised their subscriptions by Rs. 2 each, the total increase in the subscriptions would be Rs. 560. How many subscribers were there?
- (A) 340 (B) 280
 (C) 360 (D) 260
33. In Hyderabad city, legal water connections form 80% of the total connections. If 90% of the houses in the city have water connections, then what percentage of the houses in the city have illegal water connections?
- (A) 28 (B) 18
 (C) 10 (D) 20
34. In the student's union election, 10% of the voters on the voter's list did not cast their votes and 60 voters cast their ballot paper blank. There were only two contestants. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. How many voters were there on the list?
- (A) 8650 (B) 7865
 (C) 6895 (D) 6200

Solution

Practice Exercise Level 1

1.(C) Required value = $0.2 \times 0.3 \times 300$

$$\Rightarrow 0.2 \times 90 = 18$$

2.(B) As the base value is the same for both, we need not find the actual values.

So, difference $60\% - 45\% = 15\%$

$$\frac{15}{45} \times 100 = 33.33\%$$

3.(A) Given, $\frac{40}{100}x = y$,

$$\frac{y}{100} \times 40 = \frac{40x}{100} \times \frac{40}{100}$$

$$\frac{40}{100} \times y = \frac{16}{100}x$$

= 16% of x

4.(B) Percentage of girls

$$= 100 - 60 = 40\%$$

Total number of students

$$= \frac{812}{40} \times 100 = 2030$$

Hence, number of boys

$$= \frac{60}{100} \times 2030 = 1218$$

5.(B) Required percentage

$$= \frac{5}{6} \times 100 = 83.33\%$$

6.(C) Given that:

$$30\% \text{ of } C = 45\% \text{ of } D$$

$$\text{Therefore, } 2C = 3D \text{ or } C = 1.5D$$

7.(A) 30% of the number is 120

$$\text{The number} = \frac{120}{30} \times 100 = 400$$

8.(B) A = 210

$$B = \frac{210}{120} \times 100 = 175$$

$$C = \frac{175}{70} \times 100 = 250$$

9.(D) Total Votes = $(1136 + 7636 + 11628)$

$$= 20400.$$

Required percentage

$$= \left(\frac{11628}{20400} \times 100 \right) \% = 57\%$$

10.(B) Let the volume of water be 1.

$$\text{Volume after freezing} = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\therefore \% \text{ decrease} = \frac{\frac{6}{5} - 1}{\frac{6}{5}} \times 100\%$$

$$= \frac{1}{6} \times 100 = 16.66\%$$

11.(A) Total number of characters = $20 \times 55 \times 65$
= 71500

Number of pages required if the report is retyped

$$= \frac{71500}{65 \times 70} = 15.70$$

Hence, 16 pages are required.

Hence, percentage reduction

$$= \frac{20 - 16}{20} \times 100 = 20\%$$

12.(B) Let the fraction be $\frac{p}{q}$.

$$\therefore \frac{p + 20\% \text{ of } p}{q - 10\% \text{ of } q} = \frac{16}{21}$$

$$\Rightarrow \frac{p + \frac{p}{5}}{q - \frac{q}{10}} = \frac{16}{21}$$

$$\Rightarrow \frac{6p}{5} \times \frac{10}{9q} = \frac{16}{21}$$

$$\Rightarrow \frac{p}{q} = \frac{16}{21} \times \frac{9}{12} = \frac{144}{252} = \frac{4}{7}$$

Increased numerator = 1.2N

Decreased denominator = 0.9D

$$\therefore \frac{1.2N}{0.9D} = \frac{16}{21}$$

$$\Rightarrow \frac{12N}{9D} = \frac{16}{21}$$

$$\Rightarrow \frac{4N}{3D} = \frac{16}{21}$$

$$\Rightarrow \frac{N}{D} = \frac{4}{7}$$

- 13.(C)** Mayur = 2(Shweta)
 Shweta = 60% of Deepika = $\frac{3}{5}$ of Deepika
 Rakesh = 50% of Vikas = $\frac{1}{2}$ of Vikas
 Vikas = 190% of Mayur = $\frac{19}{10}$ of Mayur
 $\therefore M > D, S < D, R < V, V > M$
 $\Rightarrow V > M > D > S, V > R$
 Vikas = $\frac{19}{10}$ of Mayur
 $\Rightarrow$ Mayur = $\frac{20}{19}$ of Rakesh
 Shweta = $\frac{10}{19}$ of Rakesh
 $\Rightarrow S < R$

- 14.(C)** Let the price of the article be x.
 $x + 4\% \times x = 260$

$$\Rightarrow x + \frac{4}{100}x = 260$$

$$\Rightarrow 104x = 260 \times 100$$

$$\Rightarrow x = \frac{260 \times 100}{104}$$

$$\Rightarrow x = \frac{260 \times 25}{26}$$

$$\Rightarrow x = 250$$

- 15.(D)** Let the income be x.
 Increased income
 $= x + 16.66\% \times x$
 $= x \left(1 + \frac{1}{6} \right)$
 $\therefore$ Increased income = $\frac{7}{6}x$

$$\text{Expenses} = \frac{8}{10}x$$

Increased expenses

$$= \frac{8x}{10} + \frac{3}{8} \times \frac{8x}{10}$$

Increased expenses

$$= \frac{8x}{10} + \frac{3x}{10} = \frac{11x}{10}$$

$$\therefore \text{Savings} = \frac{7x}{6} - \frac{11x}{10}$$

$$= \frac{35x - 33x}{30} = \frac{2x}{30} = \frac{x}{15}$$

$$\therefore \% \text{ saving} = \frac{x/15}{7x/6} \times 100\%$$

$$= \frac{6}{7 \times 15} \times 100\%$$

$$= \frac{2}{7 \times 5} \times 100\%$$

$$= \frac{2 \times 20}{7} \% = 5.7\%$$

- 16.(B)** Let the whole income of family be x.

$\therefore$ Income saved

$$= x \times 0.58 \times 0.65 \times 0.79 - 0.21x$$

$$= 0.297 - 0.21$$

$$= 0.087$$

$$\therefore \% \text{ income saved} = 0.087 \times 100 = 8.7\%$$

- 17.(A)** Population in 2002

$$= 100000 \left(1 + \frac{20}{100} \right)^2 \Rightarrow 1,44,000$$

- 18.(A)** $40\% = \frac{2}{5}$

Here, 5 is the initial cost and 2 is the increment in the cost, so new cost is 7.

As we know,

Expenditure = Cost of surfing $\times$ Surfing hours

	Cost	Surfing hours	Exp.
Initial	5	7	$5 \times 7 = 35$
Final	7	5	$7 \times 5 = 35$

$$\text{Required percentage} = \frac{2}{7} \times 100$$

$$= 28.56\% \text{ decrease}$$

- 19.(D)** Revenue

= Price $\times$ Production

Change in revenue will be given by

$$-12 - 4 + \frac{-12 \times -4}{100}$$

$$= -16 + 0.48 = -15.52$$

So, the revenue will decrease by 15.52%.

- 20.(B)** **Method 1:**

Increase in each edge of cube

$$= 30\%$$

$$\text{Volume} = \text{Edge} \times \text{Edge} \times \text{Edge}$$

So, net change

$$= 30 + 30 + \frac{30 \times 30}{100}$$

$$= 30 + 30 + \frac{900}{100} = 69\%$$

$$\text{Again, } 69 + 30 + \frac{2070}{100} = 119.7\%$$

Method 2:

$$\text{Volume} = \text{Edge} \times \text{Edge} \times \text{Edge} = a$$

$$\text{Net change} = a \times 1.3 \times 1.3 \times 1.3$$

$$\Rightarrow a \times 2.197$$

So, volume increased by 119.7%.

21.(A) $1.1 \times 1.2 \times 1.3 = 1.716$

Hence, percentage increase
= 71.6%

22.(A) By applying the formula of successive percentage change, we get

$$\text{Net change} = a + b + \frac{ab}{100}$$

$$0 = 60 - b - \frac{(60 \times b)}{100}$$

$$0 = 600 - 10b - 6b$$

$$16b = 600$$

$$\text{Hence, } b = \frac{600}{16} = 37.5\%$$

23.(D) Percentage change in area

$$= l + b + \frac{lb}{100}$$

$$= 10 + 15 + \frac{10 \times 15}{100}$$

$$= 10 + 15 + 1.5$$

$$= 26.5$$

24.(D) $A = \frac{1}{2} \times 1.1b \times 0.8h$

$$= \frac{1}{2}bh \times (0.88)$$

So, area is decreased by 12%.

Net change in Area

$$= +10 - 20 - \frac{10 \times 20}{100} = -12\%$$

Hence, the area decreases by 12%.

25.(D) $A = l(1.15) \times b(1.20)$

$$A = 1.38 lb$$

So, percentage increase in area is 38%.

Net percentage change

$$= 15 + 20 + \frac{15 \times 20}{100} = 38\%$$

Hence, the area increases by 38%.

26.(D) In such case, there is always a decrease.

Given that, a = (common increase or decrease) = 7%

According to the formula,

$$\text{Net change} = a - a - \frac{a^2}{100}$$

Decreased percentage

$$= \frac{a^2}{100} \% = \frac{7^2}{100} \% = 0.49\%$$

27.(D) Price has changed from 24 to 27. So, consumption should change from 27 to 24 to keep the expenditure fixed.

Required percentage

$$= \frac{3}{27} \times 100 = \frac{1}{9} \times 100 = 11.11\%$$

28.(B) Required percentage

$$\text{correction} = \frac{1.25 - 1}{1.25} \times 100 = 20\%$$

Note: This question can be solved, even if his height is not given because there is no need of his height.

29.(B) Marks obtained = 45% of 200 = 90

As the student failed by 10 marks, the marks required for passing will be 100 marks.

Required percentage

$$= \frac{100}{200} \times 100 = 50\%$$

30.(D) Suppose monthly fellowship = Rs. 100

Amount spent on monthly expenses = Rs. 70

Amount spent on books = Rs. 20

$\therefore$ Monthly amount saved

$$= \text{Rs. } 100 - (70 + 20)$$

$$= \text{Rs. } 10$$

Actual yearly savings = Rs. 4800

Actual monthly savings

$$= \frac{4800}{12} = \text{Rs. } 400$$

If the amount saved is Rs. 10, monthly fellowship = Rs. 100

∴ If the amount saved is Rs. 400 monthly fellowship

$$= \frac{100}{10} \times 400 = \text{Rs. } 4000$$

Let the yearly fellowship be x.

Total expenses = 90% of x

Savings = 100% of x – 90% of x = 10% of x

$$\therefore 10\% \text{ of } x = 4800$$

$$x = 48000$$

∴ Monthly fellowship

$$= \frac{48000}{12} = 4000$$

31.(D) Let the total number of votes polled = x

∴ The winning candidate got 70% of the votes polled.

Hence, 40% of x = 16000

$$\Rightarrow x = 40000$$

32.(C) Suppose total number of students = 100

Seniors = 20 and Non Seniors = 80

∴ Number of seniors who attended the play = 20

Total number of students who attended the play = 60

∴ Number of non-seniors who attended the play = 60 – 20 = 40

∴ Required percentage

$$= \frac{40}{80} \times 100 = 50\%$$

33.(C) Quantity of salt in 6 litres of solution

$$= \frac{5}{100} \times 6 = 0.3 \text{ litre}$$

Percentage of salt in 5 litres of solution

$$= \frac{0.3}{5} \times 100 = 6\%$$

34.(C) $G = B + 15$ (1)

$G + 10\% \text{ of } G = B + 16\% \text{ of } B + 9$

$$\text{i.e. } \frac{11G}{10} = \frac{116B}{100} + 9$$

$$\text{i.e., } 110G - 116B = 900 \quad \dots (2)$$

Using (2) in (1), we get

$$B = 125, G = 140$$

∴ Total number of students

$$= B + G = 265$$

35.(C) Let the present price be x.

$$\text{Then, } x = \frac{32000}{64} \times 100 = \text{Rs. } 50000$$

Practice Exercise Level 2

1.(D) The percentage error

$$= \frac{1000 - 950}{1000} \times 100 = 5\%$$

2.(B) Let the number is x.

$$\Rightarrow (0.18 \times 650) - (0.25 \times x) = 19$$

$$\Rightarrow \frac{x}{4} = (117 - 19)$$

$$\Rightarrow x = (98 \times 4) = 392$$

3.(B) New price of each pen

$$= \frac{20\% \text{ of } 100}{10} = \frac{20}{10} = \text{Rs. } 2$$

4.(C) Salary of B = 100

Salary of A = 80

$$A's \text{ new salary} = 80 \times 1.2 = 96$$

$$B's \text{ new salary} = 100 \times 0.7 = 70$$

Required percentage

$$= \frac{96}{70} \times 100 = \frac{960}{7} = 137.14\%$$

5.(D) Let required value of mixture be x kg.

According to the questions,

$$26\% \text{ of } x = 91$$

$$\Rightarrow x = \frac{91 \times 100}{26} = 350 \text{ kg}$$

6.(B) Let the capacity of bucket be x litre.

$$\text{Then, } 0.80x = 0.667x + 2$$

$$\Rightarrow x = \frac{2}{0.133} = 15 \text{ litres}$$

7.(B) Suppose the shopkeeper had x eggs, originally.

Number of broken eggs

$$= 5\% \text{ of } x = \frac{x}{20}$$

$$\text{Balance} = x - \frac{x}{20} = \frac{19x}{20}$$

$$7\% \text{ of } \frac{19x}{20} = 266 \Rightarrow x = 4000$$

- 8.(D)** Expenditure of Mohit, Jayesh and Nandu are $60\%x$, $70\%y$ and $50\%z$.

$\therefore$ Savings of Mohit, Jayesh and Nandu are 40% , 30% and 50% .

i.e. 40% of $x = 8$, 30% of $y = 15$,

50% of $z = 10$

$\therefore x = 20, y = 50, z = 20$

$\therefore$ Ratio of incomes = $20 : 50 : 20$

Let their incomes be $20a$, $50a$ and $20a$ respectively.

Then, $20a + 50a + 20a = 54000$

$a = 600$

Jayesh's salary = 600×50

= Rs. 30000

- 9.(C)** Curved surface area of a cylinder = $2\pi rh$.
Now, r is increased by 20% while the height is decreased by 25% , Overall change

$$20 - 25 - \frac{500}{100} = -10\%$$

Hence, 10% decrease.

- 10.(A)** Assume percentage increase in side of square is $x\%$ by applying successive percentage formula

$$\Rightarrow x + x + \frac{x^2}{100} = 96$$

Hence, $x = 40\%$

- 11.(C)** Assume the initial price of a mango be x .

$$\frac{60}{x} - \frac{60}{1.2x} = 5 \Rightarrow x = 2$$

Hence, original price of a mango = Rs. 2

- 12.(C)** Let the price of sugar be Rs. x per kg.

$\therefore$ Initial expenditure = Rs. $30x$

New expenditure = Rs. $33x$

$\therefore$ New monthly consumption

$$= \frac{33x}{1.32x} = 25 \text{ kg}$$

- 13.(D)** Birth rate per 1000 = 53.2

Death rate per 1000 = 32.2

$\therefore$ Net increase rate per 1 = 0.021

$\therefore$ Net increase in population

$$= 1530000 \times 0.021$$

$$= 32130$$

- 14.(C)** Let the number of questions attempted be x and total number of questions be y .

$$\frac{x-3}{y} = 0.72 \text{ and } \frac{x+6-4}{y} = 0.82$$

$$\Rightarrow x - 3 = 0.72y \dots (1)$$

$$x + 2 = 0.82y \dots (2)$$

On solving both equations, we get $y = 50$ and $x = 39$.

- 15.(C)** Increase in price of sugar

$$= (8 + 2) = 10\%$$

$\therefore$ Price of sugar on Jan 1, 1996

$$= 20 \times 1.1 \times 1.1 = \text{Rs. } 24.20$$

- 16.(D)** Let the number of men and women be 40 and 60 respectively.

$\therefore$ Number of men earning more than Rs.

$$25,000 = 40 \times \frac{3}{4} = 30$$

Total number of employees earning more than Rs. 25000 = 45

$\therefore$ Number of women earning more than Rs.

$$25,000$$

$$= (45 - 30) = 15$$

Now fraction of the women earning Rs. 25000 or

$$\text{less} = \frac{60-15}{60} = \frac{45}{60} = \frac{3}{4}$$

- 17.(B)** Assume total votes polled

$$= x$$

$$\text{Valid votes} = 0.8x$$

$$\text{Votes polled to winning candidate} = 0.8x \times 0.6 = 0.48x$$

$$\text{Votes polled to losing candidate}$$

$$= 0.8x - 0.48x = 0.32x$$

$$\text{Hence, } 0.48 - 0.32x = 200$$

$$\Rightarrow 0.16x = 200$$

$$x = \frac{200}{0.16} \times 100 = 50 \times 25 = 1250$$

Ans.(18-19) Let maximum marks = N

Passing marks for first candidate

$$= 25\% \text{ of } N + 60$$

Passing marks for second candidate

$$= 42\% \text{ of } N - 8$$

$$\text{So, } 25\% \text{ of } N + 60 = 42\% \text{ of } N - 8$$

$$17\% \text{ of } N = 68$$

$$N = 400$$

So, Maximum marks = 400

$$\text{Passing marks} = 25\% \text{ of } 400 + 60 = 160$$

$$\text{Pass percentage} = \frac{160}{400} \times 100 = 40\%$$

18.(A)

19.(B)

20.(A) Total number of votes = 6000

Total number of valid votes

$$= 6000 \times 0.75 = 4500$$

Total valid votes that Bhiku gets

$$= 4500 \times 0.65 = 2925$$

Total valid votes that Mhatre gets

$$= 4500 - 2925 = 1575$$

21.(D) Total votes = x

Votes polled = $0.92x$

Votes by Winner = $0.48x$

Votes by Loser = $(0.92x - 0.48x) = 0.44x$

Now, Winner - Loser = 1100

$$0.48x - 0.44x = 1100$$

$$0.04x = 1100$$

$$x = 1100 \times \frac{100}{4} = 27500$$

22.(C) Let total amount = N

Remaining amount = 9% of N ($100 - (23 + 33 + 19 + 16)$)

$$9\% \text{ of } N = 504$$

$$N = \frac{504 \times 100}{9} = 5600$$

Amount spent on Food and Insurance

$$= (23 + 33)\% \text{ of } N$$

$$= 56\% \text{ of } N$$

$$= \frac{56}{100} \times 5600 = 3136$$

$$\text{Required value} = \frac{504}{9} \times 56 = 3136$$

23.(C) 15% of 800 = 120 coats for full length. 500 shorter length coats are removed.

$\therefore$ Percentage of full length coats out of the remaining 300 coats

$$= \frac{120}{300} \times 100 = 40\%$$

24.(A) Let his monthly salary be Rs. x .

He spends Rs. $0.4x$ on educational expenses, Rs. $0.24x$ on purchasing books and Rs. $0.08x$ on purchasing stationary items.

Remaining amount

$$= 0.4x - (0.24x + 0.08x)$$

$$= \text{Rs. } 0.08x$$

$$\text{Also, } \frac{1}{4} \times 0.08x = 160$$

$$x = \frac{160 \times 4}{0.08} = 8000$$

25.(C) $R = \frac{1}{3}B$, $G = \frac{1}{2}R$, $G = Y$

Since, $B = 42$, $R = 14$, $G = 7$ and $Y = 7$,

Total number of caps = $42 + 14 + 7 + 7 = 70$

$\therefore$ Percentage of blue caps

$$= \frac{42}{70} \times 100 = 60\%$$

26.(C) Let the total number of workers be 100.

Number of skilled workers = 75% of 100 = 75

Number of unskilled workers

$$= 100 - 75 = 25$$

Number of permanent workers

$$= 80\% \text{ of } 75 + 20\% \text{ of } 25$$

$$= \frac{80}{100} \times 75 + \frac{20}{100} \times 25$$

$$= 60 + 5 = 65$$

Number of temporary workers

$$= 100 - 65 = 35$$

The number of temporary workers are 35, then total workers = 100

Number of total workers when number of temporary workers is 126.

$$\frac{100}{35} \times 126 = 360$$

- 27.(C)** The number of goats remains the same. If the percentage that is added every time is equal to the percentage that is sold, then there should be a net decrease. The same will be the case if the percentage added is less than the percentage sold.

The only way, the number of goats will remain the same is if $p > q$.

- 28.(B)** Let company received x ball bearings in first shipment and $2x$ ball bearings in 2nd shipment.

$$\therefore 1\% \text{ of } x + 4.5\% \text{ of } 2x = 100$$

$$\Rightarrow \frac{x}{100} + \frac{9x}{100} = 100$$

$$\Rightarrow x = 1000$$

- 29.(B)** Let total marks be 100.

Then, minimum marks to be passed 40.

$\therefore$ A obtained marks

$$= 40 - 40 \times \frac{10}{100} = 40 - 4 = 36$$

$\therefore$ B obtained marks

$$= 36 - \frac{100}{9} \times \frac{36}{100}$$

$$= 36 - 4 = 32 \text{ marks}$$

$\therefore$ C obtained marks

$$= (36 + 32) - (36 + 32) \times \frac{700}{17 \times 100}$$

$$= 68 - 28 = 40 \text{ marks}$$

- 30.(C)**

	Votes in Favour	Votes in Against
Last Year	$(260000 - x)$	x
Current Year	$(260000 - 1.25x)$	$1.25x$

$$(\text{Against} - \text{Favour})_{\text{Current year}}$$

$$= 2(\text{Favour} - \text{Against})$$

$$\Rightarrow 1.25x - (260000 - 1.25x)$$

$$= 2 [(260000 - x) - x]$$

By solving we get $x = 120000$

So, $(\text{Votes in Favour})_{\text{Last year}}$

$$= 260000 - 120000$$

$$= 140000$$

- 31.(C)** Let the monthly rent be x .

Amount of rent he puts aside $12.5\%x$.

$$\text{Remaining amount} = x - 12.5x$$

$$= 87.5\%x$$

$$12 \left(\frac{87.5x}{100} \right) - 325 = \frac{5.5}{100} \times 10000$$

$$\Rightarrow 12 \left(\frac{875x}{1000} \right) = 550 + 325$$

$$\Rightarrow 875x = \frac{1000}{12} \times 875$$

$$\Rightarrow x = \frac{1000}{12}$$

$\therefore$ Monthly rent is 83.33.

- 32.(B)** Let the number of subscribers be x .

$$\frac{x}{4} \text{ gave } 20, \frac{x}{8} \text{ gave } 10 \text{ and } \frac{5x}{8} \text{ gave } 1.$$

As per the question,

Total increase in subscription is 560.

$$\text{i.e. } \frac{x}{4} \times 2 + \frac{x}{8} \times 2 + \frac{5x}{8} \times 2 = 560$$

$$\Rightarrow \frac{2x}{4} + \frac{x}{4} + \frac{5x}{4} = 560$$

$$\Rightarrow 8x = 560 \times 4$$

$$\Rightarrow x = \frac{560}{2} = 280$$

$\therefore$ Number of subscribers is 280.

- 33.(B)** Legal water connections

$$= 0.8 \times 90\% = 72\%$$

Illegal water connections

$$= 90 - 72 = 18\%$$

- 34.(D)** Total Votes = $100x$

$$\text{Voters} = 0.9 \times 100x = 90x$$

$$\text{Valid Votes} = 90x - 60$$

$$\text{Winner} = 47x$$

$$\text{Loser} = 47x - 308$$

$$\text{So, } 47x + 47x - 308 = 90x - 60$$

$$\Rightarrow x = 62$$

$$\text{Votes} = 62 \times 100 = 6200$$

Simple & Compound Interest

03

If a person A" borrows some money from another person "B" for a certain period, then after the specified period, the borrower has to return the money borrowed, as well as some additional money. This additional money that the borrower has to pay is called "Interest". The money borrowed by "A" is called the "Principal". The principal and the interest together is called the "Amount". The interest that the borrower has to pay for every 100 rupees borrowed each year is known as rate percent per annum. It is denoted by "r%" per annum. The time for which the money has been borrowed is called the "Time period". It is denoted by "t" (usually in years).

There are two ways in which the interest can be calculated i.e. Simple Interest (SI) and Compound Interest (CI).

COMPARISON BETWEEN SIMPLE INTEREST AND COMPOUND INTEREST

In Simple Interest, the interest for any time period is calculated as a percentage of the initial amount invested.

In Compound Interest, the interest for any time period is calculated as a percentage of the amount outstanding at the start of the time period and not the initial investment. The amount outstanding at the start of any time period will be equal to the initial amount invested plus the interest earned so far.

So, the interest earned in earlier time periods gets added to the initial investment and this new sum is reinvested and the interest is calculated on this new sum.

Let us see the difference between the simple interest and compound interest with the help of the given table. In the table the principal at the start of the 1st

year is Rs. 1000 and the rate of interest is 10% per annum.

Year	Simple Interest			Compound Interest		
	P	SI	A	P	CI	A
1	1000	100	1100	1000	100	1100
2	1000	100	1200	1100	110	1210
3	1000	100	1300	1210	121	1331

P → Principal ; A → Amount

Important observations from the table

- The principal in the case of SI is always the same, while the principal for CI is increasing constantly by 10% every year. In CI, the interest earned each year is added to the principal at the end of that year and the resultant amount is the principal for the next year.
- The simple interest for each of the year is constant and equal to 100 which is 10% of the principal (Rs. 1000) whereas every year the compound interest earned is increasing by 10% (which is the rate of interest).
- For the first year, values of both Simple Interest and Compound Interest are the same.
- In the case of Compound Interest, Amount increases by r% each year and Compound Interest also increases by r% each year.

SIMPLE INTEREST

Simple Interest after T years = $\frac{PRT}{100}$

Where P is the principal

And R is the rate of interest per annum.

The above formula can also be written as

$$SI = \frac{PR}{100} + \frac{PR}{100} + \frac{PR}{100} + \dots T \text{ times}$$

Or SI in T years = SI in 1 year $\times$ Number of years

If the rate of interest is 6% per annum and the principal is invested for 5 years, then the SI will be 30% (6×5) of the principal invested.

As $SI = \frac{PRT}{100}$ can be re-written as

$$SI = P \times \left(\frac{RT}{100} \right) = (RT)\% \text{ of } P$$

SI at 12% for 5 years = SI at 6% for 10 years

= SI at 5% for 12 years = SI at 2% for 30 years and so on, but only if the Principal is same.

Example 1: If Rs. 750 amounts to Rs. 1000 in 5 years, then how much will it amount to in 10 years at SI?

Solution:

$$P = 750, A = 1000$$

$$SI = 1000 - 750 = 250$$

$$SI = \frac{PRT}{100}$$

$$250 = \frac{750 \times R \times 5}{100}$$

$$\text{So, } R = \frac{20}{3}\%$$

$$\text{So, amount in 10 years} = 750 + 750 \times \frac{20}{3} \times \frac{10}{100}$$

$$= 750 + 500 = \text{Rs. } 1250$$

Example 2: In how many years will a sum of money get doubled in simple interest at the rate of 5% per annum?

Solution:

Let, Principal (P) = x

So, Amount (A) = 2x

$$SI = 2x - x = x$$

And R = 5%

$$SI = \frac{PRT}{100}$$

$$x = \frac{x \times 5 \times T}{100}$$

$$\text{So, } T = \frac{100}{5} = 20 \text{ years}$$

Example 3: A sum becomes 7 times in 11 years. Then in how many years will it become 21 times of itself in the case of Simple Interest?

Solution:

Let Principal be P.

So, Amount = 7P

and Simple Interest = 6P

$$\text{Now, } SI = \frac{P \times R \times T}{100}$$

$$6P = \frac{P \times R \times 11}{100}$$

$$R = \frac{600}{11}$$

Now, the required amount is 21P.

$\therefore$ Interest required = 20P

$$\text{So, } 20P = \frac{P \times R \times T}{100}$$

$$20P = \frac{P \times \frac{600}{11} \times T}{100}$$

$$\Rightarrow T = \frac{220}{6} = 36\frac{2}{3} \text{ years}$$

Example 4: What is the simple interest to be paid on a principal of Rs.14,000 borrowed at the rate of 15% per annum after a period of 3 years?

Solution:

$$P = 14,000 ; R = 15 ; T = 3$$

$$SI = \frac{PRT}{100}$$

$$= \frac{14000 \times 15 \times 3}{100} = ₹ 6300$$

Example 5: The rate of interest per annum on a sum of money is 4% for the first 3 years, 6% for the next 2 years; 8% for the next 4 years. If the simple interest accrued is 13440, then what is the sum?

Solution:

Let the sum be x, then

$$3(4\% \text{ of } x) + 2(6\% \text{ of } x) + 4(8\% \text{ of } x) = 13440$$

$$12\% \text{ of } x + 12\% \text{ of } x + 32\% \text{ of } x = 13440$$

$$56\% \text{ of } x = 13440$$

$$\frac{56}{100} \text{ of } x = 13440$$

$$x = \frac{13440 \times 100}{56} = ₹ 24000$$

COMPOUND INTEREST

Let us consider a principal P kept at compound interest of r% per annum, so the interest earned in

the first year will be r% of P which is equal to $P \times \frac{r}{100}$.
For the second year, we have to first calculate the outstanding amount i.e. the principal plus the interest.

Thus, the amount outstanding at the start of second

$$\text{year will be } P + \frac{P \times r}{100} = P \left(1 + \frac{r}{100} \right).$$

Interest for second year will be calculated as r% of this amount and using the same result, the amount at the end of the second year will be

$$P \left(1 + \frac{r}{100} \right) \left(1 + \frac{r}{100} \right) = P \left(1 + \frac{r}{100} \right)^2$$

Similarly, amount after 3 years = $P \left(1 + \frac{r}{100} \right)^3$ and so on.

$$\text{So, amount after } n \text{ years} = P \left(1 + \frac{r}{100} \right)^n$$

Please note that this formula is for the amount and if one needs to calculate the compound interest, one must deduct principal from the amount.

i.e. Compound Interest = Amount – Principal

$$= P \left(1 + \frac{r}{100} \right)^n - P$$

$$= P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right]$$

In the above formula, we are calculating compound interest annually i.e. we have added the interest back to the principal at the end of the year and this interest also starts earning interest in succeeding years. But, if the interest in the first half of the year is added to the principal at the end of first half year, it will earn some interest in the second half of the year. This process of adding interest back to the principal every six months is called as semi-annual compounding (or half yearly).

Compounding can be done with any frequency and not just annually or semi annually. The interest earned every quarter or every 3 months can be added back to the principal and the new sum is considered as principal for the next quarter. This process of adding interest back to the principal every three months (quarter) is called as quarterly compounding.

The same formula i.e. $A = P \left(1 + \frac{r}{100} \right)^n$ can be used for any kind of compounding i.e. annual, semi-annual, quarterly, monthly etc. The thing that needs to be taken care of in the formula is 'r' which refers to the

rate of interest per annum and 'n' which refers to the number of years.

Let's say the money is invested at 12% per annum for 2 years, then for quarterly compounding, the value of 'r' to be used in the formula will become $\frac{12}{4}\% = 3\%$

and the value of 'n' to be used in the formula will become $2 \times 4 = 8$ quarters.

So, $A = P \left(1 + \frac{r}{100} \right)^n$ is for compounding annually.

Where r is the rate of interest per annum and n is the number of years.

$A = P \left(1 + \frac{r}{2 \times 100} \right)^{2n}$ is for compounding half yearly.

$A = P \left(1 + \frac{r}{4 \times 100} \right)^{4n}$ is for compounding quarterly.

If the rates of interest are different for different years, like r_1 , r_2 and r_3 for 1st, 2nd and 3rd year respectively then, amount (A) after 3 years is given as

$$A = P \left(1 + \frac{r_1}{100} \right) \left(1 + \frac{r_2}{100} \right) \left(1 + \frac{r_3}{100} \right)$$

Example 6: What will be the compound interest after 4 years on the loan of Rs. 2000 at the rate of 10% per annum?

Solution:

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$P = 2000$, $r = 10$ and $n = 4$

$$\text{So, } A = 2000 \left(1 + \frac{10}{100} \right)^4$$

$$= 2000 \times (1.1)^4$$

$$= 2000 \times 1.4641$$

$$= \text{Rs. } 2928.2$$

So, compound interest = Rs. 2928.2 – Rs. 2000

$$= \text{Rs. } 928.2$$

Example 7: A sum of money was invested in a fund at compound interest. The interest in the second year was Rs. 550 and the interest in the third year was Rs. 605. What are the rate of interest and the amount invested?

Solution:

CI for 2nd year = 550

CI for 3rd year = 605

As we know that, every year the compound interest increases by r%.

$$\text{So, } r\% = \frac{605 - 550}{550} \times 100 = 10\%$$

$$\text{CI for 1st year} = \frac{PR}{100}$$

$$\text{CI for 2nd year} = \frac{PR}{100} + R\% \left(\frac{PR}{100} \right)$$

$$= \frac{PR}{100} + \frac{PR^2}{100^2}$$

$$\text{So, } \frac{PR}{100} \left(1 + \frac{R}{100} \right) = 550$$

$$= \frac{P \times 10}{100} \left(1 + \frac{10}{100} \right) = 550$$

$$P = \frac{550 \times 100 \times 100}{110 \times 10} = 5000$$

$P = \text{Rs. } 5000$

Alternate Method:

$$\text{CI for first year} \times \frac{110}{100} = \text{CI for second year} = 550$$

$$\text{CI for first year} = \frac{550}{110} \times 100$$

$$\text{CI} = \frac{PR}{100}$$

$P = \text{Rs. } 5000$

Example 8: Rs. 24000 amounts to Rs. 41472 in 3 years at compound interest. What is the rate of interest?

Solution:

Here Principal (P) = Rs. 24000

Amount (A) = Rs. 41472

Time (T) = 3

$$\text{So, } A = P \left(1 + \frac{r}{100} \right)^n$$

$$41472 = 24000 \left(1 + \frac{r}{100} \right)^3$$

$$\frac{41472}{24000} = \left(1 + \frac{r}{100} \right)^3$$

$$\frac{1728}{1000} = \left(1 + \frac{r}{100} \right)^3$$

$$\left(\frac{12}{10} \right)^3 = \left(1 + \frac{r}{100} \right)^3$$

$$\frac{12}{10} = 1 + \frac{r}{100}$$

$$r = 20\%$$

Example 9: A sum becomes two times of itself in 7 years, then in how many years will it become 16 times of itself if it is kept at compound interest?

Solution:

Let Principal be P.

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$2P = P \left(1 + \frac{R}{100} \right)^7$$

$$2 = \left(1 + \frac{R}{100} \right)^7 \quad \dots (1)$$

Now, new Amount = 16P

$$16P = P \left(1 + \frac{R}{100} \right)^n$$

$$16 = \left(1 + \frac{R}{100} \right)^n \quad \dots (2)$$

Now, equation (1) can be rewritten as

$$\left(\left(1 + \frac{R}{100} \right)^7 \right)^4 = (2)^4$$

$$\left(1 + \frac{R}{100} \right)^{28} = 16 \quad \dots (3)$$

By comparing equations (2) and (3),

$$n = 28 \text{ years}$$

Alternate Method:

$$P \xrightarrow{7 \text{ yrs}} 2P \xrightarrow{7 \text{ yrs}} 4P \xrightarrow{7 \text{ yrs}} 8P \xrightarrow{7 \text{ yrs}} 16P$$

So, the Principal value will become 16 times of itself in 28 years (7×4).

Example 10: An amount is invested in a fund at the rate of compound interest. The amount after 1st and 3rd years are Rs. 1200 and Rs. 1587 respectively. What is the rate of interest?

Solution:

Let the amount invested be P and rate of interest be r.

$$\text{So, } 1200 = P \left(1 + \frac{r}{100} \right)^1 \quad \dots (1)$$

$$\text{and } 1587 = P \left(1 + \frac{r}{100} \right)^3 \quad \dots (2)$$

By dividing equation (2) by equation (1), we get

$$\frac{1587}{1200} = \left(1 + \frac{r}{100} \right)^2$$

$$\Rightarrow \left(1 + \frac{r}{100} \right)^2 = \frac{529}{400} \Rightarrow 1 + \frac{r}{100} = \frac{23}{20}$$

$$\frac{r}{100} = \frac{3}{20} \Rightarrow r = 15\%$$

Example 11: One bacteria splits into eight bacteria of the next generation. But due to environment, only 50% of one generation can produce the next generation. If the seventh generation number is 4096 million, then what is the number in the first generation?

Solution:

Let the number of bacteria in the first generation be x .

Only $x/2$ will split.

Second generation = $x/2 \times 8 = 4x$

Third generation = $4x/2 \times 8 = 16x$

Fourth generation = $16x/2 \times 8 = 64x$

Fifth generation = $64x/2 \times 8 = 256x$

Sixth generation = $(256x/2) \times 8 = 1024x$

Seventh generation = $(1024x/2) \times 8 = 4096x$

So, $4096x = 4096$ million

$\therefore x = 1$ million

Alternate Method:

One bacteria splits into four bacteria.

Let the first generation be x .

So, $x \times 4^6 = 4096$ million

So, $x = 1$

SAGR AND CAGR

If sales increases from x to y in n years, then there are two ways to calculate the annual growth rate of sales:

1. Simple Annual Growth Rate (SAGR)

SAGR is somewhat similar to simple interest.

Let us understand it through an example,

We are given sales of company A in the years 1990, 1991, 1992 and 1993 and we need to find the simple annual growth rate from year 1990 to 1993 in sales.

So, first of all, Growth rate in 1993 from 1990

$$= \frac{\text{Sales (1993)} - \text{Sales (1990)}}{\text{Sales (1990)}} \times 100$$

Now, what we actually need to find is SAGR,

$$\text{i.e. SAGR} = \frac{\text{Growth rate}}{\text{Number of time periods}}$$

Here, Number of time periods = [Number of years] - 1

2. Compound Annual Growth Rate (CAGR)

CAGR is similar to compound interest.

Let us consider the example discussed in SAGR, here we will find CAGR from year 1990 to 1993 in sales and consider sales in 1990 as ' x ' and sales in 1993 as ' y '. So, the principal is considered as x and the amount as y . But ' n ' will be the number of time period and not the number of years. Hence, CAGR can be calculated by using the

$$\text{formula } A = P \left(1 + \frac{r}{100} \right)^n$$

$$\text{as } y = x \left(1 + \frac{r}{100} \right)^n$$

Here, the value of r will be the CAGR.

Example 12: The turnover of ABC Corp. went up from 200 crores to 345 crores from 1996 to 1999. What will be the simple annual growth rate and compound annual growth rate.

Solution:

Here amount = Rs. 345 crores

Principal = Rs. 200 crores

Time period = $4 - 1 = 3$

Simple Annual Growth Rate (SAGR)

$$\text{Growth rate} = \frac{345 - 200}{200} \times 100 = 72.5$$

$$\text{SAGR} = \frac{72.5}{3} = 24.2\%$$

Compound Annual Growth Rate (CAGR)

$$345 = 200 \left(1 + \frac{R}{100} \right)^3$$

$$\frac{345}{200} = \left(1 + \frac{R}{100} \right)^3$$

$$1.725 = \left(1 + \frac{R}{100} \right)^3$$

If $R = 20\%$ then $(1.2)^3 = (1.728)$

So, $R = 20\%$ (approx.)

Note:

A lot of calculations are involved in finding the exact value of CAGR, as we have to find the cube root or fourth root (or may be some other root) of a value, which is not an easy task. So, to find the CAGR, first we find the value of SAGR and eliminate some options as CAGR has to be smaller than the value of SAGR after that we can use the values given in the options to find the correct answer.

Example 13: What is the present net value of property which will be valued at Rs. 3 crores at the end of 2 years? (Annual rate of increase in Value = 5%).

Solution:

$$\text{Future Value} = \text{Present Value} \left(1 + \frac{r}{100} \right)^n$$

$$\begin{aligned} \text{Present Value} &= \frac{\text{Future Value}}{\left(1 + \frac{r}{100} \right)^n} \\ &= \frac{3}{\left(1 + \frac{5}{100} \right)^2} = 3 \times \frac{100}{105} \times \frac{100}{105} = \text{` 2.72 crores} \end{aligned}$$

Note:

Sometimes, few questions regarding depreciating/appreciating the value of a property/item are asked, so it is nothing. It is just similar to Compound Interest.

RELATION BETWEEN THE VALUE OF CI AND SI FOR 2 YEARS

Let us consider an amount kept at SI and the same amount is kept at CI for two years.

Let, the simple interest for 1st year = I

So, compound interest for 1st year = I

Now, for 2nd year the Simple Interest will be the same as ' I ' but the Compound Interest will be $I + r\%$ of I , as every year the compound interest increases by $r\%$.

Year	SI	CI
1 st	I	I
2 nd	I	$I + r\% I$

$$\begin{aligned} \text{SI for first 2 years} &= 2I \\ \text{CI for first 2 years} &= \frac{2I}{2I + r\% \times I} = \frac{2}{2 + r\%} \end{aligned}$$

DIFFERENCE BETWEEN SI AND CI FOR 2 AND 3 YEARS

Year	SI	CI
1	$\frac{PR}{100}$	$\frac{PR}{100}$
2	$\frac{PR}{100}$	$\frac{PR}{100} + \frac{R}{100} \left(\frac{PR}{100} \right) = \frac{PR}{100} + \frac{PR^2}{100^2}$
3	$\frac{PR}{100}$	$\frac{PR}{100} + \frac{PR^2}{100^2} + \frac{R}{100} \left(\frac{PR}{100} + \frac{PR^2}{100^2} \right)$ $= \frac{PR}{100} + \frac{2PR^2}{100^2} + \frac{PR^3}{100^3}$

$$\text{So, } CI_{2\text{yrs}} - SI_{2\text{yrs}} = \left(\frac{2PR}{100} + \frac{PR^2}{100^2} \right) - \left(\frac{2PR}{100} \right) = \frac{PR^2}{100^2}$$

$$CI_{2\text{yrs}} - SI_{2\text{yrs}} = \frac{PR^2}{100^2}$$

$$\text{Now, } CI_{3\text{yrs}} - SI_{3\text{yrs}} = \left[\frac{3PR}{100} + \frac{3PR^2}{100^2} + \frac{PR^3}{100^3} \right] - \frac{3PR}{100}$$

$$\therefore CI_{3\text{yrs}} - SI_{3\text{yrs}} = \frac{3PR^2}{100^2} + \frac{PR^3}{100^3}$$

Example 14: The difference between the SI and CI on a sum of Rs. 400000 at R% per annum for two years is Rs. 1000. What is the value of R?

Solution:

As we know,

$$CI_{2\text{yr}} - SI_{2\text{yr}} = \frac{PR^2}{100^2}$$

$$1000 = \frac{PR^2}{100^2}$$

$$1000 = \frac{400000R^2}{100 \times 100}$$

$$R^2 = 25$$

$$R = 5\%$$

Example 15: The simple interest for the first two years is Rs. 600 and the compound interest for the first two years is Rs. 660 for the same sum with the same rate of interest. Find the rate of interest and the principal?

Solution:

$$\text{As, } \frac{SI_{2\text{yrs}}}{CI_{2\text{yrs}}} = \frac{2}{2 + r\%}$$

$$\text{So, } \frac{600}{660} = \frac{2}{2 + r\%}$$

$$\frac{10}{11} = \frac{2}{2 + r\%}$$

$$20 + 10 \times r\% = 22$$

$$10 \times r\% = 2$$

$$r = \frac{2 \times 100}{10} = 20$$

Now, at 20% rate of interest, the first year interest is

$$= 300 \text{ i.e. } \left(\frac{600}{2} \right)$$

So, 20% of principal = 300

Hence, Principal = Rs. 1500

Alternate Method: 1

$$CI_{2\text{yrs}} - SI_{2\text{yrs}} = \frac{PR^2}{100^2}$$

$$660 - 600 = \left(\frac{PR}{100} \right) \frac{R}{100}$$

$$\left[\frac{PR}{100} = 300 (\text{first year interest}) \right]$$

$$60 = 300 \times \frac{R}{100}$$

$$r = 20\%$$

And 20% of Principal = 300

So, Principal = Rs. 1500

Alternate Method: 2

Year	SI	CI
1 st	300	300
2 nd	300	360

As every year CI increases by r%

$$\text{So } r\% = \frac{360 - 300}{300} \times 100 = 20\%$$

and 20% of Principal = 300

Principal = Rs. 1500

Example 16: Akash borrows Rs. 65000 at 10% per annum simple interest for 3 years and lends it at 10% per annum, compound interest for 3 years. What will be his gain after three years?

Solution:

As we had seen in the table above,

$$CI_{3 \text{ yrs}} - SI_{3 \text{ yrs}}$$

$$\left[\frac{3PR}{100} + \frac{3PR^2}{100^2} + \frac{PR^3}{100^3} \right] - \frac{3PR}{100}$$

$$\text{i.e. } \frac{3PR^2}{100^2} + \frac{PR^3}{100^3}$$

Now, we know $P = 65000$; $R = 10\%$;

Then, gain will be the difference between SI and CI for 3 years.

$$\text{So, Gain} = \frac{3 \times 65000 \times 100}{10000} + \frac{65000 \times 1000}{1000000}$$

$$= 1950 + 65$$

$$= \text{Rs. 2015}$$

Practice exercise Level 1

- In what time will the simple interest on Rs. 780 at 5% per annum be equal to the simple interest on Rs. 600 at $6\frac{1}{2}\%$ per annum?

(A) 2 years (B) $3\frac{1}{2}$ years
(C) 5 years (D) Always equal
- An amount of Rs. 200 becomes Rs. 221 in three years at SI. What is the rate of interest?

(A) 10% (B) 3.5%
(C) 14% (D) 7%
- Rs. 2000 amount to Rs. 2600 in 5 years at simple interest. If the interest rate were increased by 3%, then it would have been amount to how much?

(A) Rs. 2900 (B) Rs. 3200
(C) Rs. 3600 (D) None of these
- If the annual rate of simple interest increases from 10% to $12\frac{1}{2}\%$, a man's annual income increases by Rs. 1250. What is the principle?

(A) Rs. 45000 (B) Rs. 50000
(C) Rs. 60000 (D) Rs. 65000
- On a certain sum, the simple interest at the end of $12\frac{1}{2}$ years becomes $\frac{3}{4}$ of the sum. What is the rate of interest per annum?

(A) 4% (B) 5%
(C) 6% (D) 8%
- At what rate of simple interest, a certain sum will be doubled in 15 years?

(A) $5\frac{1}{2}\%$ p.a. (B) 6% p.a.
- (C) $6\frac{2}{3}\%$ p.a. (D) $7\frac{1}{2}\%$ p.a.

A certain sum of money triples itself in 5 years at simple interest. In how many years will it be five times of itself?

(A) 5 (B) 8
(C) 10 (D) 15
- In 4 years, the simple interest on a certain sum of money is $\frac{7}{25}$ of the principal. What is the rate of interest per annum?

(A) 4% (B) 4.5%
(C) 7% (D) 9%
- Three-fourths of a sum is lent at 12% per annum and the rest is lent at 24% per annum, both at simple interest. If the interest earned on the smaller sum for two years is Rs. 4800, then what is the interest earned on the larger sum for three years?

(A) Rs. 14400 (B) Rs. 10800
(C) Rs. 7200 (D) Rs. 18000
- The amount received at 10% per annum compound interest after 3 years is Rs. 5324. What was the principal?

(A) Rs. 4100 (B) Rs. 4200
(C) Rs. 4000 (D) Rs. 4300
- Rajeev invested Rs. 4500 for 2 years at compound interest in a company which paid him interest of Rs. 1980. What will be the rate of interest at which Rajeev invested his money?

(A) 10% (B) 15%
(C) 20% (D) 18%
- Compound interest compounded annually on a certain sum of money 'P' is 0.21 P after a

- period of 2 years. What is the rate of interest per annum?
- (A) 20% (B) 10%
(C) 15% (D) 7.5%
13. If a certain sum becomes 4 times in 4 years at compound interest, then in how many years, it will become 64 times?
- (A) 5 (B) 12
(C) 16 (D) 24
14. What is the compound interest on Rs. 5000 at 16% per annum for one year compounded half-yearly?
- (A) Rs. 532 (B) Rs. 738
(C) Rs. 832 (D) Rs. 621
15. The amount of Rs. 10000 after 2 years, when compounded annually with the rate of interest being 10% per annum during the first year and 12% per annum during the second year, would be:
- (A) Rs. 11320 (B) Rs. 12000
(C) Rs. 12320 (D) Rs. 12500
16. An amount was lent for two years at the rate of 20% per annum compounding annually. Had the compounding been done half yearly, the interest would have increased by 241. What was the amount lent?
- (A) Rs. 10000 (B) Rs. 12000
(C) Rs. 20000 (D) Rs. 24000
17. A certain sum amounts to Rs. 1008 in 2 years and to Rs. 1112 in 3 years. What is the rate of interest if compounded annually?
- (A) 9.5% (B) 10.3%
(C) 13% (D) 15.2%
18. If interest is compounded half-yearly and quarterly, then what is the difference between compound interest on Rs. 800 for one year at 20% per annum?
- (A) Nil (B) Rs. 2.50
(C) Rs. 4.40 (D) Rs. 6.60
19. Rs. 8000 amounts to Rs. 10648 in three years at compound interest. What is the rate of interest?
- (A) 8% (B) 9%
(C) 10% (D) 12%
20. If the rate of interest is 16% per annum payable half-yearly, then what is the rate of interest that one would get for the entire year?
- (A) 16.64% (B) 16%
(C) 8% (D) 8.64%
21. The CI in the ninth year is Rs. 990 and the CI in the tenth year is Rs. 1080. What is the rate of interest per annum?
- (A) 11.11% (B) 10%
(C) 12.5% (D) 9.09%
22. What is the effective annual rate of interest corresponding to a nominal rate of 8% per annum payable half yearly?
- (A) 8% (B) 8.01%
(C) 8.13% (D) 8.16%
23. If a sum of money at a certain rate of compound interest doubles in 11 years. In how many years will the same sum be 8 times?
- (A) 15 years (B) 22 years
(C) 28 years (D) 33 years
24. In how many years will a sum of money, when invested at 25% per annum compounded annually, becomes approximately one and half times itself?
- (A) 1 (B) 2
(C) 3 (D) 4
25. The simple and compound interest that can be earned in two years at the same rate is Rs. 1500 and Rs. 1575 respectively. What is the rate of interest?
- (A) 8% per annum (B) 10% per annum
(C) 12% per annum (D) 5% per annum
26. The difference between the SI and the CI on a certain sum of money at 6% per annum at the end of two years is Rs. 1296. What is the principal.
- (A) Rs. 36000 (B) Rs. 3600
(C) Rs. 7200 (D) Rs. 360000
27. The SI for five years is Rs. 2400 while the CI for two years is Rs. 1000. What is the rate of interest per annum?
- (A) 12% (B) 9.83%
(C) 8.33% (D) Can't say
28. The difference between the SI and CI on a sum of Rs. 400000 at R% per annum for two years is Rs. 1000. What is the value of R?
- (A) 10% (B) 8%
(C) 5% (D) 12%
29. What will be the difference between compound interest and simple interest for 4

years on a principal of Rs. 12000 at the rate of 20% per annum?

- (A) Rs. 3324.8 (B) Rs. 2818.4
(C) Rs. 3576.6 (D) Rs. 3283.2

30. Amit borrowed a certain sum of money for 2 years at 8% per annum on simple interest and immediately lent it to Ravi but on compound interest at the same rate and gained Rs. 16. What amount did Amit borrow?

- (A) Rs. 1600 (B) Rs. 2500
(C) Rs. 2400 (D) Rs. 1800

31. If the simple interest on a certain sum of money for 3 years at 5% per annum is Rs. 150, then what is the corresponding compound interest?

- (A) Rs. 147.75 (B) Rs. 148.90
(C) Rs. 151.89 (D) Rs. 157.62

Practice exercise Level 2

1. The difference between simple interest received from two different banks on Rs. 500 for 2 years, is Rs. 2.50. What is the difference between their rates?

- (A) 1% (B) 0.5%
(C) 2.5% (D) 0.25%

2. If a certain sum of money becomes four times of itself in eight years at SI, in how many years will it become 28 times of itself?

- (A) 56 years (B) 50 years
(C) 72 years (D) 80 years

3. A certain sum of money becomes six times of itself in seven years at SI. How many times of itself will it become in 35 years?

- (A) 30 times (B) 25 times
(C) 26 times (D) 3 times

4. A person took a loan of Rs. 3500 from his friend and agreed to pay it back with 4% SI per annum. At the end of 4 years, he gave an item valued at Rs. 3000. What is the additional amount to be paid to be clear of the debt at the end of four years?

- (A) Rs. 1060 (B) Rs. 500
(C) Rs. 640 (D) Rs. 100

5. A man deposited Rs. 400 for 2 years, Rs. 550 for 4 years and Rs. 1200 for 6 years all at same rate of interest. He received Rs. 1020 as the

total simple interest. What is the rate of interest per annum?

- (A) 8% (B) 10%
(C) 15% (D) 20%

6. A man invests $\frac{1}{3}$ of his capital at 7% p.a., $\frac{1}{4}$ at 8% p.a. and the remaining at 10% p.a. If his annual income is Rs. 561, then what is the capital?

- (A) Rs. 5400 (B) Rs. 6000
(C) Rs. 6600 (D) Rs. 7200

7. With a given rate of simple interest, the ratio of principal and amount for a certain period of time is 4 : 5. After 3 years, with the same rate of interest, the ratio of the principal and amount becomes 5 : 7. What is the rate of interest per annum?

- (A) 4% (B) 5%
(C) 6% (D) 7%

8. Rs. 1000 is invested at 5% p.a. simple interest. If the interest is added to the principal after every 10 years, then after how much time the amount will become Rs. 2000?

- (A) 15 years (B) $16\frac{2}{3}$ years

- (C) 18 years (D) 20 years

9. Ravi gave Rs. 1200 on loan. Some amount he gave at 4% per annum simple interest and remaining at 5% per annum simple interest. After two years, he got Rs. 110 as interest. Then what are the amounts given at 4% and 5% per annum simple interest respectively?

- (A) Rs. 500, Rs. 700 (B) Rs. 400, Rs. 800
(C) Rs. 800, Rs. 300 (D) Rs. 1100, Rs. 1100

10. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is Rs. 1120, then what is the sum?

- (A) Rs. 1500 (B) Rs. 2000
(C) Rs. 2500 (D) Rs. 4000

11. Mr. Kishore deposited a total amount of Rs. 65000 in three different schemes A, B and C with rates of interest 12% per annum, 16% per annum and 18% per annum, respectively and earned a total interest of Rs. 10180 in one year. If the amount invested in Scheme A was 72% of

the amount invested in Scheme C, then what was the amount invested in Scheme B?

- (A) Rs. 25000 (B) Rs. 22000
(C) Rs. 18000 (D) Can't say

12. A man divided his share to his sons A and B in such a way that the simple interest received by A at 15% per annum for 3 years is double the simple interest received by B at 12% per annum for 5 years. At what ratio was his share divided?

- (A) $\frac{2}{3}$ (B) $\frac{8}{3}$
(C) $\frac{3}{5}$ (D) $\frac{3}{2}$

13. A certain sum of money is invested at an interest rate of 5% per annum and a second sum, twice as large as the first, is invested at 5.5% per annum. The total amount of interest earned from the two investments together is Rs. 1000 per year and the interest is withdrawn every year. What is the second sum invested?

- (A) Rs. 6250 (B) Rs. 10500
(C) Rs. 12500 (D) Rs. 15000

14. A certain sum was lent at simple interest for a period, which in terms of years is numerically equal to the rate of interest per annum. If the interest earned was equal to $\frac{9}{16}$ th part of the sum, then what is the rate of interest per annum?

- (A) 15% (B) 7.5%
(C) 22.5% (D) 21.25%

15. A sum of Rs. 47000 is divided into three parts such that the interest accrued on them at simple interest after three, four and five years respectively are equal. What is the smallest part, if the rate of simple interest is 6% p.a. in all the cases.

- (A) Rs. 12000 (B) Rs. 10000
(C) Rs. 15000 (D) Rs. 8000

16. Gita invests an amount of Rs. 15860 in the names of her three daughters A, B, and C in such a way that they get the same amount after 2, 3, and 4 years respectively. If the rate of simple interest is 5% per annum, then what is the ratio in which the amount was invested for A, B, and C.

- (A) $\frac{1}{115} : \frac{1}{110} : \frac{1}{120}$ (B) $\frac{1}{110} : \frac{1}{115} : \frac{1}{120}$

- (C) $\frac{1}{120} : \frac{1}{110} : \frac{1}{115}$ (D) $\frac{1}{120} : \frac{1}{115} : \frac{1}{110}$

17. The rate of interest on a sum of money for the first two years is 6% p.a., for the next two years it is 7% p.a. and it is 8% p.a. for the next three years; all at simple interest. If a person earns an interest of Rs. 7536 by the end of the seven years, then what is the amount at the time of investment?

- (A) Rs. 15072 (B) Rs. 11304
(C) Rs. 22608 (D) Rs. 21308

18. When the bank reduces its rate from 4.5% p.a. to 4% p.a. a man withdraws Rs. 600 from his deposit. If he now gets Rs. 39.5 less as interest, then how much the money he initially deposited?

- (A) Rs. 2100 (B) Rs. 3130
(C) Rs. 3100 (D) Rs. 2130

19. If a sum of Rs. 12000 deposited at compound interest doubles itself after five years, then what will be the amount after twenty years?

- (A) Rs. 120000 (B) Rs. 192000
(C) Rs. 124000 (D) Rs. 96000

20. A sum of amount at r% compound interest doubles in 3 years. In 9 years it will be k times of the original principal. What is the value of k?

- (A) 10 (B) 9
(C) 6 (D) 8

21. What is the least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled?

- (A) 5 years (B) 6 years
(C) 3 years (D) 4 years

22. A sum is being lent at 20% per annum compound interest. What is the ratio of interest of 4th year to 5th year?

- (A) 4 : 5 (B) 5 : 4
(C) 5 : 6 (D) Can't say

23. Mr. Duggal invested Rs. 20000 with rate of interest at 20% per annum. The interest was compounded half yearly for first one year and in the next year it was compounded yearly. What will be the total interest earned at the end of two years?

- (A) Rs. 8800 (B) Rs. 9040
(C) Rs. 8040 (D) Rs. 9800

24. Three years back, the population of a colony was 3600 and the current population is 4800.

What will be the population three years down the line. If the rate of growth of population has been constant over the years and has been compounding annually?

- (A) 6000 (B) 6400
(C) 7200 (D) 9600

25. Rs. 5887 is divided between Shyam and Ram, such that Shyam's share at the end of 9 years is equal to Ram's share at the end of 11 years, compounded annually at the rate of 5%. What is the share of Shyam?

- (A) Rs. 2088 (B) Rs. 2000
(C) Rs. 3087 (D) None of these

26. A sum of Rs. 18750 is left by a father to be divided between two sons, 12 and 14 years of age, so that when they attain maturity at 18, the amount (principal + interest) received by each at 5% simple interest will be the same. What is the share of each son.

- (A) Rs. 9500, Rs. 9250 (B) Rs. 8000, Rs. 1750
(C) Rs. 9000, Rs. 9750 (D) None of these

27. Mr. Jeevan wanted to give some amount of money to his two children, so that although today they may not be using it, in the future the money would be of use to them. He divides a sum of Rs. 18750 between his two sons of age 10 years and 13 years respectively in such a way that each would receive the same amount at 3% per annum compound interest when they attain the age of 30 years. What would be the original share of the younger son?

- (A) Rs. 8959.60 (B) Rs. 8559.60
(C) Rs. 8969.80 (D) Rs. 8995.80

28. A certain sum of money invested at CI becomes Rs. 3380 in 2 years and Rs. 3515.20 in three years. What is the sum invested and what is the rate of interest per annum?

- (A) 3125, 4% (B) 3225, 5%
(C) 2125, 8% (D) 2315, 6%

29. The difference between the compound interest and simple interest for two years on a sum of money is Rs. 160. If the simple interest is Rs. 2880 then what is the rate of interest per annum?

- (A) $5\frac{5}{9}\%$ (B) $12\frac{1}{2}\%$

- (C) $11\frac{1}{9}\%$ (D) 9%

30. A man invested Rs. 5000 at 10% per annum SI for two years. At the end of two years, he withdrew the entire amount and invested it at 10% CI for two years. What is the final value of the matured amount?

- (A) Rs. 6660 (B) Rs. 7000
(C) Rs. 7260 (D) Rs. 6000

31. The difference between the compound interest and simple interest earned at the end of second year on a sum of money at 10% per annum is Rs. 20. What is the sum?

- (A) Rs. 4000 (B) Rs. 2000
(C) Rs. 1500 (D) Data inadequate

32. The ratio of the amount for two years under CI annually and for one year under SI is 6 : 5. If the rate of interest and principal is same in both cases, then what is the rate of interest?

- (A) 12.5% (B) 18%
(C) 20% (D) 16.66%

33. A man gets a simple interest of Rs. 1000 in a certain principal at the rate of 5% per annum in 4 years. What compound interest will the man get on twice the principal in two years at the same rate?

- (A) Rs. 1050 (B) Rs. 1005
(C) Rs. 1025 (D) Rs. 1125

34. A person invested some amount at the rate of 12% simple interest and a certain amount at the rate of 10% simple interest. He received yearly interest of Rs. 130. But if he had interchanged the amounts invested, he would have received Rs. 4 more as interest. How much did he invest at 12% simple interest?

- (A) Rs. 700 (B) Rs. 500
(C) Rs. 800 (D) Rs. 400

35. Mungeri Lal has two investment plans: A and B, to choose from plan A offers interest of 10% compounded annually while plan B offers simple interest of 12% per annum. Till how many years is plan B a better investment?

- (A) 3 (B) 4
(C) 5 (D) 6

36. What annual payment will discharge a debt of Rs. 580 due in 5 years at 8 % p.a.?

- (A) Rs. 166.40 (B) Rs. 65.60
(C) Rs. 100 (D) Rs. 120

Solution

Practice Exercise Level 1

- 1.(D)** $I_1 = \frac{780 \times 5 \times T_1}{100} = 39 T_1$
 $I_2 = \frac{600 \times 13 \times T_2}{200} = 39 T_2$
 So, it is always equal.
- 2.(B)** Interest received = Rs. 21
 $\therefore 21 = \frac{200 \times R \times 3}{100} \Rightarrow R = 3.5\%$
- 3.(A)** Sum = Rs. 2000, SI = Rs. 600, Time = 5 years.
 $\therefore \text{Rate} = \left(\frac{120}{2000} \right) \times 100 = 6\% \text{ p.a.}$
 Now, sum = Rs. 2000
 Time = 5 years
 Rate = 6 + 3 = 9
 $SI = \left(\frac{2000 \times 9 \times 5}{100} \right) = 900.$
 Amount = Rs. (2000 + 900)
 = Rs. 2900
- 4.(B)** Let the principal be Rs. x. Then, net increase in percentage
 $= \frac{25}{2}\% - 10\% = \frac{5}{2}\%$
 So, $x \times \frac{2.5}{100} = 1250$
 $x = \text{Rs. } 50000$
- 5.(C)** Let the sum be Rs. x. Then, $SI = \frac{3x}{4}$,
 Time = $\frac{25}{2}$ years,
 $\therefore \text{Rate} = \left(100 \times \frac{3x}{4} \times \frac{1}{x} \times \frac{2}{25} \right) \% \text{ p.a.}$
 $= 6\% \text{ p.a.}$
- 6.(C)** Sum doubled means $I = P$
 $\Rightarrow P = \frac{P \times R \times 15}{100}$
 $\Rightarrow R = \frac{100}{15} = 6\frac{2}{3}\% \text{ p.a.}$
- 7.(C)** Rate of interest
 $\Rightarrow 2P = \frac{P \times R \times 5}{100}$
 Rate = 40%
 Required time $\Rightarrow 4P = \frac{P \times 40 \times T}{100}$
 $T = 10 \text{ years}$
- 8.(C)** Interest = $\frac{P \times R \times T}{100}$
- 9.(B)** $\frac{7}{25}x = \frac{x \times r \times 4}{100}$
 $r = 7\%$
 Let total sum = 4P
 $4800 = \frac{P \times 24 \times 2}{100}$
 $P = 10000$
 $\therefore 3P = 30000$
 $SI \text{ (large sum)} = \frac{30000 \times 12 \times 3}{100}$
 $= \text{Rs. } 10800$
- 10.(C)** $5324 = P \left(1 + \frac{10}{100} \right)^3$
 $P = 5324 \times \frac{1000}{1331} = 4000$
- 11.(C)** $6480 = 4500 \left(1 + \frac{R}{100} \right)^2$
 $\Rightarrow \left(\frac{100 + R}{100} \right)^2 = 1.44$
 So, $R = 20\%$
- 12.(B)** Sum = P
 $CI = 0.21P$
 $\therefore \text{Amount} = \text{Sum} + CI$
 $= P + 0.21P$
 $\therefore 1.21P = P \left(1 + \frac{R}{100} \right)^2$
 $\Rightarrow \frac{121}{100} = \left(1 + \frac{R}{100} \right)^2$
 $\Rightarrow \left(\frac{11}{10} \right)^2 = \left(1 + \frac{R}{100} \right)^2$
 $\Rightarrow \frac{11}{10} = \frac{R}{100} + 1$
 $\Rightarrow \frac{R}{100} = \frac{11}{10} - 1$
 $\Rightarrow R = 10\%$
- 13.(B)** $4P = P \left(1 + \frac{R}{100} \right)^4$
 $\Rightarrow \left(1 + \frac{R}{100} \right)^4 = 4$
 $64P = P \left(1 + \frac{R}{100} \right)^n$
 $(4)^3 = (4)^{n/4}$
 So, $n = 12 \text{ years}$

- 14.(C) For compounding half yearly, $R = \frac{16}{2} = 8\%$

$$T = 2n ; T = 1 \times 2 = 2$$

Compound interest

$$= 5000 \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right]$$

$$5000 \times (1.08)^2 - 5000$$

Compound interest = Rs. 832

- 15.(C) Amount

$$= \left(1 + \frac{R_1}{100} \right) \times \left(1 + \frac{R_2}{100} \right) \times \text{Principal}$$

$$A = 1.1 \times 1.12 \times 10000$$

Hence, amount = 12320

- 16.(A) $(1.1)^4 - (1.2)^2 = 241$

$$(1.4641) - (1.44) = 241$$

$$2.41x = 24100$$

$$x = \text{Rs. } 10000$$

- 17.(B) By applying formula

$$R = \frac{\left(\frac{\text{Amount in 3 years} - \text{Amount in 2 years}}{\text{Amount in 2 years}} \right) \times 100}{1}$$

$$R = \frac{1112 - 1008}{1008} \times 100$$

$$R = 10.3\%$$

- 18.(C) Compound interest when compounded half yearly

$$(800 \times 1.1 \times 1.1) - 800 = 168$$

Compound interest when compounded quarterly

$$= [800 \times (1.05)^4] - 800 = 172.40$$

Difference = Rs. 4.40

- 19.(C) $A = P \times \left(1 + \frac{R}{100} \right)^n$

$$\frac{10648}{8000} = \left(1 + \frac{R}{100} \right)^3$$

$$\left(\frac{22}{20} \right)^3 = \left(1 + \frac{R}{100} \right)^3$$

$$1.1 - 1 = \frac{R}{100}$$

$$R = 10\%$$

- 20.(A) The effective rate of interest received

$$= 8 + 8 + \frac{64}{100} = 16.64\%$$

- 21.(D) The rate % can be calculated as

$$R\% = \frac{Cl_{10} - Cl_9}{C_9} \times 100$$

$$R\% = \frac{90}{990} \times 100 = 9.09\%$$

- 22.(D) Rate of 8% per annum payable half yearly.

So, effective rate = 4%

Effective annual rate

$$= 4 + 4 + \frac{4 \times 4}{100} = 8.16\%$$

- 23.(D) Doubles in 11 years,

Four times in 22 years and Eight times in 33 years.

- 24.(B) $1.5P = P(1.25)^t$

$$\Rightarrow 1.5 = (1.25)^t$$

Using options, $t = 2$ years

- 25.(B) $= \frac{75}{750} \times 100 = 10$

	SI	CI
1 st year	750	750
2 nd year	750	825

Required rate of interest

- 26.(D) $\frac{PR^2}{100^2} = 1296$

$$\frac{P \times 36}{100^2} = 1296$$

$$P = \text{Rs. } 360000$$

- 27.(C) The SI for 5 years is Rs. 2400. Therefore, the SI for 1 year is Rs. 480.

The SI for 2 years will be Rs. 960.

The CI for 2 years will be Rs. 1000.

The difference between the two values, that is, Rs. 40 is because of the interest earned on the first year's interest. Rs. 40 interest has been earned on Rs. 480.

$$\therefore \frac{40}{480} \times 100 = 8.33\%$$

- 28.(C) Total SI received in 2 years

$$= \frac{2PR}{100}$$

Total CI received in 2 years

$$= \frac{2PR}{100} + \frac{PR^2}{100^2}$$

Difference in the two amounts

$$= \frac{PR^2}{100^2}$$

$$\therefore R^2 \times \frac{400000}{100 \times 100} = 1000$$

$$\Rightarrow R^2 = 25 \Rightarrow R = 5\%$$

$$Cl_2 - Sl_2 = 1000$$

Difference in CI and SI of two years

$$= \frac{PR^2}{100^2}$$

$$\Rightarrow 1000 = \frac{400000 \times R^2}{100 \times 100}$$

$$\Rightarrow R^2 = 25$$

$$\Rightarrow R = 5\%$$

29.(D) Required value = $[(1.2)^4 - (1.8)] = 0.2736 \times 12000 = \text{Rs. } 3283.2$

30.(B) Let Amit borrow Rs. x.

$$x \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right] - \frac{x \times 8 \times 2}{100} = 16$$

$$\Rightarrow 0.1664x - 0.16x = 16$$

$$\Rightarrow x = \frac{16}{0.0064} = \text{Rs. } 2500$$

$$CI_{2\text{yrs}} - SI_{2\text{yrs}} = \frac{PR^2}{100^2}$$

$$\frac{P \times 8 \times 8}{100 \times 100} = 16$$

$$\Rightarrow P = \text{Rs. } 2500$$

31.(D) SI (1 year) = Rs. 50

$$\text{So, } 5 = \frac{50}{P} \times 100$$

$$\therefore P = 1000$$

$$CI (3 \text{ years}) = 1000[(1.05)^3 - 1] = \text{Rs. } 157.62$$

$$CI (1^{\text{st}} \text{ year}) = SI (1^{\text{st}} \text{ year}) = 50$$

$$CI (2^{\text{nd}} \text{ year}) = 50 + 5\% \text{ of } 50 = 52.5$$

$$CI (3^{\text{rd}} \text{ year}) = 50 + 5\% \text{ of } 50 + 5\% \text{ of } 50 + 5\% \text{ of } 50$$

$$= 55.125$$

$$\text{Total} = 50 + 52.5 + 55.125$$

$$= \text{Rs. } 157.62$$

Practice Exercise Level 2

1.(D) $I_1 = \frac{500 \times R_1 \times 2}{100} = 10 R_1$

$$I_2 = \frac{500 \times R_2 \times 2}{100} = 10 R_2$$

$$I_1 - I_2 = 10 R_1 - 10 R_2 = 2.5$$

$$\frac{2.50}{10} = R_1 - R_2$$

$$0.25\% = R_1 - R_2$$

2.(C) The required number of years is

$$3P = \frac{P \times R \times 8}{100}$$

$$R = 37.5\%$$

$$27P = P \times \frac{3}{8} \times T$$

$$T = 72 \text{ years}$$

3.(C) Let x be the number of times, then

$$5P = \frac{P \times R \times 7}{100}$$

$$R = \frac{500}{7}\%$$

$$xP = P \times \frac{500}{700} \times 35; x = 25$$

$$\text{Required amount} = 25 + 1$$

$$= 26 \text{ times}$$

4.(A) Total interest to be paid in 4 years

$$= 3500 \times \frac{4 \times 4}{100} = \text{Rs. } 560$$

$$\text{Total amount to be paid back}$$

$$= \text{Rs. } 4060. \text{ As the item is valued at Rs. } 3000, \text{ the remaining amount to be paid} = \text{Rs. } 1060$$

5.(B) Let rate of interest be r

$$SI = \frac{400 \times 2 \times r}{100} + \frac{550 \times 4 \times r}{100} + \frac{1200 \times 6 \times r}{100}$$

$$1020 = 8r + 22r + 72r$$

$$1020 = 102r$$

$$r = 10\%$$

6.(C) Let capital be Rs. x. Then,

$$\left(\frac{x}{3} \times \frac{7}{100} \times 1 \right) + \left(\frac{x}{4} \times \frac{8}{100} \times 1 \right) + \left(\left\{ x - \left(\frac{x}{3} + \frac{x}{4} \right) \right\} \times \frac{10}{100} \times 1 \right) = 561$$

$$\Rightarrow \frac{7x}{300} + \frac{x}{50} + \frac{5x}{120} = 561$$

$$\Rightarrow 14x + 12x + 25x = 336600$$

$$\Rightarrow 51x = 336600 \Rightarrow x = 6600.$$

7.(B)

$$\text{After } t \text{ years, let } P = \text{Rs. } 4x$$

$$\text{and Amount} = \text{Rs. } 5x$$

$$P + SI \text{ for } t \text{ years} = \text{Rs. } 5x \dots (1)$$

$$P : [P + SI \text{ for } (t + 3) \text{ years}] = 5 : 7$$

$$= 1 : \frac{7}{5}$$

$$\text{As the principal is } 4x, \text{ so}$$

$$4x : \left(\frac{7}{5} \times 4x \right) = 4x : \frac{28x}{5}$$

$$\therefore P + SI \text{ for } (t + 3) \text{ years}$$

$$= \frac{28x}{5} \dots (2)$$

$$\text{On subtracting (1) from (2), we get:}$$

$$SI \text{ for 3 years} = \left(\frac{28x}{5} - 5x \right) = \frac{3x}{5}$$

$$SI \text{ on } 4x \text{ for 3 years} = \frac{3x}{5}$$

$$\therefore \text{Rate} = \left(\frac{100 \times \frac{3x}{5}}{4x \times 3} \right) \% \text{ p.a.}$$

= 5% p.a.

8.(B)

SI for 10 years

$$= \left(1000 \times \frac{5}{100} \times 10 \right) = \text{Rs. } 500$$

Principal after 10 years becomes = Rs. (1000 + 500) = Rs. 1500

SI on it = Rs. (2000 - 1500) = Rs. 500

$$\text{Time} = \left(\frac{100 \times 500}{5 \times 1500} \right) = 6\frac{2}{3} \text{ years}$$

$$\text{Total time} = \left(10 + 6\frac{2}{3} \right) = 16\frac{2}{3} \text{ years}$$

9.(A)

Let the amount given at 4% per annum be Rs. x.

$\therefore$ Amount given at 5% per annum = Rs. (1200 - x)

$$\Rightarrow \frac{x \times 4 \times 2}{100} + \frac{(1200 - x) \times 5 \times 2}{100} = 110$$

$$\Rightarrow x = \text{Rs. } 500$$

Amount given at 5% = Rs. 700

10.(B)

Suppose sum = Rs. P

Total interest earned in 9 years

$$\frac{P \times 4 \times 2}{100} + \frac{P \times 6 \times 4}{100} + \frac{P \times 8 \times 3}{100}$$

According to the question,

$$\frac{8P}{100} + \frac{24P}{100} + \frac{24P}{100} = 1120$$

$$\text{or, } 14P = 1120 \times 25 = 28000$$

$$\text{or } P = \text{Rs. } 2000$$

11.(B)

Suppose Amount invested in scheme A = Rs. x

Amount invested in scheme B = Rs. y

Amount invested in scheme C = Rs. z

$$\therefore x + y + z = 65000 \quad \dots (1)$$

$$x = 72\% \text{ of } z = \frac{18}{25}z \quad \dots (2)$$

$$\text{Also, } 12x + 16y + 18z = 1018000$$

$$\text{i.e., } 6x + 8y + 9z = 509000 \quad \dots (3)$$

Using (2) in (1) and (3), we get

$$\frac{43}{25}z + y = 6500 \quad \dots (4)$$

$$\frac{108}{25}z + 8y + 9z = 509000$$

$$\frac{333}{25}z + 8y = 509000 \quad \dots (5)$$

From (4), we have

$$\frac{344}{25}z + 8y = 520000 \quad \dots (6)$$

$$\therefore (6) - (5)$$

$$\Rightarrow \frac{11z}{25} = 11000$$

$$11z = 275000$$

$$z = 25000 ; x = 18000 ; y = 22000$$

12.(B)

Let A and B received x and y amount, respectively.

$$\text{Then, } \frac{x \times 15 \times 3}{100} = 2 \times \frac{y \times 12 \times 5}{100}$$

$$\Rightarrow \frac{x}{y} = \frac{2 \times 12 \times 5}{15 \times 3} = \frac{8}{3}$$

13.(C)

Let the sum of money invested be x and 2x.

Interest on both the sums

$$= \frac{x \times 5 \times 1}{100} + \frac{2x \times 5.5 \times 1}{100}$$

$$\frac{5x}{100} + \frac{11x}{100} = \frac{16x}{100}$$

According to the question,

$$\frac{16x}{100} = 1000$$

$$\Rightarrow x = \frac{1000 \times 100}{16} = \text{Rs. } 6250$$

Hence, second sum invested = 2x = Rs. 12500

14.(B)

Let time = rate = x

$$\frac{9}{16}P = \frac{P \times x \times x}{100}$$

$$x = 7.5$$

Rate = 7.5%

15.(A)

Let the parts be x, y and z.

$$\text{So, } \frac{6 \times 3 \times x}{100} = \frac{6 \times 4 \times y}{100} = \frac{6 \times 5 \times z}{100}$$

$$\Rightarrow 3x = 4y = 5z$$

$$\Rightarrow x : y : z = 20 : 15 : 12$$

$$\text{Smallest part} = z = \frac{12}{47} \times 47000$$

$$= \text{Rs. } 12000$$

16.(B)

Let shares of A, B and C be a, b and c respectively.

$$\text{Hence, } a + \frac{2 \times 5 \times a}{100} = b + \frac{3 \times 5 \times b}{100}$$

$$= c + \frac{4 \times 5 \times c}{100}$$

$$1.1a = 1.15b = 1.2c$$

$$\Rightarrow a:b:c = \frac{1}{110} : \frac{1}{115} : \frac{1}{120}$$

17.(A) Let the amount invested be P.

$$7536 = \frac{P \times 6 \times 2}{100} + \frac{P \times 7 \times 2}{100} + \frac{P \times 8 \times 3}{100}$$

$$P = \text{Rs. } 15072$$

18.(C) Let the initial amount be x.

$$\text{Interest at } 4.5\% = \frac{4.5x}{100}$$

$$\text{Interest at } 4\% = \frac{(x-600) \times 4}{100}$$

$\therefore$ As per the question,

$$\frac{4.5x}{100} - \frac{(x-600) \times 4}{100} = 39.5$$

$$\Rightarrow \frac{4.5x - 4x + 2400}{100} = 39.5$$

$$\Rightarrow 0.5x = 3950 - 2400$$

$$\Rightarrow 0.5x = 1550$$

$$\Rightarrow x = \text{Rs. } 3100$$

19.(B) First we have to calculate r%.

$$2P = P \left(1 + \frac{r}{100} \right)^5$$

$$2 = \left(1 + \frac{r}{100} \right)^5$$

$$2^4 = \left(1 + \frac{r}{100} \right)^{5 \times 4}$$

$$16 = \left(1 + \frac{r}{100} \right)^{20}$$

Thus, P will become 16 times in 20 years.

So, required amount

$$= 12000 \times 16 = \text{Rs. } 192000$$

20.(D) $P \xrightarrow{3} 2P \xrightarrow{3} 4P \xrightarrow{3} 8P$

In 9 years it becomes 8 times. So, K = 8

21.(D) $100 \xrightarrow{20\%} 120 \xrightarrow{20\%} 144 \xrightarrow{20\%} 172.8$
 $\xrightarrow{20\%} 207.3$

So, we require 4 years

22.(C) The interest always increases by 20% of the previous years.

So, interest of 4th year : interest of 5th year

$$= x : 1.2x = 5 : 6$$

23.(B) When interest was compounded half-yearly then

$$R = \frac{20}{2} = 10\%$$

T = 2 units for 1 year

Accumulated interest in 2 years

$$= \left\{ 20000 \left(1 + \frac{10}{100} \right)^2 \left(1 + \frac{20}{100} \right) \right\} - 20000$$

$$= \left\{ 20000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{6}{5} \right\} - 20000$$

$$= 29040 - 20000$$

$$= \text{Rs. } 9040$$

24.(B) Let rate of interest = R

$$\text{Then, } 4800 = 3600 \left[1 + \frac{R}{100} \right]^3$$

$$\Rightarrow \frac{4}{3} = \left[1 + \frac{R}{100} \right]^3 \quad \dots (1)$$

Now, the population after 3 years

$$= 4800 \left[1 + \frac{R}{100} \right]^3$$

From equation (1),

$$4800 \times \frac{4}{3} = 6400$$

25.(C) Let Shyam's share = x

Then,

$$x \left[1 + \frac{5}{100} \right]^9 = (5887 - x) \left[1 + \frac{5}{100} \right]^{11}$$

$$\Rightarrow \frac{x}{5887 - x} = \left[1 + \frac{5}{100} \right]^2$$

$$\Rightarrow \frac{x}{5887 - x} = 1.1025$$

$$\Rightarrow x = \text{Rs. } 3087$$

26.(C) Let principal of two sons are x and y, respectively.

$\therefore$ (P + I) are equal

$$\therefore \frac{x \times 130}{100} = \frac{y \times 120}{100} \Rightarrow \frac{x}{y} = \frac{12}{13}$$

$$\therefore x = \frac{12}{25} \times 18750 = 9000$$

$$y = \frac{13}{25} \times 18750 = 9750$$

27.(A) Let the amount with the young son be x, time be 20 years rate 3%

The amount with the elder son be (18750 - x) time be 17 years, rate 3%

$$\text{We know that, } A = P \left(1 + \frac{R}{100} \right)^t$$

Since, both receive the same amount.

$$\therefore x \left(1 + \frac{3}{100} \right)^{20} = (18750 - x) \left(1 + \frac{3}{100} \right)^{17}$$

$$\Rightarrow \left(1 + \frac{3}{100} \right)^3 = \frac{18750 - x}{x}$$

$$\Rightarrow \left(\frac{103}{100} \right)^3 = \frac{18750}{x} - 1$$

$$\Rightarrow 2.092727 = \frac{18750}{x}$$

$$\Rightarrow x = \frac{18750}{2.092727} = \text{Rs. } 8959.60$$

28.(A) Amount after 2 years

$$= 3380 = P \left(1 + \frac{R}{100} \right)^2$$

Amount after 3 years

$$= 3515.20 = P \left(1 + \frac{R}{100} \right)^3$$

$\therefore$ Equating value of P,

$$\frac{\left(1 + \frac{R}{100} \right)^3}{\left(1 + \frac{R}{100} \right)^2} = \frac{3515.20}{3380}$$

$$\Rightarrow \left(1 + \frac{R}{100} \right) = \frac{3515.20}{338000}$$

$$\Rightarrow \frac{R}{100} = \frac{3515.20}{338000} - 1$$

$$\Rightarrow \frac{R}{100} = \frac{13520}{338000}$$

$$\Rightarrow R = 4\%$$

$$\therefore 3380 = P(1.04)^2$$

$$\therefore P = \frac{3380 \times 100 \times 100}{104 \times 104}$$

$$\therefore P = \text{Rs. } 3125$$

29.(C) Simple interest = Rs. 2880

T = 2 years

$$2880 = \frac{P \times R \times 2}{100}$$

$$\Rightarrow \frac{2880 \times 100}{2} = PR$$

$$PR = 144000$$

By applying formula,

$$\frac{PR^2}{100^2} = (CI)_2 - (SI)_2$$

$$\frac{PR \times R}{100^2} = 160$$

$$R = \frac{160 \times 100 \times 100}{144000}$$

$$R = 11.11\% \text{ or } 11\frac{1}{9}\%$$

30.(C) As per the question, we have SI for 2 years

$$= \frac{5000 \times 10 \times 2}{100} = \text{Rs. } 1000$$

$$\therefore A = 5000 + 1000 = \text{Rs. } 6000$$

Amount has been invested at 10% CI for 2 years.

$$\therefore A = 6000 \left(1 + \frac{10}{100} \right)^2$$

$$= 6000 \times 1.21 = 60 \times 121 = \text{Rs. } 7260$$

31.(B) Principal

$$= \text{Difference} \times \left(\frac{100}{R} \right)^2$$

$$= 20 \times \left(\frac{100}{10} \right)^2 = 20 \times 100 = \text{Rs. } 2000$$

32.(C)

$$\frac{P \left(1 + \frac{r}{100} \right)^2}{\left(P + \frac{Pr}{100} \right)} = \frac{6}{5}$$

$$\Rightarrow \left(1 + \frac{r}{100} \right) = \frac{6}{5}$$

$$\Rightarrow r = 20\%$$

33.(C)

$$\text{Principal} = \frac{1000 \times 100}{5 \times 4}$$

$$= \text{Rs. } 5000$$

$$CI = 10000 \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right]$$

$$= 10000 \times \frac{41}{400} = \text{Rs. } 1025$$

34.(B)

Amount invested at 12%

$$= \text{Rs. } x$$

Amount invested at 10% = Rs. y

$$130 = \frac{x \times 12 \times 1}{100} + \frac{y \times 10 \times 1}{100}$$

$$\Rightarrow 13000 = 12x + 10y \dots (1)$$

$$\text{and } 134 = \frac{x \times 10 \times 1}{100} + \frac{y \times 12 \times 1}{100}$$

$$\Rightarrow 13400 = 10x + 12y \dots (2)$$

Solving equations (1) and (2), we get

$$x = \text{Rs. } 500$$

So, amount invested at 12% is Rs. 500.

35.(B)

Till fourth year plan B will fetch Rs. 48 and plan A will fetch Rs. 46. Fifth year interest from plan B will be Rs. 60 and from plan A will be Rs. 61.

Let the initial amount be 100.

SI	CI
$SI = 100 \times \frac{12}{100} = 12$	$CI = 100(1.1) = 110$
$A_1 = 112$	$A_1 = 110$
$A_2 = 124$	$A_2 = 110(1.1) = 121.1$
$A_3 = 136$	$A_3 = 121.1(1.1) = 133.1$
$A_4 = 148$	$A_4 = 133.1(1.1) = 146.1$

$A_5 = 160$	$A_5 = 146.1(1.1) = 160.71$
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∴ Plan B (i.e. SI) is best for the first four years.

- 36.(C)** Let the required annual installment be Rs. x .
Then, (Amount of Rs. x for 4 years) +
(Amount of Rs. x for 3 years) + (Amount of
Rs. x for 2 years) + (Amount of Rs. x for 1
year) + Rs. x
= 580.

$$\Rightarrow \left(x + \frac{x \times 8 \times 4}{100} \right) + \left(x + \frac{x \times 8 \times 3}{100} \right) +$$

$$\left(x + \frac{x \times 8 \times 2}{100} \right) + \left(x + \frac{x \times 8 \times 1}{100} \right) + x$$

$$= 580$$

$$\Rightarrow \left(x + \frac{8x}{25} \right) + \left(x + \frac{6x}{25} \right) + \left(x + \frac{4x}{25} \right) + \left(x + \frac{2x}{25} \right) + x = 580$$

$$\Rightarrow \frac{33x}{25} + \frac{31x}{25} + \frac{29x}{25} + \frac{27x}{25} + x = 580$$

$$\Rightarrow (33x + 31x + 29x + 27x + 25x) = 14500$$

$$\Rightarrow 145x = 14500 \Rightarrow x = 100$$

Required installment = Rs. 100

Profit, Loss & Discount

04

Let us see some of the basic terms before going into the details of Profit, Loss and Discount.

- 1. Cost Price (CP):** The Cost Price of an article is the price at which an article is bought, to be sold.
- 2. Selling Price (SP):** The Selling Price of an article is the price at which an article is sold.
- 3. Profit (or Gain):** If the Selling Price (SP) of an article is more than the Cost Price (CP) of that article, then there is a Profit given by:
Profit = SP – CP
- 4. Loss:** If the Selling Price (SP) of an article is less than the Cost Price (CP), then there is a loss given by,
Loss = CP – SP
- 5. Profit% or Loss%:** Both are always calculated on the cost price and are given by:

$$\text{Profit\%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$\text{Loss\%} = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$$

RELATION BETWEEN SP AND CP

If an article is sold at a profit of P%, then

$$\text{SP} = \left(1 + \frac{P}{100}\right) \text{CP}$$

And, if an article is sold at a loss of L%, then

$$\text{SP} = \left(1 - \frac{L}{100}\right) \text{CP}$$

For example, if there is a profit of 20%, then

$$\text{SP} = \left(1 + \frac{20}{100}\right) \times \text{CP}$$

or SP = 120% of CP

or SP = 1.2 CP

And if there is a loss of 30%, then

$$\text{SP} = \left(1 - \frac{30}{100}\right) \times \text{CP}$$

or SP = 70% of CP

or SP = 0.7 CP

Here, we again use the concept of multiplying factor (MF) and we have the relation that SP = MF × CP

The MF in case of profit will be greater than 1 and in case of loss will be less than 1.

Also, by re-arranging the expression as SP/CP = MF, we can easily deduce the profit or loss percentage.

For example, if

CP = 45 and SP = 60 then

$$\text{MF} = \frac{\text{SP}}{\text{CP}} = \frac{60}{45} = \frac{4}{3} = 1.33$$

So, Profit = 33%

Also, if SP = 45, CP = 60

$$\text{then } \frac{\text{SP}}{\text{CP}} = \frac{45}{60} = \frac{3}{4} = 0.75 = 25\% \text{ loss}$$

Example 1: If SP = 800 and CP = 700. What is the profit percentage?

Solution:

SP = 800 and CP = 700

$$\text{Profit\%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$= \frac{800 - 700}{700} \times 100 = \frac{1}{7} \times 100 = 14.28\%$$

Alternate Method:

$$SP = 800, CP = 700$$

$$\text{So, } \frac{SP}{CP} = \frac{8}{7} = 1.1428$$

$$\therefore \text{Profit \%} = 14.28\%$$

Example 2: If $\frac{SP}{CP} = \frac{7}{5}$, then what is profit/loss percentage?

Solution:

As $MF > 1$, there will be a profit

$$\text{Profit \%} = \left(\frac{SP}{CP} - 1 \right) \times 100$$

$$= \left(\frac{7}{5} - 1 \right) \times 100$$

$$= \frac{2}{5} \times 100 = 40\%$$

So, Profit % = 40%

Alternate Method:

$$\frac{SP}{CP} = \frac{7}{5} \Rightarrow \frac{SP}{CP} = 1.4$$

Hence, 40% profit.

Example 3: If the CP is Rs. 480 and the profit is 20%, then what is the selling price of an article?

Solution:

$$CP = \text{Rs. } 480$$

$$\text{Profit \%} = 20\%$$

$$\text{So, } SP = 120\% \text{ of } CP$$

$$\text{or } SP = 1.2 \times CP$$

$$SP = 1.2 \times 480$$

$$= \text{Rs. } 576$$

Alternate Method:

$$\frac{SP}{CP} = 1.2$$

$$SP = 1.2 \times 480 = 576$$

Example 4: Find the cost price of an article if the selling price is Rs. 425 and loss is 15%.

Solution:

$$\text{Loss \%} = 15\%$$

$$\text{Then, } SP = 85\% \text{ of } CP$$

$$\text{or } SP = 0.85 \times CP$$

$$\text{or } CP = \frac{SP}{0.85} = \frac{425}{0.85} = 500$$

Alternate Method:

$$\frac{SP}{CP} = 0.85$$

$$CP = \frac{425}{0.85} = 500$$

Example 5: If the selling price is doubled, the profit triples. Find the profit percent.

Solution:

$$\text{Let } CP = \text{Rs. } x \text{ and } SP = \text{Rs. } y$$

$$\text{Profit} = \text{Rs. } (y - x)$$

$$\text{If, } SP = 2y, \text{ then profit} = 3(y - x)$$

$$\therefore 2y - x = 3(y - x)$$

$$\Rightarrow y = 2x$$

$$\therefore \text{Profit on Rs. } x \text{ is Rs. } x ; \text{ i.e., } 100\%$$

Example 6: If the selling price of a product is increased by 162, then the business would make a profit of 17% instead of a loss of 19%. What is the cost price of the product?

Solution:

$$\text{Let } CP \text{ be } 100x.$$

$$SP \text{ at the loss of } 19\% = 100 \times 0.81 = 81x$$

$$SP \text{ at the profit of } 17\% = 100 \times 1.17 = 117x$$

Difference in both the SPs = 162 $\Rightarrow 117x - 81x = 162$

$$\Rightarrow 36x = 162$$

$$\Rightarrow x = 4.5$$

$$\text{So, CP} = 100x = 100 \times 4.5 = 450$$

Alternate Method:

$$\text{So, CP} = \frac{162}{36} \times 100 = 450$$

Difference between two SP's in terms of cost price

$$= (117\% \text{ of CP}) - (81\% \text{ of CP}) = 36\% \text{ of CP} = 162$$

Example 7: I sold a dairy at loss of 15%. Had I sold it at a gain of 20%, I would have fetched Rs. 875 more. Find the cost price of the dairy?

Solution:

Difference between the two selling prices is given as Rs. 875.

$$\text{So, } (120\% \text{ of CP}) - (85\% \text{ of CP}) = 875$$

$$\text{CP} = 875 \times \frac{100}{35}$$

$$\text{CP} = \text{Rs. } 2500$$

Example 8: Reena sells her car for Rs. 5000 and incurred some loss. Had she sold it for Rs. 5600, her gain would have been twice the former loss. Find the cost of the car?

Solution:

Let the cost of the car be Rs. x .

$$\text{Profit} = 2 \times \text{Former loss}$$

$$\therefore (5600 - x) = 2(x - 5000)$$

$$\Rightarrow 3x = 15600$$

$$\text{or, } x = 5200$$

Example 9: A dealer sells an article at a profit of 20%. If he had bought it at 20% less and sold for Rs. 10 less, he would have gained 25%. Find the cost price of the article.

Solution:

$$\text{Let CP} = x$$

$$\text{Profit} = 20\%$$

$$\text{SP} = 1.2x$$

$$\text{New CP} = 80\% \text{ of } x = 0.8x$$

$$\text{Gain} = 25\%$$

$$\text{New SP} = 1.25(0.8x) = x$$

According to the question,

$$\text{SP}_{\text{old}} - \text{SP}_{\text{new}} = 10$$

$$1.2x - x = 10$$

$$\Rightarrow \frac{x}{5} = 10$$

$$\Rightarrow x = \text{Rs. } 50$$

Example 10: A trader purchases a watch and a wall clock for Rs. 390. He sells them, making a profit of 10% on the watch and 15% on the wall clock. He earns a profit of Rs. 51.50. What is the difference between the original prices of the wall clock and the watch?

Solution:

Let cost of the watch be Rs. x ,

The cost of wall clock = Rs. $(390 - x)$

$$\text{Then, } 10\% \text{ of } x + 15\% \text{ of } (390 - x) = 51.5$$

On solving, we get,

$$x = 140$$

$$\text{Hence, cost of wall clock} = 390 - 140 = \text{Rs. } 250$$

$$\therefore \text{Difference} = 250 - 140 = \text{Rs. } 110$$

Example 11: A man buys two radios each for Rs. 1200 each. He sells one at a profit of 20% and sells the other at a loss of 20%. Find the net profit/loss?

Solution:

$$\text{Profit on first radio} = 20\% \text{ of } 1200 = 240$$

$$\text{Loss on second radio} = 20\% \text{ of } 1200 = 240$$

$$\text{Net profit/loss} = 240 - 240 = 0$$

Example 12: A man sells two watches each for Rs. 1200. He sells one at a profit of 20% and sells the other at a loss of 20%. Find the net profit/loss?

Solution:

One important thing to be noted in this question is that the selling price of both the watches is same and not the cost price.

So,

$$SP_1 = 120\% \text{ of } CP_1$$

$$CP_1 = \frac{1200 \times 100}{120} = 1000$$

$$CP_1 = 1000$$

$$SP_2 = 80\% \text{ of } CP_2$$

$$CP_2 = \frac{1200 \times 100}{80} = 1500$$

$$CP_2 = 1500$$

$$\text{Total CP} = 1000 + 1500 = 2500$$

$$\text{Total SP} = 1200 + 1200 = 2400$$

$$\text{Loss} = 2500 - 2400 = 100$$

$$\text{Loss}\% = \frac{100}{2500} \times 100 = 4\%$$

Note:

- I. If cost price of two articles is same and one article is sold at R% profit and other at R% loss, then overall there will be no profit and no loss.
- II. When two articles are sold at same selling price, one at R% profit and one at R% loss. So, there will always be loss and the loss percentage will be given by $\left(\frac{R^2}{100}\right)\%$.

Example 13: 'A' bought a new bicycle from the market and after using it for one month sold it to 'B' at a loss of 20%. 'B' sells it to 'C' at a gain of 25%.

Then C sold it to 'D' at a loss of $33\frac{1}{3}\%$. Finally 'D' paid Rs. 480 to 'C' for the cycle. At what price 'A' had bought the cycle?

Solution:

Let the cost at which A bought the cycle be x.

$$\text{Then } \left(66\frac{2}{3}\%(125\% \text{ of } (80\% \text{ of } x))\right) = 480$$

$$\text{or } \frac{2}{3} \times \frac{5}{4} \times \frac{4}{5} \times (x) = 480$$

$$\text{or } x = 480 \times \frac{5}{4} \times \frac{4}{5} \times \frac{3}{2}$$

$$x = 720$$

So, 'A' bought the cycle for Rs. 720.

Involvement of Number of articles

In these questions, the information about selling price and cost price of certain articles is given. Generally the question statement talks about number of articles at a cost price and the number of articles at an equivalent selling price.

Example 14: The cost price of 5 articles is equal to the selling price of 7 articles. Find the gain/loss percentage?

Solution:

In these type of questions, we will always assume that the cost price of each article is same and also the selling price of each item is same.

Then, we will have loss in this case as the cost price of 5 articles is realized after selling 7 articles.

$$\text{Let cost price of one article} = CP_1$$

$$\text{And selling price of one article} = SP_1$$

$$\text{So, } 5 \times CP_1 = 7 \times SP_1$$

$$\Rightarrow \frac{SP_1}{CP_1} = \frac{5}{7}$$

$$\text{So, Loss} = \frac{2}{7} \times 100 = 28.56\%$$

Shortcut Method:

Profit/Loss% in the questions involving number of articles can be found out by the following method:

Let the number of articles corresponding to CP be x and the number of articles corresponding to SP be y
i.e. $x \text{ CP} = y \text{ SP}$

Case I: If $x > y$, then we gain on the total transaction

$$\text{i.e., Profit \%} = \frac{x-y}{y} \times 100$$

Case II: If $x < y$, then we lose on the total transaction

$$\text{i.e., Loss \%} = \frac{y-x}{y} \times 100$$

Simply put, the Profit/Loss% can be calculated by

$$= \frac{\text{difference in the number of articles}}{\text{number of articles corresponding to SP}} \times 100$$

Example 15: By selling 88 metres of cloth, a man incurs a loss equivalent to the selling price of 11 metres of cloth. Find the loss percentage.

Solution:

Let the cost price of 1 m of cloth be CP_1

and the selling price of 1 m of cloth be SP_1 .

As per the question,

$$88 \times CP_1 - 88 \times SP_1 = 11 \times SP_1$$

$$\Rightarrow 88 \times CP_1 = 99 SP_1$$

$$\therefore \text{Loss \%} = \frac{\text{difference in the number of articles}}{\text{number of articles corresponding to SP}} \times 100$$

$$= \frac{1}{9} \times 100 = 11.11\%$$

Example 16: Aman buys 8 toffees for a rupee and sells only 6 toffees for a rupee. Find his profit percentage.

Solution:

Cost price of 1 toffee = $1/8$

and Selling price of 1 toffee = $1/6$

Taking ratio of SP and CP

$$\frac{SP}{CP} = \frac{1/6}{1/8} = \frac{8}{6} = 1.33$$

So, Profit = 33.33%

Alternate Method: 1

Let Aman buy 24 toffees. [(LCM of 8 and 6) = 24]

$$\text{CP of 24 toffees} = \frac{24}{8} = 3$$

$$\text{SP of 24 toffees} = \frac{24}{6} = 4$$

$$\frac{SP}{CP} = \frac{4}{3} = 1.33$$

So, profit = 33.33%

Note:

Here, we have assumed the total number of toffees as 24, which is the LCM so that the cost price as well as the selling price come out to be an integer.

Alternate Method: 2

As per the question,

$$8 \times CP_1 = 6 \times SP_1$$

$$\therefore \text{Profit \%} = \frac{\text{difference in the number of articles}}{\text{number of articles corresponding to SP}} \times 100$$

$$= \frac{2}{6} \times 100 = 33.33\%$$

Example 17: Akshay bought two varieties of oranges. One variety at the rate of Rs. 200 for 4 kgs and second variety at the rate of Rs. 400 for 10 kgs. He mixed the two varieties and sold them at the rate of Rs. 50 per kg. Find the profit/loss percentage for Akshay, if:

I. He bought equal quantity of both the varieties

II. He spent equal amount on both the varieties

Solution:

- I. Assume that Akshay bought 20 kgs (LCM of 4 and 10 = 20) of both the varieties

$$\text{CP of 20 kgs of first variety} = 200 \times 5 = 1000$$

$$\text{CP of 20 kgs of second variety} = 400 \times 2 = 800$$

$$\text{Total CP} = 1800$$

$$\text{SP of 40 kgs oranges} = 50 \times 40 = 2000$$

$$\text{Profit \%} = \frac{2000 - 1800}{1800} \times 100$$

$$= \frac{200}{1800} \times 100 = 11.11\%$$

- II. Assume that Akshay spent Rs. 400 (LCM of 200 and 400 = 400) on each type of Oranges.

Quantity purchased of first variety for Rs. 400.

$$= \frac{400}{200} \times 4 = 8 \text{ kgs}$$

Quantity purchased of second variety for Rs. 400

$$= \frac{400}{400} \times 10 = 10 \text{ kgs}$$

$$\text{Total Quantity} = 18 \text{ kgs}$$

$$\text{Total cost price} = 400 + 400 = 800$$

$$\text{Selling Price of 18 kgs} = 18 \times 50 = \text{Rs. } 900$$

$$\text{So, Profit \%} = \frac{900 - 800}{800} \times 100 = 12.5\%$$

Example 18: Amritanshu has 240 mangoes with him. He sells x at a profit of 10% and remaining at a loss of 10%. He gains 5% on the whole. What is the value of x ?

Solution:

Let the cost price of 1 mango be Rs. 1.

So, Total cost price = Rs. 240

Now 10% of x - 10% of $(240 - x)$ = 5% of (240)

$$\Rightarrow \frac{x}{10} - \frac{1}{10}(240 - x) = 12$$

$$\Rightarrow \frac{x}{10} - 24 + \frac{x}{10} = 12$$

$$\Rightarrow \frac{x}{5} = 36$$

$$\Rightarrow x = 180$$

FAULTY WEIGHTS

Sometimes a dealer intentionally or unintentionally may be selling items using faulty weights, where he professes to sell the amount at cost price at an intended weight but he uses a weight lighter than that.

$$\text{In such cases, Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

In the above formula Numerator shows the difference between the reading of weighing balance shown to the customer and the faulty weight that the dealer uses and the denominator shows the faulty weight that the dealer uses.

Example 19: A dishonest dealer professes to sell his goods at the cost price but uses a false weight of 950 grams for a 1 kg weight. Find the gain % for the dealer?

Solution:

Let us say the shopkeeper is selling almonds which cost him Rs. 1000 per kg. Now if a customer asks for 1 kg of almonds, the shopkeeper will weigh the almonds with the faulty weight and would actually give him 950 grams.

So, the cost price for the shopkeeper is Rs. 950 and he is charging the customer Rs. 1000.

$$\text{So, profit \%} = \frac{1000 - 950}{950} \times 100$$

$$= \frac{50}{950} \times 100 = 5.26\%$$

Alternate Method:

$$\text{Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$\text{Gain \%} = \frac{50}{950} \times 100 = 5.26\%$$

Example 20: A meter scale is rigged to measure 90 cm. However, in summers it expands by 10%. Find the profit/loss percentage in summers of the dealer who professes to sell his goods at the cost price?

Solution:

For 100 cm, the dealer will give 90 cm.

In summers, the meter scale will expand by 10%.

So, now it will read = $1.1 \times 90 = 99$ cm

As in summers, for 100 cm, the dealer will give 99 cm of cloth.

$$\text{So, Profit} = \frac{1}{99} \times 100 = 1.01\%$$

MARKED PRICE AND DISCOUNT

Marked Price

The Marked Price or the List Price is the price that is indicated or marked on the item at which the product is intended to be sold.

Markup

The process of tagging an item at higher value than CP is called the 'Markup' i.e. the percentage by which the MP is higher than the CP i.e.

$$\text{Markup \%} = \frac{\text{MP} - \text{CP}}{\text{CP}} \times 100$$

$$\text{Marked price} = \left(1 + \frac{m}{100} \right) \times \text{CP}$$

Where m is the markup %.

Discount

Discount is the reduction given on the Marked Price.

i.e. Value of Discount = Marked Price – Selling Price

Discount is expressed as a percentage of the marked price as

$$\text{Discount \%} = \frac{\text{Marked Price} - \text{Selling Price}}{\text{Marked Price}} \times 100$$

$$= \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

$$\text{So, Selling Price} = \left(1 - \frac{d}{100} \right) \times \text{Marked Price}$$

Where d is the discount percent.

Now, let us see the process of the discount and profit carefully.

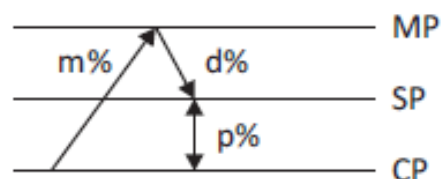
The shopkeeper marks an item up by m% (of CP) and puts a price tag of MP

$$\text{CP} \xrightarrow{m\%} \text{MP}$$

Now, the customer bargains a discount of d% on the MP and thus arrives at the SP

$$\text{CP} \xrightarrow{m\%} \text{MP} \xrightarrow{d\%} \text{SP}$$

We also know m% is that of CP and d% is that of M.P. Thus m% and d% are successive percentage changes or more specifically percentage increase and decrease. Also, the net percentage change between CP and SP is profit percentage.



As successive change for two changes is $a + b + (ab/100)$.

$$p = m - d - \frac{md}{100}$$

We can say that

Where m, d and p are the markup, discount and profit percentage, respectively.

Also from the concept of Multiplying Factor we can say that

$$MF_{\text{Profit}} = MF_{\text{Markup}} \times MF_{\text{Discount}}$$

Note:

$$SP = \frac{100 - D}{100} \times MP \quad \dots (1)$$

$$SP = \frac{100 \pm \text{Profit or Loss}}{100} \times CP \quad \dots (2)$$

Equating (1) and (2)

$$\frac{100 - D}{100} \times MP = \frac{100 \pm \text{Profit or Loss}}{100} \times CP$$

$$\frac{MP}{CP} = \frac{100 \pm \text{Profit / Loss}}{100 - D},$$

where D is the discount percent.

Example 21: A trader marks his product 30% above the cost price and then offers a discount of 20%. Find the profit percentage earned by the trader.

Solution:

Let the cost price of the article = 100

Then Marked price = 130

Discount = 20%

So, selling price = 80% of 130

$$= 0.8 \times 130 = 104$$

$$\text{Now, Profit\%} = \frac{SP - CP}{CP} \times 100$$

$$= \frac{104 - 100}{100} \times 100 = 4\%$$

Alternate Method: 1

$$MF_{\text{Profit}} = MF_{\text{Markup}} \times MF_{\text{Discount}}$$

As Markup = 30% ; So, $MF_{\text{Markup}} = 1.3$

and Discount = 20% ; So, $MF_{\text{Discount}} = .8$

So, $MF_{\text{Profit}} = 1.3 \times .8$

$$= 1.04$$

So, profit = 4%

Alternate Method: 2

$$\text{As } P\% = m - d - \frac{md}{100}$$

and $m = 30$ and $d = 20$

$$\text{So, } P\% = 30 - 20 - \frac{30 \times 20}{100}$$

$$P\% = 10 - 6 = 4\%$$

So, Profit % = 4%

Example 22: After giving 20% discount, gain is 20%. Find the profit percentage when there is a discount of 10%.

Solution:

$$\text{As, } P\% = m - d - \frac{md}{100}$$

$P\% = 20$ and $d = 20$

$$20 = m - 20 - \frac{m \times 20}{100}$$

$$20 = m - 20 - \frac{m}{5}$$

$$40 = \frac{4}{5}m$$

$$m = 50$$

Here, markup = 50%

Let CP = 100

So, MP = 150

Now, if discount is 10%

Then, SP = $0.9 \times 150 = 135$

So, Profit = 35%

Alternate Method:

$$\text{As we know, } \frac{\text{Marked Price}}{\text{Cost Price}} = \frac{100 + P}{100 - D}$$

where P is profit % and D is discount %.

$$\text{So, } \frac{\text{MP}}{\text{CP}} = \frac{120}{80} = \frac{3}{2}$$

If discount of 10% is given, then

$$\frac{3}{2} = \frac{100 + P}{100 - 10}$$

$$\Rightarrow \frac{3}{2} = \frac{100 + P}{90}$$

$$P = 35\%$$

Example 23: At what percentage above the cost price must an article be marked so as to gain 33% after allowing the customer a discount of 5%?

Solution:

$$m - d - \frac{md}{100} = P\%$$

$$m - 5 - \frac{m \times 5}{100} = 33$$

$$\frac{19}{20}m = 38$$

$$m = 40$$

Alternate Method:

As we know,

$$\frac{\text{MP}}{\text{CP}} = \frac{100 \pm P / L\%}{100 - D\%}$$

$$\frac{\text{MP}}{\text{CP}} = \frac{100 + 33}{100 - 5}$$

$$\frac{\text{MP}}{\text{CP}} = \frac{133}{95}$$

$$\text{Here, MP} = 133$$

$$\text{CP} = 95$$

$$\text{Markup \%} = \frac{133 - 95}{95} \times 100 = 40\%$$

SUCCESSIVE DISCOUNT

When two or more discounts are allowed one after the other, then such discounts are known as Successive Discounts.

If two successive discounts of $x\%$ and $y\%$ are allowed on the Marked Price of an item, then

$$\text{Selling Price} = \left(1 - \frac{x}{100}\right) \left(1 - \frac{y}{100}\right) \times \text{Marked Price}$$

$$\text{Selling Price} = \left[1 - \frac{y}{100} - \frac{x}{100} + \frac{xy}{100^2}\right] \times \text{Marked Price}$$

$$= \left[1 - \frac{x + y - \frac{xy}{100}}{100}\right] \times \text{Marked Price}$$

So, a single equivalent discount of two successive

$$\text{discounts of } x\% \text{ and } y\% = \boxed{x + y - \frac{xy}{100}}$$

Example 24: The price of an article is raised by 30% and then two successive discounts of 10% each are allowed. What will be the profit percentage on the article?

Solution:

Net discount for two successive discounts of 10% and 10%.

$$\left[10 + 10 - \frac{10 \times 10}{100}\right] = 19\%$$

Now, markup % is given i.e. 30%.

$$\text{So, } P\% = m - d - \frac{m \times d}{100}$$

$$P\% = 30 - 19 - \frac{570}{100}$$

$$P\% = 5.3\%$$

Example 25: Which is a better bargain for a customer.

- I. Successive discount of 30% and 10%
- II. Successive discount of 20% and 20%

Solution:

$$\text{Successive discount} = x + y - \frac{xy}{100}$$

$$\text{I. } 30 + 10 - \frac{300}{100} = 37\%$$

$$\begin{aligned} \text{II. } 20 + 20 - \frac{20 \times 20}{100} \\ = 40 - 4 = 36\% \end{aligned}$$

So, (I) is a better bargain for the customer.

Example 26: What is the discount percentage on "Buy three get one Free" offer?

Solution:

Let the cost price of the article be Rs. 100. If we buy 4 articles, then the total we have to pay is Rs. 300 (as one article is free with three). The total Marked Price of the 4 articles is Rs. 400. So, we got a discount of Rs. 100 on Rs. 400.

$$\text{So, Discount\%} = \frac{100}{400} \times 100 = 25\%$$

Note:

Buy x get y free i.e. x + y articles are sold at the price

of x articles, then discount percentage is $\frac{y}{x+y} \times 100$.

Practice exercise Level 1

- Monika purchases a watch for Rs. 2000 and after two months she sells the watch at 20% profit. What is the sum received by Monika for the watch?
(A) Rs. 2400 (B) Rs. 1600
(C) Rs. 2200 (D) None of these
- If by selling 3 diamonds, the owner of MG Diamond Works gains the cost price of 2 diamonds, What is the profit or loss percentage of MG Diamond Works?
(A) 60% profit (B) 40% profit
(C) 50% profit (D) 66.67% profit
- The difference between cost price and selling price of an article is Rs. 100. If profit percentage is 20%, what is the selling price?
(A) Rs. 600 (B) Rs. 500
(C) Rs. 400 (D) Rs. 800
- There would be 10% loss if a toy is sold at Rs. 10.80 per piece. At what price should it be sold in order to earn a profit of 20%?
(A) Rs. 12 (B) Rs. 12.96
(C) Rs. 14.40 (D) None of these
- A man sells 320 mangoes at the cost price of 400 mangoes. What is his gain percent?
(A) 10% (B) 15%
(C) 20% (D) 25%
- A box of Alphonso mangoes were purchased by a fruit seller for Rs. 300. However, he had to sell them for Rs. 255 because they began to ripe. What was the loss percentage?
(A) 15% (B) 10%
(C) 20% (D) 18%
- Harish bought a second-hand typewriter for Rs. 1200 and spent Rs. 200 on its repairs. He sold it for Rs. 1680. What was his profit or loss percentage?
(A) 10% loss (B) 10% gain
(C) 20% loss (D) 20% gain
- An item is sold for Rs. 240 at a loss of 20%. At what price it needs to be sold in order to make a profit of 20%?
(A) Rs. 300 (B) Rs. 288
(C) Rs. 260 (D) Rs. 360
- What is the profit percentage if 34% of cost price is equal to 26% of the selling price?
(A) 30.77% (B) 74%
(C) 25.16% (D) 88.40%

10. 5% more is gained by selling a watch for Rs. 350 than by selling it for Rs. 340. The cost price of the watch is:
 (A) Rs. 110 (B) Rs. 140
 (C) Rs. 200 (D) Rs. 250
11. A man sells an article at 5% profit. If he had bought it at 5% less and sold it for Rs. 1 less, he would have gained 10%. What is the cost price of the article?
 (A) Rs. 200 (B) Rs. 150
 (C) Rs. 250 (D) Rs. 240
12. Pankaj purchased an item for Rs. 7500 and sold it at the gain of 24%. From that amount he purchased another item and sold it at the loss of 20%. What is his overall gain/loss?
 (A) Loss of Rs. 140 (B) Gain of Rs. 60
 (C) Loss of Rs. 60 (D) Neither gain or loss
13. A man sells two articles each for Rs. 240. He sells one at a profit of 20% and sells the second at a loss of 25%. What is the net profit or loss in the whole transaction?
 (A) 9.09% Profit (B) 7.69% Profit
 (C) 7.69% Loss (D) 9.09% Loss
14. An article when sold for Rs. 960 fetches 20% profit. What would be the percentage profit/loss if 5 such articles are sold for Rs. 825 each?
 (A) 3.125% profit (B) 3.125% loss
 (C) Neither profit nor loss (D) $16 \times 5\%$ profit
15. A man purchases two clocks A and B at total cost of Rs. 650. He sells A at a 20% profit and B at a loss of 25% and gets the same selling price for both the clocks. What are the purchasing price of A and B respectively?
 (A) Rs. 225 ; Rs. 425 (B) Rs. 250 ; Rs. 400
 (C) Rs. 275 ; Rs. 375 (D) Rs. 300 ; Rs. 350
16. A milkman buys milk at Rs. 25 per litre and adds water which is $\frac{1}{4}$ of the milk and sells the mixture at Rs. 26 per litre. His gain is:
 (A) 25% (B) 20%
 (C) 30% (D) 15%
17. If the manufacturer gain 10%, the whole-seller 15% and the retailer 25%, then what is the cost of production of a bicycle, the retail price of which is Rs. 1265.
 (A) 845 (B) 800
 (C) 850 (D) 825
18. A person sold a book for Rs. 21 and got a loss percentage which was numerically equal to the cost price. What is the cost price (in Rs.) of the book?
 (A) 30 (B) 70
 (C) Both 30 and 70 (D) Cannot be determined
19. A man buys 100 bedsheets for Rs. 4000 and sells 20 of them at a gain of 5%. At what gain % must he sell the remainder so as to gain 20% on the whole?
 (A) 24.25% (B) 24.75%
 (C) 25.25% (D) 23.75%
20. A shopkeeper professes to sell his goods at cost price but uses a 960 gm weight instead of 1 kilogram weight which is basically what weighing balance is showing. What is the profit percentage of the shopkeeper?
 (A) 3.16% (B) 5.26%
 (C) 4.16% (D) 6.36%
21. A shopkeeper claims to sell his goods at cost price but uses a weight of 225 grams for a 250 grams weight which is basically what weighing balance is showing. What is the profit percentage for the shopkeeper?
 (A) 12.5% (B) 10%
 (C) 11.11% (D) 9.09%
22. MG Education Pvt. Ltd. is a leading educational software company in Hyderabad. If it offers

- 20% discount on its software and still earns a profit of 50%, then how much percentage, softwares are being marked up?
- (A) 90% (B) 60%
(C) 80% (D) 87.5%
23. A merchant changed his trade discount from 25% to 15%. This would increase selling price by:
- (A) $3\frac{1}{3}\%$ (B) $6\frac{1}{6}\%$
(C) $13\frac{1}{3}\%$ (D) $16\frac{1}{3}\%$
24. An article is sold for Rs. 6552 after a discount of 22%. What is the marked price of the article?
- (A) Rs. 8450 (B) Rs. 8425
(C) Rs. 8400 (D) Rs. 8750
25. An article is sold at a discount of 35%. If the selling price of the article is Rs. 2275, then what is the marked price of the article?
- (A) Rs. 3250 (B) Rs. 3300
(C) Rs. 3000 (D) Rs. 3500
26. A shop keeper earns a profit of 12% on selling a book at 10% discount on the printed price. The ratio of cost price to the printed price of the book is:
- (A) 45 : 56 (B) 50 : 61
(C) 99 : 125 (D) None of these
27. A shopkeeper marks his goods at Rs. 2000 and after allowing discount of 25%, he still gains 50%. What is the cost price of the article?
- (A) Rs. 1000 (B) Rs. 1200
(C) Rs. 800 (D) Rs. 1500
28. I sold an item at a discount of 20%. If the % mark-up is 30%, then what is the overall profit percentage?
- (A) 4% (B) 10%
(C) 14% (D) No profit, no loss
29. By selling an article at 80% of its marked price, a merchant makes a loss of 12%. What will be the percent profit or loss made by the merchant if he sells the article at 95% of its marked price?
- (A) 5.5% profit (B) 1% loss
(C) 5% profit (D) 4.5% profit
30. The marked price of an article is 60% more than its cost price. What should be the discount offered by the shopkeeper so that he earns a profit of 12%?
- (A) 12% (B) 25%
(C) 30% (D) 60%
31. The cost price of an article is x. It is marked up by 150%. It is sold at Rs 600 after giving 40% discount. What is x?
- (A) Rs. 494.44 (B) Rs. 400
(C) Rs. 840 (D) Rs. 942.66
32. After two successive discounts of 20% and 35%, an article is sold for Rs. 50700. What is the marked price of the article?
- (A) Rs. 92500 (B) Rs. 98500
(C) Rs. 97500 (D) Rs. 94000
33. Which among the following is a better option for the buyer?
- I. Single discount of 45%.
II. Two successive discounts of 25% and 20%.
- (A) Only I (B) Both are equal
(C) Only II (D) Can't say
34. A trader fixed the price of an article in such a way that by giving a rebate of 10% on the price fixed he made a profit of 15%. If the cost of the article is Rs. 72, the price fixed on it is:
- (A) Rs. 82.80 (B) Rs. 90.00
(C) Rs. 92.00 (D) Rs. 97.80
35. The MP of an article is 30% higher than its CP and 20% discount is allowed on this article then the profit percentage?
- (A) 10% (B) 14%
(C) 4% (D) 25%
36. The difference between a discount of 35% and two successive discounts of 20% and 20% on a

certain bill was Rs. 22. What is the amount of the bill?

- (A) Rs. 1100 (B) Rs. 200
(C) Rs. 2200 (D) Data inadequate

37. Sunder purchased an office bag with a price tag of Rs. 600 in a sale where 25% discount was being offered on the tag price. He was given a further discount of 10% on the amount arrived at after giving usual 25% discount. What was the final amount paid by Sunder?

- (A) 210 (B) 540
(C) 405 (D) 450

38. A shopkeeper sold a TV set for Rs. 17940, with a discount of 8% and gained 19.6%. If no discount is allowed, then what will be his gain percentage?

- (A) 25% (B) 26.4%
(C) 24.8% (D) None of these

39. An article is listed at Rs. 1800 and two successive discounts of 8% and 8% are given on it. How much would the seller gain or loss, if he gives a single discount of 16% instead of two discounts?

- (A) Rs. 11.52 loss (B) Rs. 11.52 gain
(C) Rs. 12.62 loss (D) Rs. 12.52 gain

Practice exercise Level 2

1. The margin percentage on a certain item is 40%. What will be the profit if cost price is Rs. 600?

- (A) 40% profit (B) 66.67% profit
(C) 80% profit (D) None of these

2. Garg H and W Company sells electronic calculators at Rs. 96 per calculator in such a way that the profit percentage is same as the cost price of a calculator. If it sells at twice the percentage of its previous profit percentage, what is the new selling price of the calculator?

- (A) Rs. 132 (B) Rs. 120

- (C) Rs. 100 (D) Rs. 110

3. The profit earned by selling an article for Rs. 317 is equal to the loss incurred when the same article is sold for Rs. 233. What should be the sale price of the article for making 20% profit?

- (A) Rs. 390 (B) Rs. 370
(C) Rs. 350 (D) Rs. 330

4. If a man purchases 7 oranges for Rs. 8 and sells 8 oranges for Rs. 7. How much profit or loss does he make?

- (A) $\frac{275}{16}$ % loss (B) $\frac{275}{16}$ % profit
(C) $\frac{375}{16}$ % loss (D) $\frac{375}{16}$ % profit

5. A milkman mixes 20 L of water to 100 L of milk. What is the overall profit or loss %?

- (A) 20% (B) 25%
(C) 33.33% (D) 16.66%

6. If selling price is doubled, the profit triples. What is the profit percentage?

- (A) $66\frac{2}{3}$ % (B) 100%
(C) $105\frac{1}{3}$ % (D) 120%

7. Vineet calculates his profit percentage on the selling price whereas Roshan calculates his profit on the cost price. They find that the difference of their profit is Rs. 275. If the selling price of both of them are the same and Vineet gets 25% profit and Roshan gets 15% profit, then what is their selling price?

- (A) Rs. 2100 (B) Rs. 2300
(C) Rs. 2350 (D) Rs. 2250

8. A horse and a carriage together cost Rs. 8000. If by selling the horse at a profit of 10% and the carriage at a loss of 10%, a total profit of 2.5% is made, then what is the cost price of the horse?

- (A) Rs. 3000 (B) Rs. 3500
(C) Rs. 4000 (D) Rs. 5000

9. A man purchases a certain number of toffees at 6 per rupee and the same number of toffees at 7 per rupee. He mixes the toffees and sells them at 6 per rupee. What is his gain or loss percent?
- (A) $6\frac{9}{13}\%$ loss (B) $7\frac{9}{13}\%$ gain
(C) $6\frac{5}{13}\%$ gain (D) $7\frac{5}{13}\%$ gain
10. Seema sold a cycle worth Rs. 9000 to Sapna at 10% loss. Sapna sells the cycle back to Seema at 10% gain. What is the next change in the price of the cycle?
- (A) Gain = 1% (B) Loss = 1%
(C) Gain = 9% (D) Loss = 9%
11. Kamal bought a certain quantity of potato at the rate of Rs. 1500 per quintal. 10% of the potato was spoiled. At what price should he sell the remaining to gain 20% of his outlay?
- (A) Rs. 1800 per quintal (B) Rs. 2000 per quintal
(C) Rs. 2200 per quintal (D) Rs. 2400 per quintal
12. Find the selling price of an article if two merchants claim to make 25% profit each, one calculating it on cost price while another on the selling price, the difference in the profits earned being Rs. 100 and selling price being the same in both the cases?
- (A) Rs. 2200 (B) Rs. 1800
(C) Rs. 1200 (D) None of these
13. Mr. Errorfree bought 16 pens and sold them gradually. Due to a calculation mistake in fixing the SP, he lost an amount equal to the SP of 4 pens. What is his loss percentage?
- (A) 33.33% (B) 20%
(C) 25% (D) 66.66%
14. A class prepared 85 kites. They bought 25 sheets of coloured paper at 30 paise each; glue worth Re. 1 and sticks worth Re. 1. They sold kites at 60 paise each but were able to sell only 75 of them. The rest 10 were sold among themselves at a cheaper rate. Their gain was 400%. At what rate did they buy the kites themselves?
- (A) 30 paise (B) 25 paise
(C) 35 paise (D) 20 paise
15. A dishonest dealer sells the goods at 10% loss on cost price but uses 20% less weight. What is his percentage profit or loss?
- (A) $12\frac{1}{2}\%$ loss (B) 12.5% gain
(C) 13.5% gain (D) 12% gain
16. A dishonest dealer sells the goods at 20% loss on cost price but uses 25% less weight. What is his percentage profit or loss?
- (A) $6\frac{11}{17}\%$ loss (B) $6\frac{1}{3}\%$ loss
(C) $6\frac{3}{2}\%$ gain (D) $6\frac{2}{3}\%$ gain
17. A trader uses a weight of 920 gram instead of 1 kg and sells the articles at the marked price which is 15% above the cost price. What is the profit percentage?
- (A) 20% (B) 23%
(C) 25% (D) 10%
18. A dishonest dealer sells articles at 10% loss on cost price but uses the weight of 16 g instead of 18 g. What is his profit or loss percentage?
- (A) $1\frac{1}{4}\%$ gain (B) $1\frac{1}{4}\%$ loss
(C) $3\frac{1}{4}\%$ loss (D) $5\frac{1}{4}\%$ gain
19. The marked price of a shirt and trousers are in the ratio 1 : 2. The shopkeeper gives 40% discount on the shirt. If the total discount on both is 30%, then what is the discount offered on the trousers?
- (A) 15% (B) 20%
(C) 25% (D) 30%
20. Left pan of a faulty balance weighs 100 gram more than its right pan. A shopkeeper keeps

the weight measure in the left pan while buying goods but keeps it in the right pan while selling his goods. He uses only 1 kg weight measure. If he sells his goods at the listed cost price, what is his gain percentage?

- (A) $\frac{200}{9}\%$ (B) $\frac{100}{11}\%$
 (C) $\frac{100}{9}\%$ (D) $\frac{200}{11}\%$

21. A milkman mixes 25% water to milk and sells the mixture at a markup of 10%. What is the overall profit %?

- (A) 25% (B) 35%
 (C) 37.5% (D) None of these

22. An item is marked at a mark-up of $x\%$. It is discounted by $y\%$ while being sold to a customer. If the shopkeeper does not gain or lose anything, which of the following is definitely true?

- (A) $x = y$ (B) $x < y$
 (C) $x > y$ (D) No definite relation exists

23. The milkman mixes x L of water to 145 L of milk and sells the mixture at a discount price of Rs. 36/L. If pure milk costs Rs. 45/L and the milkman still makes an overall profit of 20%, what is the value of x ?

- (A) 29 L (B) 36 L
 (C) 72.5 L (D) Can't say

24. A trader has some watches in his stock. He marks his watches 20% above the cost price. He sold half the stock at marked price, one quarter at a discount of 20% on the marked price and rest on discount of 40% on the marked price. What is his gain percentage?

- (A) 2% (B) 5%
 (C) 3% (D) 8%

25. The marked price of a watch is Rs. 800. A shopkeeper gives two successive discounts and

sells the watch for Rs. 612. If the first discount is 10%, then what is the second discount?

- (A) 12% (B) 20%
 (C) 15% (D) 10%

26. A tradesman gives 4% discount on the marked price and 1 article free with every 15 articles bought and still gains 35%. The marked price is more than the cost price by:

- (A) 40% (B) 39%
 (C) 20% (D) 50%

27. Instead of listing an article at 20% above its cost price and selling it at a discount of 25%, a trader listed it 25% above its cost price and then sold it at a discount of 20%. What is the gain or loss to the trader because of the changed selling price in terms of the original selling price?

- (A) 10% loss (B) 10% gain
 (C) No loss, No gain (D) $11\frac{1}{9}\%$ gain

28. A trader marks up an article by $x\%$ and gives a discount of $y/2\%$ and gets a profit of $y\%$. Another trader marks up another article by $y\%$ and gives a discount of $x/2\%$ and gets a loss of $x/4\%$. If I mark up an article by $2x\%$ which I bought for Rs. 500 and give a discount of $y\%$ on the article, then what would be my profit?

- (A) Rs. 250 (B) Rs. 500
 (C) Rs. 100 (D) Can't say

29. A cycle merchant allows 25% commission on his advertised price and still makes a profit of 20%. If he gains Rs. 60 over the sale of one cycle, what is the advertised price of a cycle?

- (A) Rs. 520 (B) Rs. 420
 (C) Rs. 500 (D) Rs. 480

30. A trader marks the CP of an article at a profit of 15% but gives 5% discount. During the month, he incurs miscellaneous expenses of Rs. 325 and still makes a net profit of 6%. What is the CP and profit?

- (A) Rs. 480, 26.8 (B) Rs. 520, 20
(C) Rs. 420, 25.2 (D) Rs. 580, 23.4
31. In his catalogue a manufacturer A prices his goods 80 percent above the cost of production. He sells to an agent B allowing him 33.33% discount off catalogue prices, while B sells to a retailer C allowing him 25% off catalogue prices. What is the percentage profit made by A and B respectively?
- (A) 25%, 37.5% (B) 25%, 12.5%
(C) 20%, 12.5% (D) 20%, 37.5%
32. It is found that in the manufacturing of an article the cost of material and that of labour are in the ratio 5 : 9. The cost of material alone is Rs. 15. The marked price of the finished product is fixed by adding 20% to the total cost. What would be

the total amount received from selling 50 articles at a discount of 10%? Also express the net profit as a percentage of the cost.

- (A) 12% (B) 15%
(C) 20% (D) 8%
33. A publisher issued 5000 copies of a book and its cost is Rs. 1.50 each book. The published price for each book was Rs. 2.75. The publisher allows the bookseller discount of 20% and counts 25 copies for 24 bought at a time. All the books sold in lots of 24, if the author is paid 50% of the profits, how much does the author get?
- (A) 26.4% (B) 25.2%
(C) 24.8% (D) 27.2%

Solution

Practice Exercise Level 1

- 1.(A) Required value
 $= 2000 \times \frac{120}{100} = 2400$
- 2.(D) 3 Selling price = 3 Cost price + 2 Cost price,
 3 Selling price = 5 Cost price
 $\Rightarrow \frac{\text{Selling price}}{\text{Cost price}} = \frac{5}{3}$
 Hence, profit % = $\frac{2}{3} \times 100 = 66.67\%$
- 3.(A) 20% equal to Rs. 100
 Hence, cost price
 $= \frac{100 \times 100}{20} = 500$
 Selling price = Rs. 600
- 4.(C) Cost price = $\frac{10.80}{0.9} = 12$
 Selling price = $12 \times 1.2 = 14.40$ Rs.
- 5.(D) Selling price of 320 mangoes = cost price of 400 mangoes = 1
- Selling price = $\frac{400}{320} = 1.25$
 Cost price = 1
 Profit % = $\frac{0.25}{1} \times 100 = 25\%$
- 6.(A) Loss = $300 - 255 = 45$
 Loss % = $\frac{45}{300} \times 100 = 15\%$
- 7.(D) Total CP = Rs. (1200 + 200)
 = Rs. 1400 and SP = Rs. 1680
 Hence, profit %
 $= \frac{1680 - 1400}{1400} \times 100 = 20\%$
- 8.(D) As per the problem:
 CP $\times 0.80$ = Rs. 240
 CP = Rs. 300
 Required SP = $300 \times 1.2 = 360$
- 9.(A) Let cost price and selling price be x and y, respectively.
 $\therefore \frac{34}{100}x = \frac{26}{100}y \Rightarrow y = \frac{17}{13}x$

$$\therefore \text{Profit \%} = \frac{y-x}{x} \times 100$$

$$= \frac{\frac{17}{13}x - x}{x} \times 100 = 30.77\%$$

10.(C) Let the cost price be x .

$$SP_1 = \text{Rs. } 340 ; SP_2 = \text{Rs. } 350$$

Extra profit in values = Rs. 10

Extra profit in percentage = 5%

So, According to the question,

$$x \times 0.05 = 10$$

$$x = \text{Rs. } 200$$

11.(A) Let the CP of the article be Rs. x .

Profit = 5%

$$\therefore SP = 1.05 \times x = \frac{21x}{20}$$

If CP would have been $(x - 5\% \text{ of } x)$, i.e.

$$\frac{19x}{20} \text{ and SP would have been } \left(\frac{21x}{20} - 1 \right),$$

then

$$\text{gain \%} = 10$$

$$\therefore \frac{\left(\frac{21x}{20} - 1 \right) - \frac{19x}{20}}{\frac{19x}{20}} \times 100 = 10$$

$$\Rightarrow \frac{2x - 20}{19x} \times 100 = 10$$

$$\Rightarrow 19x = 20x - 200$$

$$\Rightarrow x = 200$$

12.(C) Net gain/loss

$$= 24 - 20 - \frac{(24 \times 20)}{100}$$

$$= 4 - 4.8 = -0.8\%$$

So, he incurs a loss of 0.8% or

$$= \frac{0.8}{100} \times 7500 = \text{Rs. } 60$$

13.(C) Let CP_1 be x .

$$SP_1 = \text{Rs. } 240$$

$$\text{So, } CP_1 \times 1.2 = \text{Rs. } 240$$

$$CP_1 = \text{Rs. } 200$$

Let CP_2 be y .

$$SP_2 = \text{Rs. } 240$$

$$y \times 0.75 = 240$$

$$y = 320 ; CP^2 = \text{Rs. } 320$$

Required loss percentage

$$= \frac{40}{520} \times 100 = 7.69\%$$

14.(A) CP of the article

$$= \frac{960}{1.2} = \text{Rs. } 800$$

$$\therefore \text{CP of 5 articles} = \text{Rs. } 800 \times 5$$

$$= \text{Rs. } 4000$$

$$\therefore \text{SP of 5 articles} = \text{Rs. } 825 \times 5$$

$$= \text{Rs. } 4125$$

$$\therefore \text{Gain \%} = \frac{4125 - 4000}{4000} \times 100$$

$$= 3.125\%$$

15.(B) $A + B = 650$ (1)

$$\text{and } 1.2A = 0.75B \text{ (2)}$$

Solving (1) and (2) we get

$$A = 250 ; B = 400$$

16.(C) Let cost price = 25

Now, as we are adding water which is equal to $\frac{1}{4}$ of the mixture.

$$\text{Hence, selling price} = 26 \times 1.25$$

Profit percentage

$$= \frac{26 \times 1.25 - 25}{25} \times 100 = 30\%$$

17.(B) Let the cost price of the bicycle be x .

As per the question,

$$x \times \frac{11}{10} \times \frac{115}{100} \times \frac{125}{100} = 1265$$

$$\Rightarrow x = \frac{1265}{1.58}$$

$$\Rightarrow x = 800$$

18.(C) Let CP be x .

$$\frac{x - 21}{x} \times 100 = x$$

$$x^2 - 100x + 2100 = 0$$

$$x = 70, 30$$

So, both 30 and 70 are possible values of cost price.

19.(D) CP of 100 bedsheets

$$= 4000$$

CP of 20 bedsheets

$$= \frac{20}{100} \times 4000 = 800$$

$$\text{SP of 20 bedsheets} = 1.05 \times 800 = 840$$

$$\text{SP of 100 bedsheets} = 1.2 \times 4000 = 4800$$

$$\therefore \text{CP of 80 bedsheets}$$

$$= 4000 - 800 = 3200$$

$$\therefore \text{SP of 80 bedsheets}$$

$$= 4800 - 840 = 3960$$

$$\therefore \text{Gain \%} = \frac{\text{SP}_{80} - \text{CP}_{80}}{\text{CP}_{80}} \times 100$$

$$= \frac{3960 - 3200}{3200} \times 100$$

$$= \frac{760}{3200} \times 100 = 23.75\%$$

20.(C) Profit percentage

$$= \frac{40}{960} \times 100 = 4.16\%$$

21.(C) Profit percentage

$$= \frac{25}{225} \times 100 = 11.11\%$$

22.(D) $M - D - \frac{MD}{100} = P$

$$\Rightarrow M - 20 - \frac{20M}{100} = 50$$

$$\Rightarrow \frac{4}{5}M = 70, M = \frac{350}{4} = 87.5\%$$

23.(C) Let the MP be Rs. 100.

$$\text{Initial SP} = 0.75 \times 100 = 75$$

$$\text{New SP} = 0.85 \times 100 = 85$$

$$\text{Percentage change} = \frac{85 - 75}{75} \times 100$$

$$= \frac{10}{75} \times 100 = 13\frac{1}{3}\%$$

24.(C) $\text{MP} = \frac{6552}{78} \times 100 = \text{Rs. } 8400$

25.(D) $\text{MP} = \frac{2275}{65} \times 100 = \text{Rs. } 3500$

26.(A) Using the formula,

$$\frac{100 \pm (\text{Profit/Loss}\%)}{100 - \text{Discount \%}} = \frac{\text{MP}}{\text{CP}}$$

$$\frac{\text{MP}}{\text{CP}} = \frac{100 + 12}{100 - 10} = \frac{112}{90} = \frac{56}{45}$$

$$\text{Hence, } \frac{\text{CP}}{\text{MP}} = \frac{45}{56}$$

27.(A) $2000 \times 0.75 = \text{Rs. } 1500$

= Selling Price

$$\text{CP} = 1500 \times \frac{100}{150} = \text{Rs. } 1000$$

28.(A) The overall profit % can be calculated as

$$30 - 20 - \frac{600}{100} = 10 - 6 = 4\%$$

29.(D) Let the cost price of article of 100.

Then, selling price of article = 88

Now, $88 = 0.8 \text{ MP}$

$$\text{MP} = 88 \times \frac{5}{4} = 110$$

Now, $\text{SP} = 0.95 \times 110 = 104.5$

Profit = 4.5%

30.(C) Let CP = 100 ; MP = 160

Discount percentage

$$= \frac{160 - 112}{160} \times 100 = 30\%$$

31.(B) Marked Price can be compared as

$$600 \times \frac{100}{60} = \frac{250}{100} \times x$$

$x = \text{Rs. } 400$

32.(C) Marked price

$$= 50700 \times \frac{100}{80} \times \frac{100}{65} = \text{Rs. } 97500$$

33.(A) Two successive discounts of 20% and 25% is equivalent to an overall discount of

$$-20 - 25 + \frac{500}{100} = -45 + 5 = -40\%$$

That is, 40% discount is definitely lower than a single discount of 45%.

34.(C) CP = 72

$$\text{SP} = 1.15 \times 72 = 82.8$$

SP = 0.9 MP

$$\text{MP} = \frac{82.8}{0.9} = \text{Rs. } 92$$

35.(C) MP = 1.3 CP

$$\text{SP} = 0.8 \times \text{MP} = 0.8 \times 1.3 \text{ CP}$$

$$= 1.04 \text{ CP}$$

$$\Rightarrow \frac{\text{SP}}{\text{CP}} = 1.04$$

$\therefore$ Profit of 4%

36.(C) Net discount for Successive discounts

$$= 20 + 20 - \frac{20 \times 20}{100} = 36$$

Difference in discounts = $36 - 35 = 1\%$

1% of Bill amount = 22

Bill amount = 2200

37.(C) Final Amount = $0.9 \times 0.75 \times 600 = 405$

38.(D) SP = Rs. 17940

Discount = 8%

$$\therefore \text{MP} = \frac{17940 \times 100}{92} = \text{Rs. } 19500$$

Gain = 19.6%

$$\text{CP} = \frac{17940 \times 100}{100 + 19.6} = \text{Rs. } 15000$$

If no discount is allowed on the Marked Price, then SP = Rs. 19500

$$\therefore \text{Gain \%} = \frac{4500}{15000} \times 100 = 30\%$$

39.(A) Net discount

$$= -8 - 8 + \left(\frac{8 \times 8}{100} \right) = -15.36\%$$

So, discount of 15.36% = Rs. 276.48

Flat 16% discount = 288

$$\therefore \text{Loss of seller} = 288 - 276.48 = 11.52$$

Practice Exercise Level 2

1.(B) Let selling price be Rs. x .

$$x - \frac{40}{100}x = 600$$

$$x = 1000$$

Profit = Rs. 400

Hence, profit percentage

$$= \frac{400}{600} \times 100 = 66.67\%$$

2.(A) Let CP = x , Profit % = $x\%$

$$\text{SP} = \frac{x(100 + x)}{100} = 96$$

$$\Rightarrow 100x + x^2 = 9600$$

$$x^2 + 100x - 9600 = 0$$

$$x^2 + 160x - 60x - 9600 = 0$$

$$(x + 160)(x - 60) = 0,$$

Cost price = Rs. 60

New SP = $60 \times 2.2 = \text{Rs. } 132$

3.(D) Let the CP be x .

Profit = $317 - x$

Loss = $x - 233$

According to the question,

$$317 - x = x - 233$$

$$2x = 550 ; x = 275$$

Required SP = $275 \times 1.2 = 330$

4.(D) LCM of quantities (7 and 8) is 56.

So, CP of 7×8 oranges = $8 \times 8 = 64$

SP of 8×7 oranges = $7 \times 7 = 49$

$$\text{Loss \%} = \frac{15}{64} \times 100 = \frac{375}{16} \%$$

5.(A) The percentage profit is equal to the percentage of impurity added to the pure component.

Therefore, the required answer is 20%.

6.(B) Let CP = Rs. x

$\therefore$ SP = Rs. y

Profit = Rs. $(y - x)$

If, SP = $2y$, then profit = $3(y - x)$

$$\therefore 2y - x = 3(y - x)$$

$$\Rightarrow y = 2x$$

$\therefore$ Profit on Rs. x = Rs. x i.e., 100%

7.(B) Let the SP of Vineet and Roshan be x .

15% of profit on CP

$$= \frac{15}{115} \times 100 = \frac{300}{23} \% \text{ profit on SP}$$

$$\therefore \frac{x \times 25}{100} - \frac{300x}{23 \times 100} = 275$$

On solving, $x = 2300$

8.(D) Suppose CP of Horse = Rs. x

$\therefore$ CP of Carriage = Rs. $(8000 - x)$

$\therefore$ 10% of x - 10% of $(8000 - x)$

= 2.5% of 8000

$$\Rightarrow 20\% \text{ of } x = 1000$$

$$\Rightarrow x = 5000$$

9.(B) Let the number of toffees of each type be 42.

(i.e. LCM of 6 and 7)

$$\text{Total CP} = \left(\frac{42}{6}\right) + \left(\frac{42}{7}\right) = ₹ 13$$

$$\text{Total SP} = \left(\frac{84}{6}\right) = ₹ 14$$

$$\text{Gain} = \frac{1}{13} \approx 7\frac{9}{13}\% \text{ gain}$$

10.(B) CP of cycle = 9000

Seema sold = 8100

Sapna sells = 8910

So, Sapna gains = 10%

$$\text{Net change} = \frac{9000 - 8910}{9000} \times 100$$

$$= 1\%$$

11.(B) Let total quantity be 100 quintal.

$$\text{Total CP} = 1500 \times 100 = 150000$$

$$\text{To gain 20\%, SP} = 1.2 \times 150000$$

$$= 180000$$

$$\text{SP per quintal} = \frac{180000}{100} = 1800$$

12.(D) Let SP = 100x

Profit in case 1:

$$= 100x - \frac{100x}{1.25} = 20x$$

Profit in case 2:

$$= 25\% \text{ of } 100x = 25x$$

$$\text{Difference} = 5x = 100$$

$$x = 20$$

$$\text{SP} = 100 \times 20 = 2000$$

13.(B) As per the question,

$$16\text{CP} - 16\text{SP} = 4\text{SP}$$

$$\Rightarrow 16\text{CP} = 20\text{SP}$$

$$\frac{\text{SP}}{\text{CP}} = \frac{4}{5}$$

$$\therefore \text{Loss \%} = \frac{5-4}{5} \times 100$$

$$= \frac{1}{5} \times 100 = 20\%$$

14.(B) CP of 85 kites

$$= 25 \times \frac{30}{100} + 1 + 1$$

$$= 7.5 + 2 = 9.5$$

$$\text{SP of 75 kits} = 75 \times \frac{60}{100} = 45$$

$$\text{Gain \%} = 400\%$$

$$\therefore \text{SP}_{85} = \text{CP}_{85} \times (100 + \text{P}\%)$$

$$= 9.5 \times \left(100 + \frac{400}{100}\right)$$

$$= 9.5 \times 5 = 47.5$$

$$\therefore \text{SP of 10 kites} = 47.5 - 45 = 2.5$$

$$\therefore \text{SP of 1 kite} = \frac{2.5}{10} = 0.25$$

$\therefore$ Kids bought the remaining kites at 25 paise each.

15.(B) Let correct weight

$$= 1000 \text{ gm}$$

$$\text{and CP of 1000 gm} = 1000 \times 1$$

$$= \text{Rs. } 1000$$

$$20\% \text{ less weight} = 1000 \times 0.8$$

$$= 800$$

Profit while using less weight = 200, 10% loss on CP will be calculated on actual CP

$$\text{Loss} = 1000 \times 0.1 = 100$$

$$\text{So, net profit} = \frac{200 - 100}{800} \times 100$$

$$= 12.5\%$$

16.(D) Let the actual weight be 1000 gm.

$$\text{Cost price of 1000 gm} = 1000 \times 1 = \text{Rs. } 1000$$

$$25\% \text{ less weight} = 1000 \times 0.75$$

$$= 750 \text{ gm}$$

$$\text{Profit while using 25\% less weight} = 250$$

$$20\% \text{ loss on cost price}$$

$$= 1000 \times 0.2 = 200$$

$$\text{So, net profit} = \frac{250 - 200}{750} \times 100$$

$$= \frac{20}{3}\% = 6\frac{2}{3}\%$$

17.(C) Let CP = Re. 1/gram

Trades sells 920 gram at Rs. 1150 (1.15 × 1000)

$$\therefore \text{Profit \%} = \frac{230}{920} \times 100 = 25\%$$

18.(B) Let CP = Rs. 1 for 1 g

$$\text{CP for 18 g} = 18$$

$$\text{SP for 18 g} = 18 \times 0.9 = 16.2$$

$$\text{Gain \%} = \frac{0.2}{16} \times 100 = 1.25\%$$

19.(C)

	Shirt	Trouser	Total
MP	x	2x	3x
Discount	40%	?	30%
SP	0.6x	?	2.1x

$$\begin{aligned}\text{So, SP of trouser} &= (2.1 - 0.6)x \\ &= 1.5x\end{aligned}$$

$$\text{Discount} = \frac{0.5}{2} \times 100 = 25\%$$

20.(A)

Let the price of 1 g be Rs. 1.

The shopkeeper buys 1100 g at the rate of Rs. 1000

He sells 900 g at the cost price of Rs. 1000.

So, CP for 900 g would be

$$= \frac{1000}{1100} \times 900 = \text{Rs. } 818.18$$

So, gain %

$$= \frac{1000 - 818.18}{818.18} \times 100 = 22.22\%$$

$$\text{or } \frac{200}{9}\%$$

21.(C)

Water added is 25% and so the profit earned by addition of impurity will be 25%. Also, 10% mark-up will additionally give a profit of 10%.

Now to find net profit, we can use successive percentage change

Overall profit %

$$= 25 + 10 + \frac{25 \times 10}{100} = 37.5\%$$

22.(C)

As y is being calculated on a higher value, (the marked price), therefore, to get the same effect, x has to be greater than y.

23.(C)

Discount offered

$$= \frac{45 - 36}{45} \times 100 = 20\% \text{ discount}$$

Let x represent the percentage profit due to addition of impurity, then

$$x - 20 - \frac{20x}{100} = 20$$

$$\Rightarrow \frac{80x}{100} = 40 \Rightarrow x = 50\%$$

Water added is 50% of the volume of pure milk. As the quantity of milk is 145 L, therefore, the quantity of water added = 50% of 145 = 72.5 L

24.(A)

Let CP of each watch = 100

MP of each watch = 120

Let the watches = 4 Total CP = 400

$$\begin{aligned}\text{Total SP} &= [120 \times 2 + 120 \times 0.8 + 120 \times 0.6] \\ &= 408\end{aligned}$$

$$\% \text{ Profit} = \left(\frac{408}{400} - 1 \right) \times 100 = 2\%$$

25.(C)

Let the second discount be x%.

Price after first discount

$$= 0.9 \times 800 = 720$$

Final Price = 612

$$\text{Second discount} = \frac{108}{720} \times 100 = 15\%$$

26.(D)

Discount on articles

$$= \frac{1}{16} \times 100 = 6.25\%$$

Overall discount

$$= -4 - 6.25 + \frac{4 \times 6.25}{100} = -10\%$$

Let cost price = Rs. 100, then

Selling price = Rs. 135

So, 90% of marked price = 135

$$\text{Marked Price} = \frac{135 \times 100}{90} = \text{Rs. } 150$$

Marked price is increased by

$$= \frac{150 - 100}{100} \times 100 = 50\%$$

27.(B)

Original gain percent

$$= 20 - 25 - \frac{20 \times 25}{100} = -10\%$$

New gain percent

$$= 25 - 20 - \frac{20 \times 25}{100} = 0\%$$

There is a 10% gain.

28.(A)

Case I:

$$x - \frac{y}{2} - \frac{xy}{200} = y$$

Case II:

$$y - \frac{x}{2} - \frac{xy}{200} = -\frac{x}{4}$$

Subtracting case (II) from (I),

$$x - y - \frac{y}{2} + \frac{x}{2} = y + \frac{x}{4}$$

$$\frac{3x}{2} - \frac{x}{4} = \frac{3y}{2} + y$$

$$\frac{5x}{4} = \frac{5y}{2}$$

$$x = 2y$$

Putting in case (I),

$$2y - \frac{y}{2} - \frac{2y^2}{200} = y$$

$$\frac{y}{2} = \frac{2y^2}{200}$$

$$y = 50$$

$$\therefore x = 100$$

Case III:

$$CP = 500$$

$$MP = 3 \times 500 = 1500 \text{ (as mark up is } 2x = 200\%)$$

$$\text{Discount} = 50\% \text{ of } 1500 = 750$$

$$SP = 1500 - 750 = 750$$

$$\text{Profit} = 750 - 500 = 250$$

29.(D) Let the advertised price be x .

$$\text{Then, } SP = \frac{3}{4}x$$

$$\text{Also, } SP = 1.2CP$$

$$\Rightarrow SP = \frac{6}{5}CP \quad \dots (1)$$

$$\text{Profit} = 60 = SP - CP$$

$$\Rightarrow SP = 60 + CP \quad \dots (2)$$

Put (2) in (1),

$$60 + CP = \frac{6}{5}CP$$

$$300 + 5CP = 6CP$$

$$\Rightarrow CP = 300 \quad \Rightarrow SP = 360$$

$$\therefore x = \frac{4 \times 360}{3}$$

$$= 4 \times 120$$

$$\therefore x = 480$$

30.(C) $d\% = 5\%$, Profit % = 15%

$$\therefore \frac{CP}{MP} = \frac{MF_d}{MF_p}$$

$$\Rightarrow \frac{CP}{MP} = \frac{95}{115}$$

Let the CP be 95.

$$\therefore SP = (CP + 325) \times 1.06$$

$$\Rightarrow SP = (95 + 325) \times 1.06$$

$$\Rightarrow SP = 420 \times 1.06$$

$$\Rightarrow SP = 445.2$$

$$CP = 420$$

$$\therefore \text{Profit} = 25.2$$

31.(C)Let the cost price of A be CP_A .Let the catalogue price of A be MP_A .

$$\therefore MP_A = 1.8 CP_A$$

$$\text{Also, } SP_A = \frac{2}{3} MP_A$$

$$\Rightarrow SP_A = \frac{2}{3} \times 1.8 CP_A = 1.2 CP_A$$

$$\Rightarrow \frac{SP_A}{CP_A} = 1.2$$

$$\therefore P\% \text{ of A} = 20\%$$

$$SP_A = CP_B = \frac{2}{3} MP_A$$

$$\text{Also, } SP_B = \frac{3}{4} MP_A$$

$$\Rightarrow \frac{3}{2} CP_B = \frac{4}{3} SP_B$$

$$\Rightarrow \frac{CP_B}{SP_B} = \frac{8}{9}$$

$$\therefore P\% \text{ of B} = \frac{9-8}{8} \times 100$$

$$= \frac{1}{8} \times 100 = 12.5\%$$

32.(D)

Cost price of material = 15

Cost price of labour = 27

Total cost price of production of 50 articles
 $= (15 + 27) \times 50 = 2100$ Marked price = $2100 \times 1.2 = 2520$ SP = $0.9 \times 2520 = 2268$

$$\therefore P\% = \frac{2268 - 2100}{2100} \times 100 = \frac{168}{2100} \times 100 = 8\%$$

33.(A)

CP of 1 book = 1.5

MP of 1 book = 2.75

$$\therefore \text{SP of 1 book} = 2.75 \times 0.8 = 2.2$$

	25	24
CP	37.5	36
SP	55	52.8

$$\therefore \text{P\%} = \frac{\text{SP}_{25} - \text{CP}_{24}}{\text{CP}_{24}} \times 100$$

$$= \frac{55 - 36}{36} \times 100 = \frac{19}{36} \times 100 = 52.8\%$$

$$\therefore \text{P\% of author} = \frac{52.8\%}{2} = 26.4\%$$

TOPRANKPREP

Ratio & Proportion

05

RATIO

A ratio is a comparison of two or more quantities of similar type. The ratio of a and b is written as $a:b = a/b$.

In the ratio $a : b$, a and b are called the terms of the ratio where 'a' is the antecedent and 'b' is the consequent.

Properties of Ratio

- In a ratio, the quantities which has to be compared must be of the same kind, i.e. they must be expressed in the same units.
- The ratio of two quantities determines how many times of one quantity is contained by the other.
- The order of the terms in a ratio ' $a : b$ ' is very important. Since $4 : 5$ is different from $5 : 4$.

Dividing a number in the given Ratio

Let 'A' be the number. The ratio given is $a_1 : a_2$. Here 'A' is to be divided in the ratio $a_1 : a_2$. It implies that A is divided in two parts such that value of first part : value of second part = $a_1 : a_2$.

Therefore, First part = $\frac{a_1}{a_1 + a_2} \times A$ and

Second part = $\frac{a_2}{a_1 + a_2} \times A$

Note:

These relations are also true when we divide a number into more than two ratios (i.e. into more than two parts).

Example 1: Two numbers are in the ratio $8 : 9$. If the sum of the numbers is 119, then what are the numbers?

Solution:

Since the sum of two numbers is 119, so 119 has to be divided in two parts in the ratio $8 : 9$.

Therefore, First number = $\frac{8}{8+9} \times 119 = 56$

Second number = $\frac{9}{8+9} \times 119 = 63$

Alternate Method:

As we know that the above ratio i.e. $8 : 9$ is divided into two sections.

First section = 8 parts

Second section = 9 parts

Total = 17 parts

17 parts = 119

1 part = 7

8 parts = 56 ; 9 parts = 63

Numbers are 56 and 63.

Example 2: If Rs. 3200 is divided among P, Q and R in the ratio $5 : 2 : 9$, then what is the amount received by Q?

Solution:

Amount received by Q = $\frac{2}{5+2+9} \times 3200 = ₹ 400$

Alternate Method:

Ratio is divided in three sections.

First section = 5 parts

Second Section = 2 parts

Third Section = 9 parts

Sum of sections = 16 parts

These 16 parts is equal to Rs. 3200.

1 part = Rs. 200

Amount received by Q = 2 parts = Rs. 400

Change in Ratio

Suppose $a : b = 2 : 3$ or $a/b = 2/3$

If we multiply/divide the numerator and denominator by the same quantity, ratio remains same.

$$\text{e.g. } \frac{a}{b} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9} = \frac{2}{3} \Rightarrow a:b = 2:3$$

If we add/subtract the same quantity from the numerator and denominator, the ratio may or may not remain same.

$$\text{e.g. } \frac{a}{b} = \frac{2+3}{3+3} = \frac{5}{6} \neq \frac{2}{3} \Rightarrow a:b = 5:3 \neq 2:3$$

Example 3: In a rare coin collection, there is one gold coin for every three non gold coins. 10 more gold coins are added to the collection and the ratio of gold coins to non gold coins is 1 : 2. What is the total number of coins in the collection now becomes?

Solution:

Ratio of gold to non gold = 1 : 3

So, assume gold coins = x

non gold coins = 3x

Now, 10 gold coins are added

$$\frac{x+10}{3x} = \frac{1}{2} \Rightarrow 2x+20=3x$$

$$\Rightarrow x = 20$$

Coins in the collection now = $x + 3x + 10 = 4x + 10$

$$= (4 \times 20) + 10 = 90$$

Example 4: A person has only Rs. 1 and Rs. 2 coins with him. If the total number of coins he has is 50 and the amount of money with him is Rs. 75, then what is the respective number of Rs. 1 and Rs. 2 coins?

Solution:

Let number of Rs. 1 and Rs. 2 coins be x and y respectively.

$$\text{then, } x + y = 50 \quad \dots (1)$$

$$x + 2y = 75 \quad \dots (2)$$

Solving (1) and (2),

$$y = 25, x = 25$$

Example 5: The first, second and third class fares between two railway stations were 10 : 8 : 3 and the number of first, second and third class passengers between the two stations was 3 : 4 : 10. If total sales of tickets, is Rs. 8050, the what is the amount of the

money obtained from the sales of second class tickets?

Solution:

Let the fares of first, second and third class be 10x, 8x and 3x respectively.

Let the passengers of first, second and third class be 3y, 4y and 10y.

Total sales = Per ticket fare $\times$ Number of passengers

Total first class sales = 30xy

Total second class sales = 32xy

Total third class sales = 30xy

$$\text{Now, } 30xy + 32xy + 30xy = 8050$$

$$xy = 87.5$$

$$\text{Required sales} = 32xy$$

$$= 32 \times 87.5 = 2800$$

VALUES IN FRACTIONS

Sometimes the ratio of the quantities can be

expressed in terms of fractions such as. 1/3:1/7

Instead of dividing the numbers in the ratio [in terms of fractions, we should convert these fractions into integers.

For example, 1/3:1/7

LCM of 3 and 7 will be 21.

$$\text{So, ratio will be } \frac{1}{3} \times 21 : \frac{1}{7} \times 21 = 7 : 3$$

In another case, we are having ratio of three terms in the form of fractions as,

$$\frac{1}{3} : \frac{1}{7} : \frac{1}{8}$$

LCM of 3, 7 and 8 will be 168.

$$\text{So, the ratio will be } \frac{1}{3} \times 168 : \frac{1}{7} \times 168 : \frac{1}{8} \times 168$$

$$= 56 : 24 : 21$$

Example 6: If $\frac{L}{9} = \frac{M}{4} = \frac{N}{3}$, then what is the value of L :

M : N?

Solution:

$$\text{Put } \frac{L}{9} = \frac{M}{4} = \frac{N}{3} = K$$

$$L = 9K ; M = 4K ; N = 3K$$

So, $L : M : N = 9K : 4K : 3K$ or $9 : 4 : 3$.

Example 7: If $3A = 5B = 9C$, then what is the value of $A : B : C$?

Solution:

$$3A = 5B = 9C = K$$

$$A = \frac{K}{3}; B = \frac{K}{5}; C = \frac{K}{9}$$

$$\text{So, } A : B : C = \frac{K}{3} : \frac{K}{5} : \frac{K}{9}$$

$$\text{or } A : B : C = \frac{1}{3} : \frac{1}{5} : \frac{1}{9}$$

$$= \frac{1}{3} \times 45 : \frac{1}{5} \times 45 : \frac{1}{9} \times 45 = 15 : 9 : 5$$

Example 8: A sum of Rs. 782 has been divided among A, B and C in the ratio of $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$. What is the share of C?

Solution:

$$A : B : C = \frac{1}{2} : \frac{2}{3} : \frac{3}{4}$$

LCM of (2, 3 and 4) = 12

Multiply all values by 12

$$12 \times \frac{1}{2} : 12 \times \frac{2}{3} : 12 \times \frac{3}{4}$$

Ratio is 6 : 8 : 9.

$$\text{Now, share of C} = \frac{9}{23} \times 782 = 306$$

RATIO WITH COMMON VARIABLE

Sometimes, we are given with two or more ratios with a common variable. To get the equivalent ratio we convert the value of common variable same in all ratios.

For example,

if $a : b = 2 : 3$ and $b : c = 4 : 5$. We see the common variable is 'b'. So, we will make the value of 'b' same in both the ratios.

So, the multiply first ratio by 4 (value of 'b' in second ratio) and second ratio be 3 (value of 'b' in first ratio).

$$\text{So, } a : b = 2 \times 4 : 3 \times 4 \\ = 8 : 12$$

$$\text{and } b : c = 4 \times 3 : 5 \times 3 \\ = 12 : 15$$

Hence, $a : b : c = 8 : 12 : 15$

Example 9: If $B : C = 5 : 6$, $C : D = 7 : 8$, then what will be the value of $B : C : D$?

Solution:

Common variable in $B : C$ and $C : D$ is C. So, first of all we will make this variable same.

$$\text{So, } B : C = 5 \times 7 : 6 \times 7$$

$$C : D = 7 \times 6 : 8 \times 6$$

$$\text{So, } B : C : D = 35 : 42 : 48$$

Shortcut Method:

- For $a : b$, $b : c$, to find $a : b : c$.

$$\frac{a}{b} = \frac{N_1}{D_1}, \frac{b}{c} = \frac{N_2}{D_2}$$

$$\text{So, } a : b : c = N_1 \times N_2 : D_1 \times N_2 : D_1 \times D_2$$

- For $a : b$; $b : c$, $c : d$, to find $a : b : c : d$.

$$\frac{a}{b} = \frac{N_1}{D_1}, \frac{b}{c} = \frac{N_2}{D_2}, \frac{c}{d} = \frac{N_3}{D_3}$$

$$\text{So, } a : b : c : d = N_1 \times N_2 \times N_3 : D_1 \times N_2 \times N_3 : D_1 \times D_2 \times N_3 : D_1 \times D_2 \times D_3$$

Example 10: If $A : B = 4 : 5$, $B : C = 3 : 4$, $C : D = 7 : 11$, then what is the value of $A : D$?

Solution:

$$\frac{A}{B} = \frac{4}{5}, \frac{B}{C} = \frac{3}{4}, \frac{C}{D} = \frac{7}{11}$$

$$\text{So, } A : B : C : D = 4 \times 3 \times 7 : 5 \times 3 \times 7 : 5 \times 4 \times 7 : 5 \times 4 \times 11$$

$$A : B : C : D = 84 : 105 : 140 : 220$$

$$\text{Required } A : D = 84 : 220 = 21 : 55$$

Alternate Method:

$$\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{A}{D}$$

$$\text{So, required value} = \frac{4}{5} \times \frac{3}{4} \times \frac{7}{11} = \frac{21}{55}$$

PROPORTION

When two ratios are equal they are known to be in Proportion to each other.

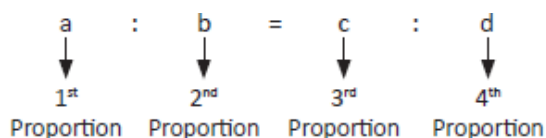
For example, $a : b = c : d$ or $a : b :: c : d$, are said to be in proportion.

Which can also be written as:

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow a \times d = b \times c$ or product of extremes (a and d) = product of middles (b and c)

Note:



Useful Results on Proportion

(i) $\frac{b}{a} = \frac{d}{c}$ (Invertendo)

(ii) $\frac{a}{c} = \frac{b}{d}$ (Alternando)

(iii) $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

(iv) $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

(v) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo and dividendo)

Example 11: What number should be subtracted from each of 23, 29, 35 and 45 so that the numbers are in proportion?

Solution:

Let the number to be subtracted be x.

$$\text{Then, } \frac{23-x}{29-x} = \frac{35-x}{45-x}$$

$$\text{As we know, if } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\text{We have, } \frac{23-x+29-x}{23-x-29+x} = \frac{35-x+45-x}{35-x-45+x}$$

$$\Rightarrow \frac{52-2x}{6} = \frac{80-2x}{10}$$

$$\Rightarrow 520 - 20x = 480 - 12x$$

$$\Rightarrow 20x - 12x = 40$$

$$\Rightarrow 8x = 40$$

$$\Rightarrow x = 5$$

Continued Proportion

When $a : b = b : c$ or $a : b :: b : c$, then the numbers are said to be in continued proportion.

$$\text{So, } \frac{a}{b} = \frac{b}{c}$$

$$\Rightarrow b^2 = ac$$

In the above case, 'b' is also known as mean proportion between 'a' and 'c'.

Example 12: Find the mean proportion between 9 and 16.

Solution:

$$\text{Required mean proportional} = \sqrt{9 \times 16} = \sqrt{144} = 12$$

TYPES OF PROPORTION

When two or more quantities are dependent on each other and then if any one of them is changed, the other (dependent) quantity is also changed.

For example,

- I. When the salary of a person increases, then his savings/expenditure increase.
- II. When the number of guests in a hotel/number of students in hostel/number of employees changes, their respective expenses change.

Basically, there are two types of proportion:

1. Direct Proportion
2. Inverse Proportion

Direct Proportion

One quantity A is said to vary directly with another quantity B if the two quantities depend on each other in such a manner that if B is increased in a certain ratio, A also increases in the same ratio and if B is decreased in a certain ratio, A also decreases in the same ratio.

This is denoted as $A \propto B$ i.e. A varies directly as B.

- If $A \propto B$, then $A = kB$, where k is a constant. It is called the "Constant of Proportion".

For example, when the quantity of sugar purchased by a housewife is double the original quantity, the total amount she spends on sugar

also doubles, i.e., the quantity and the amount increases (or decreases) in the same ratio.

From the above definition of direct proportion, we can see that when two quantities A and B vary directly with each other, then $A/B = k$ or the ratio of the two quantities is a constant.

Conversely, when the ratio of two quantities is a constant, we can conclude that they vary directly with each other.

If X varies directly with Y and we have two sets of values of the variables X and Y, X_1 corresponding to Y_1 and X_2 corresponding to Y_2 , then, since $X \propto Y$, we can write down:

$$\frac{X_1}{X_2} = \frac{Y_1}{Y_2} \text{ or } \frac{X_1}{Y_1} = \frac{X_2}{Y_2}$$

Inverse Proportion

A quantity A is said to vary inversely with another quantity B if the two quantities depend on each other in such a manner that if B is increased in a certain ratio, A gets decreased in the same ratio and if B is decreased in a certain ratio, then A gets increased in the same ratio.

- It is the same as saying that A varies directly with $\frac{1}{B}$. It is denoted as $A \propto \frac{1}{B}$ i.e. $A = \frac{k}{B}$, where k is the constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.

From the definition of inverse proportion, we can see that when two quantities A and B vary inversely with each other, then $A \times B = k$ constant, i.e., the product of the two quantities is a constant. Conversely, if the product of two quantities is a constant, we can conclude that they vary inversely with each other.

If X varies inversely with Y and we have two sets of values of X and Y, X_1 corresponding to Y_1 and X_2 corresponding to Y_2 , then since X and Y are inversely related to each other, we can write down.

$$X_1 Y_1 = X_2 Y_2 \text{ or } \frac{X_1}{X_2} = \frac{Y_2}{Y_1}$$

Example 13: The value of a coin varies directly to the square of its radius, when its thickness is constant. The radius of a coin is 1.5 cm and its value is Rs. 2. What will be the radius of the coin if its value is Rs. 5?

Solution:

Let the value of coin be 'V'

Let the radius of the coin be 'r'

$$V \propto r^2$$

$$V = Kr^2$$

$$2 = K(1.5)^2$$

$$K = \frac{2}{2.25}$$

$$K = \frac{8}{9}$$

$$5 = \frac{8}{9} \times r^2$$

$$r^2 = \frac{5 \times 9}{8}$$

$$r = \frac{3}{2} \times \sqrt{\frac{5}{2}} = 1.5 \times \sqrt{2.5} = 1.5 \times 1.6$$

$$r \approx 2.4 \text{ cm}$$

Hence, required radius = 2.4 cm (approximately)

Example 14: According to a law, at a constant temperature, the pressure of a definite mass is inversely proportional to its volume. If pressure is increased by 25%, then what will be the net change in volume?

Solution:

As pressure is inversely proportional to volume,

$$P \propto \frac{1}{V}$$

$$P = \frac{K}{V}$$

So, $K = P.V$

Now, as we know that this value of K will have to be same throughout the question.

According to question,

$K = P \times \frac{5}{4} \times V_{\text{new}}$, as we know the value of K has to be $P.V$.

So, new volume (V_{new}) = $\frac{4}{5}V$

So, $K = P \times \frac{5}{4} \times V \times \frac{4}{5}$

$K = P \times V$

So, to keep K 's value same, we have to reduce V by 20%.

Alternate Method:

Let pressure be P and volume be V .

$$P \propto \frac{1}{V}$$

$$P = \frac{K}{V}$$

So, $K = PV$, where K is constant.

Let the initial pressure be 4 and initial volume be 5.

$$25\% \text{ increase} = \frac{1}{4} \times 4 = 1$$

$\therefore$ Final pressure = $1 + 4 = 5$

	Pressure	Volume	Value of constant
Initial	4	5	$4 \times 5 = K$
Final	5	4	$5 \times 4 = K$

Example 15: The speed of a railway engine is 42 km per hour when no compartment is attached and the reduction in speed is directly proportional to the square root of the number of compartments attached. If the speed of the train of this engine is 24 km per hour when 9 compartments are attached, what is the maximum number of compartments that can be carried by engine?

Solution:

$S = V - K\sqrt{x}$, where V = Original Speed

$K\sqrt{x}$ = Reduced Speed

x = Number of attached compartments

Given, $V = 42$

So, $S = 42 - K\sqrt{x}$

Also given,

$S = 24$, when $x = 9$

Hence, $24 = 42 - K\sqrt{9}$

$$\Rightarrow 3K = 18$$

$$\Rightarrow K = 6$$

So, $S = 42 - 6\sqrt{x}$

Let $S = 0$

$$\Rightarrow 42 = 6\sqrt{x}$$

$$\Rightarrow \sqrt{x} = 7$$

$$\Rightarrow x = 49$$

Hence, at 49 compartments the engine will stop, so it can carry a maximum of 48 compartments.

DIRECT RELATION

Till now, what we had seen in directly proportional case is that if one quantity increases, then the other quantity corresponding to it also increases proportionally, but in the case of direct relation, if a quantity increases/decreases, then other quantity corresponding to it will also increase/decrease but not proportionally.

Let us take two equations to understand this difference.

$$z = 2r$$

z	r
2	1
4	2

$$z = 2r + 7$$

z	r
2	1
13	3
19	6

Now, let us see the reason behind the increment of 'z' but not in a proportional way. The reason is the value of constant i.e. 7 as there will be no change in this constant value. So, we can conclude that if r will

increase, then z will also increase but not proportionally.

Example 16: The expenses of a college hostel are partly fixed and partly variable depending upon the number of hostellers. When there were 50 hostellers, the charges were Rs. 140 per head, but when there were 75 hostellers, the charges were Rs. 120 per head. What will be the charge per head if there were 100 hostellers?

Solution:

As we know,

Expenses = Fixed Cost + Variable Cost

Let fixed cost be x and variable be y .

So, $E = x + Ky$ [K is the number of hostellers]

According to question,

$$140 \times 50 = x + 50y \quad \dots (1)$$

$$120 \times 75 = x + 75y \quad \dots (2)$$

By equating (1) and (2),

We get $x = 3000$ and $y = 80$

So, the total expenses for 100 hostellers.

$$E = x + 100y$$

$$E = 3000 + 100 \times 80$$

$$E = 11000$$

$$\text{Charge per head} = \frac{11000}{100} = ₹ 110$$

Alternate Method:

Number of hostellers	Charge per head	Total expense
50	140	7000
75	120	9000

So, from the above table, we can say that expense of extra 25 hostellers will be $9000 - 7000 = 2000$

Now, the hostellers became 100 from 75, so the total expense will become $9000 + 2000 = 11000$.

$$\text{Hence, charge per head} = \frac{11000}{100} = ₹ 110$$

AGES

Most of the questions based on ages involve the concept of ratio e.g., the age of Ravi is 16 years, the age of her mother is twice etc. or the ratio of the ages of father and son at present is 3 : 1. 4 years earlier, the ratio was 4 : 1. What are the present ages of the father and son etc. So, these kind of questions you see are related to ratio.

Some important points related to age:

1. If the present age of a person is x years, then after n years his age will be $(x + n)$ years.
2. The difference between the ages of two persons will always remains the same, whether the calculations are done on them few years back from now or few years after from now.
3. If there are ' n ' number of members in a family and their average is ' m ', then after k years the average age of the family will be ' $m + k$ '.

Example 17: The ratio of ages of Krishna and Balram is 3 : 4. Four years earlier, the ratio of the ages of Krishna and Balram was 5 : 7. What are their present ages?

Solution:

Let the present age of Krishna and Balram be $3x$ and $4x$, respectively, then

Four years ago their ages be $(3x - 4)$ and $(4x - 4)$

$$\text{So, } \frac{(3x - 4)}{(4x - 4)} = \frac{5}{7}$$

$$\Rightarrow 7(3x - 4) = 5(4x - 4)$$

$$\Rightarrow 21x - 28 = 20x - 20$$

$$\Rightarrow x = 8$$

$\therefore$ Present age of Krishna = $3 \times 8 = 24$ years.

and age of Balram = $4 \times 8 = 32$ years.

Example 18: The ratio of ages of Amar and his mother is 3 : 11. The difference of their ages is 24 years. What will be the ratio of their ages after 3 years?

Solution:

Let the present age of Amar and his mother be $3x$ and $11x$ respectively. Then 3 years later, their ages will be $(3x + 3)$ and $(11x + 3)$ respectively.

$$\text{Again } 11x - 3x = 24$$

$$\Rightarrow 8x = 24 \Rightarrow x = 3$$

Therefore, the ratio of their ages after 3 years

$$= \frac{3x+3}{11x+3} = \frac{12}{36} = \frac{1}{3} \text{ i.e. } 1:3$$

Example 19: The age of Sachin is 4 times that of his son. Five years ago, Sachin was nine times as old as his son was at that time. What is the present age of Sachin?

Solution:

Let the present age of son is x years, then the present age of Sachin is $4x$ years.

5 years ago,

$$\therefore (4x - 5) = 9(x - 5) \Rightarrow x = 8$$

$$\therefore \text{Age of Sachin is } 32 \text{ years.}$$

Example 20: The ratio of Varun's age and his mother's age is $5 : 11$. The difference of their ages is 18 years, then what will the ratio of their ages after 5 years?

Solution:

Let their ages be $5x$ and $11x$.

$$\therefore 11x - 5x = 18 \Rightarrow x = 3$$

So, their present ages are 15 and 33 years. Therefore, ratio of their ages after 5 years = $20 : 38 = 10 : 19$.

PARTNERSHIP

When two or more than two people runs a business jointly by investing their money/resources then it is called a joint venture or business in partnership.

All these people, who have invested their resources mainly money, are called as Partners.

Partners are basically of two types:

1. **Working Partner:** A partner who is directly involved with day-to-day activities of business is called a working partner.
2. **Sleeping Partner:** A partner who just invests his or her money is called a sleeping partner.

General rules of Partnership:

1. If the partners invest different amounts for the same period of time, then profits of all the partners are shared in the ratio of their investments.
2. If the partners invest same amount for different time periods, then the profits of all the partners are shared in the ratio of time periods for which their amounts were invested.
3. If the partners invest different amounts for different time periods, then their profits are shared in the ratio of product of respective investments with the time period of each partner, individually.

Thus gain or loss is divided in the ratio of 'money-time' capitals.

Example 21: Bhanu and Shafeeq started a business by investing Rs. 36000 and Rs. 63000 for the same time. What is the share of each person, out of an annual profit of Rs. 5500?

Solution:

As Bhanu and Shafeeq invest money for the same time hence profit of Bhanu and Shafeeq will be divided in the ratio of their capitals or amount they invested.

$$\text{Hence, } \frac{\text{Profit of Bhanu}}{\text{Profit of Shafeeq}} = \frac{36000}{63000} = \frac{4}{7}$$

Now, total profit = Rs. 5500

$$\text{Share of Bhanu} = \frac{4}{7+4} \times 5500 = ₹ 2000$$

$$\text{Share of Shafeeq} = \frac{7}{7+4} \times 5500 = ₹ 3500$$

Example 22: The ratio of investments of L, M and N is $4 : 5 : 6$ and the ratio of their profits is $7 : 8 : 9$ respectively. What will be the respective ratio of their time periods?

Solution:

As we know when investment and time periods are different then, Profit = Investment $\times$ Time period

$$L's \text{ investment} = 4x ; M's \text{ investment} = 5x ;$$

N's investment = $6x$

L's profit = $7y$; M's Profit = $8y$; N's profit = $9y$

$$\text{L's time period} = \frac{7y}{4x}$$

$$\text{M's time period} = \frac{8y}{5x}$$

$$\text{N's time period} = \frac{9y}{6x}$$

$$\text{Ratio of L's, M's and N's time period} = \frac{7}{4} : \frac{8}{5} : \frac{3}{2}$$

$$\text{Ratio} = \frac{7}{4} \times 20 : \frac{8}{5} \times 20 : \frac{3}{2} \times 20 = 35 : 32 : 30$$

Example 23: A started a business with Rs. 52000 and after 4 months B joined him with Rs. 39000. At the end of the year, out of the total profits B received total Rs. 20000 including 25% profit as commission for managing the business. What amount did A receive?

Solution:

As in this case, Profit = Investment $\times$ Time

$$\text{Profit share of A and B} = 52000 \times 12 : 39000 \times 8 \\ = 2 : 1$$

Let the profit be Rs. x , then B receives 25% as commission for managing business. The remaining 75% of the total profit x is shared between A and B in the ratio 2 : 1. Hence, B will get $1/3^{\text{rd}}$ part of this in addition to his commission. Hence, his total earning.

$$= 0.25x + \frac{1}{3} \times 0.75x$$

$$\Rightarrow 0.5x = 20000$$

$$\Rightarrow x = 40000$$

$$\text{Now, A will receive } \frac{2}{3} \times 0.75x$$

$$\text{i.e. } \frac{2}{3} \times 0.75 \times 40000 = \text{` } 20000$$

Example 24: Sohan, Gopi and Juhi enter into a partnership with an amount of Rs. 10000 each. After 4 months, Sohan invests an additional amount of Rs. 2000. Three months later, Gopi invests Rs. 4000, and Juhi at the same time withdraws Rs. 2000 from business. Juhi is allowed Rs. 2000 as monthly salary. Calculate their respective profit if the profit for the year is Rs. 217000?

Solution:

$$\text{Sohan's capital investment in that year} \\ = 10000 \times 4 + 12000 \times 8 = 136000$$

$$\text{Gopi's capital investment in that year} \\ = 10000 \times 7 + 14000 \times 5 = 140000$$

$$\text{Juhi's capital investment in that year} \\ = 10000 \times 7 + 8000 \times 5 = 110000$$

$$\text{Ratio in which profits are to be shared} = 68 : 70 : 55$$

$$\text{Annual Salary of Juhi} = 2000 \times 12 = \text{Rs. } 24000$$

$$\text{Available profits at the end after Juhi's salary} \\ = 217000 - 24000 = \text{Rs. } 193000$$

$$\text{Sohan's share} = \frac{68}{193} \times 193000 = \text{` } 68000$$

$$\text{Gopi's share} = \frac{70}{193} \times 193000 = \text{` } 70000$$

$$\text{Juhi's share} = \frac{55}{193} \times 193000 = \text{` } 55000$$

Practice exercise Level 1

- Rs. 3200 is divided among A, B and C in the ratio of 3 : 5 : 8 respectively. What is the difference between the share of B and C?
(A) Rs. 400 (B) Rs. 600
(C) Rs. 800 (D) Rs. 900
- Rs. 94000 is divided among A, B and C such that 20% of A's share = 25% of B's share = 15% of C's share. What is the share of C?
(A) Rs. 23500 (B) Rs. 29500
(C) Rs. 40000 (D) Rs. 42000
- If $a : b = 2 : 3$, $b : c = 1 : 2$ and $c : d = 3 : 1$, then what is the value of $\frac{ab}{cd}$?
(A) 2 : 1 (B) 1 : 3
(C) 3 : 4 (D) 1 : 2
- Rs. 6400 is divided among three workers in the ratio $\frac{3}{5} : 2 : \frac{5}{3}$. What is the share of the second worker?
(A) Rs. 2560 (B) Rs. 3000
(C) Rs. 3200 (D) Rs. 3840
- If 32 students in a class are females and the ratio of female to male students is 16 : 9, then what percentage of the class is female?
(A) 32% (B) 36%
(C) 56.25% (D) 64%
- Which of the following is the lowest ratio?
(A) 7 : 13 (B) 17 : 25
(C) 7 : 15 (D) 15 : 23
- Rs. 680 is divided among A, B, C such that A gets $\frac{2}{3}$ rd of what B gets and B gets $\frac{1}{4}$ th of what C gets. Then, what are their respective shares?
(A) Rs. 75, Rs. 325, Rs. 280
(B) Rs. 80, Rs. 120, Rs. 480
(C) Rs. 90, Rs. 210, Rs. 380
(D) Rs. 100, Rs. 200, Rs. 380
- In a rare coin collection, there is one gold coin for every three non gold coins, 10 more gold coins are added to the collection and the ratio of gold coins to non-gold coins would be 1 : 2. Based on the information, the total number of coins in the collection now becomes?
(A) 90 (B) 80
(C) 60 (D) 50
- A bag contains Rs. 216 in the form of one-rupee, 50 paise and 25 paise coins in the ratio of 2 : 3 : 4. What is the number of 50 paise coins?
(A) 96 (B) 114
(C) 141 (D) 144
- The three numbers are in the ratio $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$. The difference between the greatest and the smallest numbers is 36. What are the numbers?
(A) 72, 84, 108 (B) 60, 72, 96
(C) 72, 84, 96 (D) 72, 96, 108
- Two numbers are in the ratio $1\frac{1}{2} : 2\frac{2}{3}$. When each of these is increased by 15, they become in the ratio $1\frac{2}{3} : 2\frac{1}{2}$. What is the value of the greater of the numbers?
(A) 27 (B) 36
(C) 48 (D) 64
- If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then what is the value of $\frac{2a+3b+c}{b}$?
(A) $\frac{1}{7}$ (B) $\sqrt{2}$
(C) 6.25 (D) 7
- A bag has Rs. 51.25 in the form of 2-rupee, 50-paise and 25-paise coins in the ratio of 3 : 5 : 7

- respectively. What is the total number of 50-paise coins?
- (A) 15 (B) 35
(C) 25 (D) 5
14. If 177 is divided into 3 parts in the ratio $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$, then what is the value of second part?
- (A) 75 (B) 45
(C) 72 (D) 60
15. Instead of dividing Rs. 117 among P, Q, R in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, by mistake it was divided in the ratio 2 : 3 : 4. Who gained in the transaction?
- (A) Only P (B) Only Q
(C) Only R (D) Both Q and R
16. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. What is the quantity of milk in the original mixture?
- (A) 16 litres (B) 22.75 litres
(C) 25 litres (D) 32.5 litres
17. Three bottles of equal capacity are containing a mixture of milk and water in ratio 2 : 1, 3 : 7 and 4 : 11 respectively. These three bottles are emptied into a large bottle. What is the ratio of milk and water respectively in this large bottle?
- (A) 37 : 53 (B) 37 : 90
(C) 37 : 30 (D) 7 : 30
18. Given that the ratio of A's money to that of B's money is 4 : 5 and B's money to C's is 2 : 3. If A has Rs. 1000, then what is the total amount of money among A, B and C?
- (A) Rs. 3500 (B) Rs. 4125
(C) Rs. 3000 (D) Rs. 4000
19. A watermelon is cut into two pieces in the ratio of 3 : 5 by weight. The bigger of the two is further cut in the ratio of 5 : 7 by weight. What is the ratio of each of the three pieces?
- (A) 3 : 5 : 7 (B) 36 : 25 : 35
(C) 15 : 25 : 56 (D) 36 : 25 : 34
20. The ratio of the present ages of Anju and Sandhya is 13 : 17, respectively. Four years ago, the respective ratio of their ages was 11 : 15. What will be the respective ratio of their ages six years hence?
- (A) 3 : 4 (B) 7 : 8
(C) 5 : 7 (D) 4 : 5
21. If a tray containing a dozen glasses is dropped, which of the following can not be the ratio of the broken glasses to unbroken glasses?
- (A) 2 : 1 (B) 7 : 5
(C) 3 : 2 (D) 3 : 1
22. If $2x = 3y = 4z$, then $x : y : z$ is:
- (A) 2 : 3 : 4 (B) 4 : 3 : 2
(C) 6 : 4 : 3 (D) 3 : 4 : 2
23. A and B are two alloys of gold and copper prepared by mixing gold and copper in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of alloys are melted to form a third alloy C then what is the ratio of gold and copper in alloy C?
- (A) 2 : 3 (B) 7 : 5
(C) 5 : 9 (D) 3 : 8
24. What is the fourth proportional to 72, 168 and 150?
- (A) 450 (B) 300
(C) 350 (D) 400
25. A person distributes his pens among four friends A, B, C and D in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$. What is the minimum number of pens that the person should have?
- (A) 42 (B) 57
(C) 58 (D) 60
26. Which among the following cannot be the ratio of the boys to girls in a school having 72 students?
- (A) 3 : 5 (B) 5 : 4
(C) 9 : 7 (D) 1 : 2
27. If $4a = 5b$ and $7b = 9c$, then $a : b : c = ?$
- (A) 45 : 36 : 28 (B) 44 : 33 : 28

- (C) 28 : 36 : 45 (D) 36 : 28 : 45
28. Out of 120 applicants for a post, 70 are male and 80 have drivers licence. What is the ratio between the minimum to maximum number of males having driver's licence?
 (A) 1 : 2 (B) 2 : 3
 (C) 3 : 7 (D) 5 : 7
29. Salaries of P, Q and R were in the ratio 3 : 5 : 7, respectively. If their salaries were increased by 50%, 60% and 50%, respectively, what will be the new ratio of their respective salaries?
 (A) 3 : 6 : 7 (B) 4 : 5 : 7
 (C) 4 : 5 : 8 (D) None of these
30. In two alloys, the ratio of Iron and Copper is 4 : 3 and 6 : 1 respectively. If 14 kg of the first alloy and 35 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to iron in the new alloy?
 (A) 38 : 11 (B) 11 : 38
 (C) 3 : 8 (D) 11 : 8
31. A bag contains one rupee, half rupee and ten paise coins. The rupee and half rupee coins are in the ratio 2 : 5 and the half rupee and ten paise coins are in the ratio 4 : 9. If the total amount is Rs. 1125, then what is the number of coins of each kind?
 (A) 400, 1000, 2250 (B) 500, 1000, 2000
 (C) 450, 1500, 2000 (D) 500, 1250, 2125
32. An employer reduces the number of his employees in the ratio 9 : 8 and increases their wages in the ratio of 14 : 15. If the old wage bill was Rs. 1890, then what is the new wage bill?
 (A) Rs. 2000 (B) Rs. 2500
 (C) Rs. 1800 (D) Rs. 2600
33. x is directly proportional to y , and $y = 7$ when $x = 15$. What is the value of x when $y = 14$?
 (A) 23 (B) 30
 (C) 33 (D) 39
34. x^2 varies directly as y^3 and when $x = 6$, $y = 3$. Which of the following equations correctly represents the relationship between x and y ?
 (A) $6x^2 = 3y^3$ (B) $3y^2 = 6x^3$
 (C) $3x^2 = 2y^3$ (D) $3x^2 = 4y^3$
35. The ratio of Mona's and Vikas's present ages is 9 : 10. After 4 years, the ratio of their ages becomes 11 : 12. What is Mona's present age?
 (A) 18 years (B) 22 years
 (C) 20 years (D) Can't say
36. A is 40 years old while B is 60 years old. How many years ago, the ratio of their ages was 3 : 5?
 (A) 5 years (B) 10 years
 (C) 20 years (D) 37 years
37. Three years ago, the ratio of the ages of a father and daughter is 3 : 1 and 2 years hence, their ages will be in the ratio 22 : 9. What is the ratio of their present ages?
 (A) 21 : 8 (B) 5 : 14
 (C) 9 : 5 (D) 5 : 9
38. Lalit's present age is three years less than twice his age 12 years ago. Also the respective ratio between Pankaj's present age and Lalit's present age in 4 : 9. What will be Pankaj's age after 5 years?
 (A) 12 years (B) 17 years
 (C) 21 years (D) 24 years
39. Three years hence, a boy's age will be 4 times the square of what it was 11 years ago. What is the present age of the boy?
 (A) 18 (B) 13
 (C) 20 (D) 17
40. Raman, Manan and Kamal are partners and invest in a business such that Raman invests $\frac{2}{5}$ th of total and Manan invest $\frac{3}{8}$ th of the total. What is the ratio of profit of Raman, Manan and Kamal respectively?
 (A) 16 : 15 : 9 (B) 16 : 15 : 31
 (C) 2 : 3 : 5 (D) 15 : 16 : 9

41. A and B started a business with initial investments in the ratio 12 : 11 and their annual profits were in the ratio 4 : 1. If A invested the money for 11 months then B invested money for:
 (A) 3 months (B) 4 months
 (C) $3\frac{2}{3}$ months (D) 6 months
42. A started a business with a capital of Rs. 6000. B joined few months later with a capital of Rs. 9000. If the profit at the end of the year was divided equally between A and B, then after how many months did B join?
 (A) 8 (B) 4
 (C) 6 (D) 3
43. A, B, C subscribe Rs. 50000 for business. A subscribes Rs. 4000 more than B and B Rs. 5000 more than C, then out of total profit of Rs. 35000, what is the share of A?
 (A) Rs. 11900 (B) Rs. 8400
 (C) Rs. 14700 (D) Rs. 13600
44. Firoz invested Rs. 65000 to start a business. Dhruv joined him six months later by investing Rs. 80000. At the end of two years from the commencement of the business, they earned a profit of Rs. 43500. What is Firoz's share of the profit?
 (A) Rs. 19500 (B) Rs. 18500
 (C) Rs. 24000 (D) Rs. 22620
45. P, Q and R invest in a business in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{12}$ respectively. Out of a total profit of Rs. 121000, what will be the share of Q?
 (A) Rs. 44000 (B) Rs. 50000
 (C) Rs. 48000 (D) Rs. 46500
46. Subhash started a business investing Rs. 50000. After one year he invested another Rs. 30000 and Yash also joined him with a capital of Rs. 70000. If the profit earned in three years from the starting of business was Rs. 87500, what is the share of Yash in the profit?
 (A) Rs. 37500 (B) Rs. 35000
 (C) Rs. 30000 (D) Rs. 32500
47. A started a business investing Rs. 70000. B joined him after six month with an amount of Rs. 105000 and C joined them with Rs. 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among A, B and C, respectively, three years after A started the business?
 (A) 42 : 45 : 56 (B) 7 : 6 : 10
 (C) 12 : 15 : 16 (D) 12 : 17 : 15

Practice exercise Level 2

1. One year ago, the ratio between Mahesh's and Suresh's salaries was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salaries for the present year are Rs. 43000, then what is the present salary of Mahesh?
 (A) Rs. 19000 (B) Rs. 18000
 (C) Rs. 16000 (D) Rs. 15500
2. The ratio between the number of passengers travelling by economy and business class by jet airways is 50 : 1, whereas the ratio of economy and business class fares is 1 : 3. If on a particular flight, Rs. 530000 were collected from the passengers travelling on this flight, then what was the amount collected from the business class passengers?
 (A) Rs. 30000 (B) Rs. 34000
 (C) Rs. 40000 (D) Rs. 50000
3. In a cricket match, India scored 232 runs without losing a wicket. The score consisted of byes, wides and runs scored by two opening batsmen: Dhawan and Vijay. The runs scored by the two batsmen are 26 times wides. There are 8 more byes than wides. If the ratio of the

runs scored by Dhawan and Vijay is 6 : 7, then how many runs did Vijay scored?

- (A) 88 (B) 96
(C) 102 (D) 112

4. Rs. 750 is divided between A, B and C such that if Rs. 20, Rs. 30 and Rs. 50 are subtracted from their shares, the remaining amount with each one in that order is in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. What is the difference between the original shares of A and C?

- (A) Rs. 140 (B) Rs. 120
(C) Rs. 180 (D) Rs. 200

5. Boxes A and B have certain number of balls in them. If three balls are transferred from box A to box B, the ratio of the balls in the two boxes is 7 : 3. However, if twice the number of balls is transferred from box A to box B, the new ratio will be 3 : 2. What is the original difference between the number of balls in A and B?

- (A) 18 (B) 24
(C) 6 (D) 12

6. In a class, the ratio of the number of students who are rank holders to those who are not is 2 : 7. If 15% of the boys are rank holders while 30% of the girls are not rank holders, what is the ratio of boys to girls in the class?

- (A) 86 : 13 (B) 5 : 1
(C) 25 : 11 (D) 13 : 5

7. Let a, b, c, d, e be integers such that $a = 6b = 12c$ and $2b = 9d = 12e$. Then which among the following pairs contains a number that is not an integer?

- (A) $\frac{a}{27}, \frac{b}{e}$ (B) $\frac{a}{36}, \frac{c}{e}$
(C) $\frac{a}{12}, \frac{bd}{18}$ (D) $\frac{a}{6}, \frac{c}{d}$

8. Three solutions having milk and water in the ratio 1 : 2, 2 : 3 and 3 : 4 are mixed in the ratio

2 : 4 : 1. What is the ratio of milk and water in the resultant mixture?

- (A) 283 : 452 (B) 452 : 283
(C) 226 : 283 (D) None of these

9. The total expenditure for a party consists of two parts. One part remains constant and the second part depends on the number of guests. The expenditure of Rs. 650 for 7 guests and Rs. 970 for 11 guests. What is the expenditure for 18 guests?

- (A) Rs. 1700 (B) Rs. 1530
(C) Rs. 1800 (D) Rs. 2000

10. If $\frac{a}{b} = \frac{1}{3}, \frac{b}{c} = 2, \frac{c}{d} = \frac{1}{2}, \frac{d}{e} = 3$ and $\frac{e}{f} = \frac{1}{4}$, then what is the value of $\frac{abc}{def}$?

- (A) $\frac{3}{8}$ (B) $\frac{27}{8}$
(C) $\frac{3}{4}$ (D) $\frac{27}{4}$

11. A man ordered 4 pairs of black socks and some pairs of brown socks. The price of a black pair is double that of a brown pair. While preparing the bill, the clerk did a mistake and interchanged the number of black and brown pairs. This increased the bill by 50%. What was the ratio of the number of black and brown pairs of socks in the original order?

- (A) 4 : 1 (B) 2 : 1
(C) 1 : 4 (D) 1 : 2

12. A change-making machine contains one-rupee, two-rupee and five-rupee coins. The total number of coins is 300. The amount is Rs. 960. If the numbers of one-rupee coins and two-rupee coins are interchanged, the value comes down by Rs. 40. What is the total number of five-rupee coins?

- (A) 100 (B) 140
(C) 60 (D) 150

13. The incomes of Kamla, Bimla and Vimla are in the ratio 7 : 9 : 12 and their spendings are in the ratio 8 : 9 : 15. If Kamla saves one-fourth of her income, then what is the ratio of the savings of Kamla, Bimla and Vimla?
 (A) 56 : 99 : 69 (B) 39 : 72 : 49
 (C) 47 : 74 : 99 (D) 49 : 56 : 48
14. A contractor employed 25 labourers on a job. He was paid Rs. 27500 for the work. After retaining 20 percent of the sum, he distributed the remaining amount amongst the labourers. If the number of men to women labourers was in the ratio 2 : 3 and their wages in the ratio 5 : 4, then what wages did each woman labourer get?
 (A) Rs. 800 (B) Rs. 1000
 (C) Rs. 1200 (D) Rs. 1500
15. The cost of an article is three times the value of the raw material used. The cost of raw material increase in the ratio of 5 : 12 and manufacturing expenses in the ratio 4 : 5. What will be the cost of The article, which originally cost Rs. 6?
 (A) Rs. 9.80 (B) Rs. 10
 (C) Rs. 17 (D) Rs. 20.50
16. Between two stations the first, second and third class fares were fixed in the ratio 8 : 6 : 3, but afterwards the first class fares were reduced by $\frac{1}{6}$ and the second class by $\frac{1}{12}$. In a year, number of first, second and third class passengers were respectively 9 : 12 : 26 and the money at the booking offices was Rs. 1088. How much was paid by the first-class passengers?
 (A) 380 (B) 220
 (C) 280 (D) 320
17. At the beginning of a term, the ratio of the number of boys in a school under 15 years to those over 15 years was 5 : 4. At the end of the term it was 7 : 8 as 20 of the boys had reached the age of 15 during the term. What is the total number of boys, given that no boy left the school or was admitted?
 (A) 225 (B) 250
 (C) 275 (D) 215
18. Rs. 430 is divided among 45 persons consisting of men, women and children. The sum of men's, women's and children's shares are in the ratio 12 : 15 : 16 but the individual shares of a man, woman and child are as 6 : 5 : 4. What is the share of each?
 (A) 55, 60, 75 (B) 60, 75, 80
 (C) 45, 60, 80 (D) 60, 55, 40
19. S Varies directly as R varies and T varies inversely as R varies. At a time $R = 20$, $S = 40$ and $T = 10$. If R is changed to 10, then what will be the value of T?
 (A) 20 (B) 10
 (C) 40 (D) 80
20. What is the third proportion of 16 and 24?
 (A) 24 (B) 36
 (C) 16 (D) None of these
21. A variable P is directly proportional to Q and inversely proportional to the square root of R. If the value of P is 5 when R is 16 and Q is 10, what is the value of P when R is 8 and Q is 20?
 (A) $\frac{20}{\sqrt{2}}$ (B) $20 \times \sqrt{2}$
 (C) 40 (D) 10
22. The value of a precious stone varies directly with the square of its weight. It had a value of Rs. 25000. It broke into two pieces whose weights are in the ratio 2 : 3. What is the loss due to breakage?
 (A) Rs. 12000 (B) Rs. 15000
 (C) Rs. 9000 (D) Rs. 18000
23. The ratio of the rate of flow of water in pipes, varies inversely as the square of the radius of the pipes. What is the ratio of flow in two pipes of diameters 4 cm and 16 cm?
 (A) 1 : 4 (B) 4 : 1

- (C) 8 : 1 (D) 16 : 1
24. Four years ago, the father's age was three times the age of his son. The total ages of the father and son after 4 years from now will be 64 years. What is the father's age at present?
(A) 32 years (B) 36 years
(C) 44 years (D) None of these
25. In a family, a couple has a son and a daughter. The age of the father is three times that of his daughter and the age of the son is half of his mother. The wife is nine years younger to her husband and the brother is seven years older than his sister. What is the age of the mother?
(A) 40 years (B) 45 years
(C) 50 years (D) 60 years
26. The ratio of the ages of Ratan and Supriya is 4 : 3. After 4 years, the ratio will be 9 : 7. If at the time of the marriage, the ratio was 5 : 3, then how many years ago were they married?
(A) 12 years (B) 8 years
(C) 10 years (D) 15 years
27. A family consists of a father, a mother, two sons and the youngest daughter. The age of the father is four times the age of the second son. The age of first son and daughter are as 3 : 1. The mother is 3.5 times as old as the second son. The age of the second son is $\frac{2}{3}$ of the age of the first son. The age of the youngest daughter is 5 years. What are their ages?
(A) 15, 10, 40, 35 (B) 10, 15, 60, 40
(C) 40, 65, 13, 20 (D) 18, 20, 50, 60
28. The combined ages of a man and his wife are six times the combined ages of their children. Two years ago their combined ages were ten times the combined ages of their children and six year hence their combined ages will be three times the combined ages of their children. How many children do they have?
(A) 2 (B) 3
(C) 1 (D) Indeterminate
29. Tarun and Vikram enter into a partnership with Rs. 50000 and Rs. 60000, respectively. Mahesh joins them after n months contributing Rs. 70000 and Vikram leaves n months before the end of the year. If they share the profit in the ratio of 20 : 18 : 21, then what is the value of n ?
(A) 3 (B) 6
(C) 8 (D) 9
30. A started a business with a capital of Rs. 6000. Two months later, B joined with a capital of Rs. 4000. After another three months, A invested Rs. 2000 more while B withdrew Rs. 2000. In what ratio will the profit get divided at the end of the year?
(A) 5 : 2 (B) 3 : 2
(C) 5 : 1 (D) 43 : 13
31. A and B are partners in a business. They invest in the ratio 5 : 6. At the end of 8 months. A withdraws. If they receive profits in the ratio of 5 : 9, for what time period B's investment was used?
(A) 12 months (B) 10 months
(C) 15 months (D) 14 months
32. Poonam is a working partner and Suman is a sleeping partner in a business. Poonam puts in Rs. 75000 and Suman puts in Rs. 90000. Poonam receives 12.5 percent of the profit for managing the business and the rest is divided in proportion to their capital. What does each get out of a profit of Rs. 44000?
(A) Rs. 20000 and Rs. 24000
(B) Rs. 23000 and Rs. 21000
(C) Rs. 22500 and Rs. 21500
(D) Rs. 23500 and Rs. 20500
33. P, Q and R invested some amount in a business in the ratio of 5 : 7 : 6, respectively. In the next year, they increased their investment by 26%, 20% and 15% respectively. The profit earned

during the second year should be distributed in what ratio among P, Q and R?

- (A) 31 : 27 : 21 (B) 21 : 28 : 23
(C) 26 : 20 : 15 (D) 21 : 29 : 31

34. Kavya and Shagun enter into a partnership, Kavya contributes Rs. 5000 while Shagun contributes Rs. 4000. After 1 month, Shagun withdraws one-fourth part of her contribution and after 3 months from the starting, Kavya puts Rs. 2000 more. When Shagun withdraws her money Ishita also joins them with Rs. 7000. If at the end of 1 year, there is profit of Rs. 4800, what will be the share of Ishita in the profit?

- (A) Rs. 1650 (B) Rs. 1925
(C) Rs. 2175 (D) None of these

35. Three partners invested capital in the ratio 2 : 7 : 9. The time period for which each of them invested was in the ratio of the reciprocals of the amount invested. What is the share of the partner who brought in the highest capital, if profit is Rs. 15300?

- (A) Rs. 2700 (B) Rs. 3900
(C) Rs. 5100 (D) Rs. 4200

36. Chevron, Shell and Exxon, three US based business partners, jointly invested in a business project to supply nuclear fuel to India. As per their share in the investment, Exxon will receive $\frac{2}{3}$ of the profits whereas Chevron and Shell divide the remainder equally. It is estimated that the income of Chevron will increase by \$ 180 million when the rate of profit rises from 4% to 7%. What is the income of Shell?

- (A) \$ 6000 million (B) \$ 3000 million
(C) \$ 5000 million (D) \$ 8000 million

37. A, B, C start in business together; A puts in Rs. 1200000. B, Rs. 1440000 and C Rs. 2160000. At the end of 3 months, C leaves, and at the end of 7 months more B leaves, both taking out

their capital with them, but the profits are not divided until the end of the year. When C receives Rs. 12000. How much will A and B receive?

- (A) Rs. 43333 (B) Rs. 53333
(C) Rs. 45550 (D) Rs. 47550

38. The marks of a student are directly related with the number of hours a student spends in the studies everyday. Atul scored 80% after studying 4 hours a day. What would be the marks of Anuj, who spends 3 hours a day on studies?

- (A) 90 (B) 80
(C) 75 (D) 60

39. The voltage across a wire is directly proportional to the current passing through it. In a particular instance, when voltage is 36 volts, the current passing through the wire is 54 amperes. At what voltage will the current passing through the wire be 90 amperes?

- (A) 72 volts (B) 90 volts
(C) 60 volts (D) 54 volts

40. If the number of runs scored by a batsman is directly proportional to the square root of the time (in minutes) he spends at the pitch, then how much longer should a batsman, who has scored 32 runs in 64 minutes, stay at the pitch to complete a century?

- (A) 9 hours 21 minutes
(B) 10 hours 25 minutes
(C) 12 hours 25 minutes
(D) 15 hours

41. Arun and Tarun started a business with capitals of Rs. 12500 and Rs. 10000 respectively. After 5 months, Arun withdrew a certain amount from his capital which was equal to 62.5% of amount that Tarun added to his capital after two more months. What amount did Arun withdraw if they receive equal profit at the end of the year?

(A) Rs. 3150 (B) Rs. 2050

(C) Rs. 3000 (D) Rs. 2000

42. Manikchand and Nanabhai start a business with respective capitals of Rs. 51000 and Rs. 33000. Manikchand withdraws an amount of Rs. 2000 while Nanabhai put in an additional amount of Rs. 2000 every month into the

business. If they close the business after 13 months after making a profit of Rs. 84000, then what will be the share of Nanabhai?

(A) Rs. 42000 (B) Rs. 45000

(C) Rs. 36000 (D) Rs. 57000

Solution

Practice Exercise Level 1

- 1.(B) Difference of share of B and C $\frac{3}{16} = \times 3200$
= Rs. 600

- 2.(C) $20A = 25B = 15C$ or $4A = 5B = 3C$
 $A : B : C = 15 : 12 : 20$

$$C's \text{ share} = \frac{94000}{47} \times 20 = \text{Rs. } 40000$$

- 3.(D) Given that,
 $a : b = 2 : 3, b : c = 1 : 2, c : d = 3 : 1$
 $a : b : c : d = 2 \times 1 \times 3 : 3 \times 1 \times 3 : 3 \times 2 \times 3 : 3 \times 2 \times 1$
 $= 6 : 9 : 18 : 6$
 $= 2 : 3 : 6 : 2$

$$\text{Now, } \frac{a \times b}{c \times d} = \frac{6}{12} = \frac{1}{2}$$

- 4.(B) Given ratio,
 $= \frac{3}{5} : \frac{2}{1} : \frac{5}{3} = 9 : 30 : 25$

Share of the second worker

$$= \left(6400 \times \frac{30}{64} \right) = \text{Rs. } 3000$$

- 5.(D) Let the females be $16x$ and males $9x$.
Then, $16x = 32 \Rightarrow x = 2$
So, there are 32 females and 18 males in the class.
Percentage of females
 $= \left(\frac{32}{50} \times 100 \right) \% = 64\%$

- 6.(C) $\frac{7}{13} = 0.538, \frac{17}{25} = 0.68$
 $\frac{7}{15} = 0.466, \frac{15}{23} = 0.652$

Clearly, $\frac{7}{15}$ is smallest.

- 7.(B) Suppose C gets Rs. x . Then, B gets Rs. $\frac{x}{4}$ and A gets

$$\left(\frac{2}{3} \times \frac{x}{4} \right) = \frac{x}{6}$$

$$\therefore \frac{x}{6} + \frac{x}{4} + x = 680$$

$$\Rightarrow 2x + 3x + 12x = 680 \times 12$$

$$\Rightarrow 17x = 8160 \Rightarrow x = 480$$

$\therefore$ A gets Rs. 80, B gets Rs. 120 and C gets Rs. 480.

- 8.(A) Ratio of gold to non gold
 $= 1 : 3$

So, gold coins = x

non gold coins = $3x$

Now, 10 gold coins are added

$$\text{So, } \frac{x+10}{3x} = \frac{1}{2} \Rightarrow 2x+20=3x$$

$$\Rightarrow x = 20$$

Coins in new collection

$$= x + 3x + 10$$

$$= (4 \times 20) + 10 = 90$$

- 9.(D) Let the number of one-rupee, 50-p and 25-p coins be $2x$, $3x$ and $4x$ respectively. Then,

$$2x + \frac{3x}{2} + \frac{4x}{4} = 216$$

$$\Rightarrow 8x + 6x + 4x = 864$$

$$\Rightarrow 18x = 864 \Rightarrow x = 48$$

$$\text{Number of 50-p coins} = (3 \times 48) = 144$$

- 10.(D) Let the number be

$$\frac{1}{2}k, \frac{2}{3}k, \text{ and } \frac{3}{4}k$$

$$\text{Largest} = \frac{3}{4}k \text{ and smallest} = \frac{1}{2}k$$

$$\therefore \frac{3}{4}k - \frac{1}{2}k = 36 \Rightarrow k = 144$$

So, The numbers are 72, 96 and 108.

- 11.(C) Let the numbers be

$$\frac{3}{2}x \text{ and } \frac{8}{3}x$$

$$\text{So, } \frac{\frac{3}{2}x + 15}{\frac{8}{3}x + 15} = \frac{5}{2} \text{ or } \frac{3x + 30}{8x + 45} = \frac{5}{2}$$

$$\text{or } \frac{3x + 30}{8x + 45} \times \frac{2}{2} = \frac{5}{3} \text{ or } \frac{3x + 30}{8x + 45} = \frac{5}{3}$$

$$\text{or } x = 18$$

$\therefore$ Greater of the numbers

$$= \frac{8}{3} \times 18 = 48$$

- 12.(C) According to the problem,

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$$

$$2a + 3b + c = 6k + 12k + 7k = 25k$$

$$\frac{2a + 3b + c}{b} = \frac{25k}{4k} = 6.25$$

- 13.(C) $6x + 2.5x + 1.75x = 51.25$

$$10.25x = 51.25$$

$$x = 5$$

$$\text{Number of 50 paise coins} = 5 \times 5 = 25$$

- 14.(D) $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$; LCM of 2, 3 and 5 is 30.

Multiplying each term by 30.

$$\frac{1}{2} \times 30 : \frac{2}{3} \times 30 : \frac{4}{5} \times 30$$

$$= 15 : 20 : 24$$

$$\text{2nd part} = \frac{20}{15 + 20 + 24}$$

$$= \frac{20}{59} \times 177 = 60$$

- 15.(D) $\frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3.$

$$P = \left(117 \times \frac{6}{13} \right) = \text{` } 54,$$

$$Q = \left(117 \times \frac{4}{13} \right) = \text{` } 36$$

$$\text{and } R = \left(117 \times \frac{3}{13} \right) = \text{` } 27$$

Again, the ratio is $2 : 3 : 4.$

$$P = \left(117 \times \frac{2}{9} \right) = \text{` } 26,$$

$$Q = \left(117 \times \frac{3}{9} \right) = \text{` } 39$$

$$\text{and } R = \left(117 \times \frac{4}{9} \right) = \text{` } 52$$

Thus, both Q and R gained in the transaction.

- 16.(C) In the given mixture, let milk = $5x$ litres and water = x litres.

$$\text{Then, } \frac{5x}{x + 5} = \frac{5}{2} \Rightarrow 10x = 5x + 25$$

$$\Rightarrow 5x = 25 \Rightarrow x = 5$$

$$\therefore \text{Milk in original mixture} = (5 \times 5) \text{ litres}$$

$$= 25 \text{ litres}$$

- 17.(A) Let the quantity of mixture in each bottle is 30 units (LCM of 3, 10 and 15)

$$\text{Total milk} = (2 \times 10) + (3 \times 3) +$$

$$(4 \times 2) = 37 \text{ units}$$

$$\text{Total water} = (1 \times 10) + (7 \times 3) + (11 \times 2) = 53 \text{ units}$$

$$\text{So, } 37 : 53$$

- 18.(B) B's money

$$= \frac{5}{4} \times 1000 = \text{` } 1250$$

$$C's = \frac{3}{2} \times 1250 = \text{` } 1875$$

$$\text{Therefore, total amount of money} = 1000 + 1250 + 1875 = 4125$$

19.(B) Let the initial pieces be $3x$ and $5x$.

Now, $5x$ broken in to two pieces

$$\text{i.e. } 5x \times \frac{5}{12} \text{ and } 5x \times \frac{7}{12}$$

So, final ratio will be $3 \times 12 : 25 : 35$

$$\text{i.e. } 36 : 25 : 35$$

20.(D) Let present age of Anju and Sandhya be $13x$ and $17x$.

$$\frac{13x-4}{17x-4} = \frac{11}{15}$$

$$195x - 60 = 187x - 44$$

$$195x - 187x = -44 + 60$$

$$8x = 16 \Rightarrow x = 2$$

Ratio of their ages after 6 years

$$= \frac{13 \times 2 + 6}{17 \times 2 + 6}$$

$$= \frac{32}{40} = \frac{4}{5} = 4 : 5$$

21.(C) Clearly, $3 : 2$ cannot be the ratio of the broken glass to unbroken glass because total glass in tray is 12 and 12 glasses can not be divided in the ratio of $3 : 2$ because number of broken glasses and unbroken glasses has to be integers.

22.(C) Let $2x = 3y = 4z = k$

$$x = \frac{k}{2}, y = \frac{k}{3}, z = \frac{k}{4}$$

$$x : y : z = \frac{k}{2} : \frac{k}{3} : \frac{k}{4}$$

$$x : y : z = 6 : 4 : 3$$

23.(B) A alloy = $\frac{\text{Gold}_1}{\text{Copper}_1} = \frac{7}{2}$

$$\text{B alloy} = \frac{\text{Gold}_2}{\text{Copper}_2} = \frac{7}{11}$$

$$\text{In alloy C} = \frac{\text{Gold}}{\text{Copper}} = \frac{\frac{7}{2} + \frac{7}{11}}{\frac{9}{2} + \frac{11}{11}}$$

$$= \frac{21}{15} = \frac{7}{5} = 7 : 5$$

24.(C) (I) $a : b$ (II) $::$ (III) $c : d$ (IV)

$$\frac{a}{b} = \frac{c}{d}$$

So, fourth proportional i.e. d .

$$d = \frac{168 \times 150}{72} = 350$$

25.(B) LCM of 3, 4, 5 and 6 is 60

So, the pens are distributed among A, B, C and D in the ratio.

$$\frac{1}{3} \times 60 : \frac{1}{4} \times 60 : \frac{1}{5} \times 60 : \frac{1}{6} \times 60$$

$$\text{i.e. } 20 : 15 : 12 : 10$$

$\therefore$ Total number of pens

$$= 20x + 15x + 12x + 10x = 57x$$

For minimum number of pens,

$$x = 1$$

$\therefore$ The person should have atleast 57 pens.

26.(C) 72 students cannot be divided in the ratio $9 : 7$.

27.(A) Given that $4a = 5b$

$$\therefore \frac{a}{b} = \frac{5}{4}$$

Also, $7b = 9c$

$$\therefore \frac{b}{c} = \frac{9}{7}$$

$$\therefore a : b = 5 : 4 = (5 \times 9) : (4 \times 9) = 45 : 36$$

$$\therefore b : c = 9 : 7 = (9 \times 4) : (7 \times 4) = 36 : 28$$

$$\therefore a : b : c = 45 : 36 : 28$$

28.(C) Total applicants = 120

Number of males = 70

Number of females = 50

Number of applicants with driving licence = 80

Maximum males with licence can be = 70 (when all have licence)

Minimum number of males with licence can be

$$= 80 - 50 = 30$$

(when all 50 females have licence)

So, required ratio = $3 : 7$.

29.(D) Suppose the salaries of P, Q and R were 300, 500 and 700, respectively.

After increment salary of

$$P = 300 + 50\% \text{ of } 300 = 450$$

$$Q = 500 + 60\% \text{ of } 500 = 800$$

$$R = 700 + 50\% \text{ of } 700 = 1050$$

Hence, new ratio of the respective salaries

$$A, B \text{ and } C = 450 : 800 : 1050 = 9 : 16 : 21.$$

30.(B)

		Proportion of iron in the alloys
1 st Alloy	Iron	$\frac{4 \times 2}{7 \times 2} = \frac{8}{14}$
	Copper	
2 nd Alloy	Iron	$\frac{6 \times 5}{7 \times 5} = \frac{30}{35}$
	Copper	

$$\text{Hence, Iron} = 8 + 30 = 38$$

$$\text{and Copper} = 14 + 35 - (8 + 30) = 11$$

So, in the new alloy total iron will be 38 kg and copper will be 11 kg.

So, Ratio of copper to iron

$$= 11 : 38$$

31.(A) Let number of Rs. 1, 50 paise and 10 paise coins be a, b and c respectively.

$$\text{Then, } \frac{a}{b} = \frac{2}{5} \Rightarrow a = \frac{2}{5}b$$

$$\text{and } \frac{b}{c} = \frac{4}{9} \Rightarrow c = \frac{9}{4}b$$

As per the question,

$$100 \times a + 50 \times b + 10 \times c = 112500$$

$$\Rightarrow 100 \times \frac{2}{5}b + 50b + 10 \times \frac{9}{4}b$$

$$= 112500$$

$$\Rightarrow \frac{200}{5}b + 50b + \frac{90}{4}b = 112500$$

$$\Rightarrow 800b + 1000b + 450b$$

$$= 2250000$$

$$\Rightarrow 2250b = 2250000$$

$$\Rightarrow b = 1000$$

$$\therefore a = \frac{2}{5} \times 1000 = 400$$

$$\therefore c = \frac{9}{4} \times 1000 = 2250$$

32.(C) Number of employees initial = 9x

Initial wages = 14y

$$\therefore 9x \times 14y = 1890$$

$$\Rightarrow xy = \frac{1890}{126} = 15$$

Reduced number of employees = 8x

Increased wages = 15y

$$\therefore \text{New wage bill} = 8x \times 15y$$

$$= (8 \times 15) xy$$

$$= 120 \times 15 = 1800$$

33.(B) $x \propto y \Rightarrow x = ky$

$$15 = k \times 7 \Rightarrow k = \frac{15}{7}$$

$$x = ky = \frac{15}{7} \times 14 = 30$$

Hence, the required value of x is 30.

34.(D) $x^2 \propto y^3$

$$x^2 = Ky^3$$

$$\text{So, } x = 6, y = 3$$

$$k = \frac{4}{3}$$

$$\text{So, } 3x^2 = 4y^3$$

35.(A) Let Mona's present age = 9x years. Then, Vikas's present age = 10x years.

$$\frac{9x+4}{10x+4} = \frac{11}{12}$$

$$\Rightarrow 12(9x+4)$$

$$= 11(10x+4)$$

$$\Rightarrow 2x = (48 - 44) \Rightarrow x = 2$$

Mona's present age = (9 × 2) years = 18 years

36.(B) Let the given ratio of ages be x years ago.

Therefore,

$$\frac{(40-x)}{(60-x)} = \frac{3}{5}$$

$$200 - 5x = 180 - 3x$$

$$2x = 20 \Rightarrow x = 10$$

37.(A) Three years ago, let the ages of the father and daughter be 3x and x, respectively. Today, their ages will be 3x + 3 and x + 3 and after another 2 years their ages will be 3x + 5 and x + 5. As per the problem

$$\frac{(3x+5)}{(x+5)} = \frac{22}{9}$$

$$27x + 45 \Rightarrow 22x + 110$$

$$5x = 65 \Rightarrow x = 13$$

Three years ago, their ages were 39 and 13 respectively.

Their present ages will be 42 and 16 and the required ratio will be 42 : 16 or 21 : 8.

38.(B)

Present age of Lalit

= x years

According to the question,

$$x + 3 = 2(x - 12)$$

$$x + 3 = 2x - 24$$

$$x = 27$$

Present age of Pankaj : Present age of Lalit = 4 : 9

So, Present age of Pankaj

$$= \frac{27}{9} \times 4 = 12$$

After 5 years age of Pankaj = 12 + 5 = 17 years

39.(B)

Let the present age of boy be x.

As per the question,

$$x + 3 = 4(x - 11)^2$$

$$\Rightarrow x + 3 = 4(x^2 + 121 - 22x)$$

$$\Rightarrow x + 3 = 4x^2 + 484 - 88x$$

$$\Rightarrow 4x^2 - 89x + 481 = 0$$

Using discriminant formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We get

$$x = \frac{-(-89) \pm \sqrt{7921 - 7696}}{2 \times 4}$$

$$x = \frac{89 \pm \sqrt{225}}{8}$$

$$x = 13 \text{ or } x = \frac{74}{8} = 9.25, \text{ which is not}$$

possible.

$\therefore$ Present age of boy is 13 years.

Use options

40.(A)

Let the total investment be 1.

Kamal's investment

$$= 1 - \frac{2}{5} - \frac{3}{8} = \frac{9}{40}$$

$$\text{Required ratio} = \frac{2}{5} : \frac{3}{8} : \frac{9}{40}$$

$$= 16 : 15 : 9$$

41.(A)

Let B invested money for x months, then

Investment made by A = 12 × 11

$$= 132$$

Investment made by B = 11 × x

$$= 11x$$

Ratio of profit = 4 : 1

$$\therefore \frac{132}{11x} = \frac{4}{1}, x = 3 \text{ months}$$

42.(B)

As per the problem, we have $6000 \times 12 = 9000 \times y$

$$\Rightarrow y = 8 \text{ months}$$

That is, B must have joined after 4 months.

43.(C)

Let C = x ; B = x + 5000,

$$A = x + 5000 + 4000$$

$$\text{So, } x + x + 5000 + x + 9000$$

$$= 50000; 3x = 36000$$

$$x = 12000$$

$$A : B : C = 21000 : 17000 : 12000$$

$$= 21 : 17 : 12$$

$$A's \text{ share} = \frac{21}{50} \times 35000 = 14700$$

44.(D)

Firoz	:	Dhruv
65000×24	:	80000×18
13	:	12

$$\text{So, share of Firoz} = \frac{13}{25} \times 43500$$

$$= \text{Rs. } 22620$$

45.(A)

$$P : Q : R = \frac{1}{2} : \frac{1}{3} : \frac{1}{12} = 6 : 4 : 1$$

$$\text{So, Q's share} = \frac{121000}{11} \times 4$$

$$= \text{Rs. } 44000$$

46.(B)

Ratio of profit (Subhash and Yash)

$$= (50000 \times 12) + (80000 \times 24) : (70000 \times 24)$$

$$= 600000 + 1920000 : 1680000$$

$$= 3 : 2$$

Hence, share of Yash in the profit earned from the business

$$= \frac{87500}{3+2} \times 2 = ₹ 35000$$

- 47.(C) Ratio of their investments = $A : B : C = (70000 \times 36) : (105000 \times 30) : (140000 \times 24)$
Hence, the profit will be distributed in ratio = 12 : 15 : 16

Practice Exercise Level 2

- 1.(B) Let the present year salary of Mahesh's and Suresh's be x , and y , respectively and the last year salary of Mahesh's and Suresh's be x' and y' respectively.

According to the given condition,

$$\frac{x'}{y'} = \frac{3}{5}, \frac{x'}{x} = \frac{2}{3}$$

$$\text{and } \frac{y'}{y} = \frac{4}{5}$$

$$\text{So, } \frac{\frac{x'}{y'}}{\frac{y'}{y}} = \frac{\frac{2}{3}}{\frac{4}{5}}$$

$$\Rightarrow \frac{x'}{y'} \times \frac{y}{x} = \frac{10}{12}$$

$$\Rightarrow \frac{3}{5} \times \frac{y}{x} = \frac{10}{12}$$

$$\Rightarrow \frac{y}{x} = \frac{50}{36}$$

Also, $x + y = 43000$

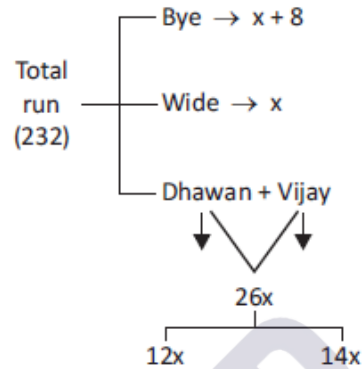
$$\Rightarrow x + \frac{50}{36}x = 43000$$

$$\Rightarrow x = \frac{43000 \times 36}{86} = ₹ 18000$$

- 2.(A) Ratio of the amount collected from the economy class and the business class passengers = $50 \times 1 : 1 \times 3$
So, amount collected from the business class passengers

$$= \frac{3}{53} \times 530000 = ₹ 30000$$

- 3.(D)



$$\text{So, } x + 8 + x + 26x = 232$$

$$28x = 232 - 8 = 224$$

$$x = 8$$

$$\text{Hence, Vijay Scored} = 14 \times 8 = 112$$

- 4.(B) As per the problem, after deduction, the remaining amount with A, B and C is in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, that is, 6 : 4 : 3

Now, Rs. 650 = $(750 - 20 - 30 - 50)$ is to be divided in the ratio 6 : 4 : 3.

$$\text{A's share} = \frac{6}{13} \times 650 = ₹ 300$$

$$\text{C's share} = \frac{3}{13} \times 650 = ₹ 150$$

A's original share = Rs. 300 + Rs. 20 = Rs. 320

C's original share = Rs. 150 + Rs. 50 = Rs. 200

Difference in shares = Rs. 120

- 5.(A) Let the number of balls in A and B be x and y , respectively.

$$\frac{x-3}{y+3} = \frac{7}{3}$$

$$3x - 9 = 7y + 21$$

$$3x - 7y = 30 \quad \dots (1)$$

$$\frac{x-6}{y+6} = \frac{3}{2}$$

$$2x - 12 = 3y + 18$$

$$2x - 3y = 30 \quad \dots (2)$$

Solving Eq. (1) and (2) we get,

$$6x - 14y = 60$$

$$6x - 9y = 90$$

$$y = 6 \text{ and } x = 24$$

Difference between x and $y = 24 - 6 = 18$

- 6.(A)** Let the ratio of boys to girls be $x : y$. Then
Number of boys who are rank holders = $0.15x$

Number of girls who are rank holders = $0.7y$

Number of rank holders

$$= 0.15x + 0.7y$$

Number of students who are not rank holders = $0.85x + 0.3y$

According to the question,

$$\frac{0.15x + 0.7y}{0.85x + 0.3y} = \frac{2}{7}$$

$$1.05x + 4.9y = 1.7x + 0.6y$$

$$0.65x = 4.3y$$

$$\frac{x}{y} = \frac{430}{65} = \frac{86}{13}$$

- 7.(D)** Given that,

$$a = 6b = 12c \text{ and } 2b = 9d = 12e$$

$$a = 6b = 12c \text{ and } 6b = 27d = 36e$$

$$a = 6b = 12c = 27d = 36e$$

$$\text{Therefore, } a = k; b = \frac{k}{6}; c = \frac{k}{12};$$

$$d = \frac{k}{27}; e = \frac{k}{36}$$

$$\text{So, } a : b : c : d : e$$

$$= 108 : 18 : 9 : 4 : 3$$

So, by checking for each option

We get that,

$$\frac{a}{6}, \frac{c}{d} \Rightarrow \frac{108}{6}, \frac{9}{4}$$

This is not an integer pair.

- 8.(A)** Let the solutions added be 2, 4 and 1 litre, respectively. Then, quantity of milk in the solution is

$$2 \times \frac{1}{3} + 4 \times \frac{2}{5} + 1 \times \frac{3}{7}$$

$$= \frac{2}{3} + \frac{8}{5} + \frac{3}{7}$$

$$= \frac{70 + 168 + 45}{105} = \frac{283}{105}$$

So, the quantity of water

$$= 7 - \frac{283}{105}$$

$$= \frac{735 - 283}{105} = \frac{452}{105}$$

So, the ratio of milk to water in the resultant solution is $283 : 452$.

- 9.(B)** Let the fixed cost be x and variable cost per person be y .

$$TC = FC + VC$$

$$\Rightarrow 650 = x + 7y$$

$$\Rightarrow 970 = x + 11y$$

Solving the above equations we get $x = 90$ and $y = 80$

$$\text{Now, } x + 18y = 90 + 18 \times 80 = 1530$$

- 10.(A)** $\frac{a}{b} = \frac{1}{3}; \frac{b}{c} = \frac{2}{1}$

$$\Rightarrow a : b : c = 2 : 6 : 3$$

Similarly, $(a : b : c) : (d : e : f)$

$$= (6 : 18 : 9) : (18 : 6 : 24)$$

$$\therefore \frac{abc}{def} = \frac{6 \times 18 \times 9}{18 \times 6 \times 24} = \frac{3}{8}$$

- 11.(C)** Let he purchased x pairs of brown socks.

Price of black socks and brown socks be Rs.

$2a$ and Rs. a per pair respectively.

$$\text{So, } \frac{3}{2} (4 \times 2a + x \times a)$$

$$= x \times 2a + 4 \times a$$

$$\Rightarrow 12a + \frac{3}{2}xa = 2xa + 4a$$

$$\Rightarrow 12 + \frac{3}{2}x = 2x + 4$$

$$\Rightarrow \frac{x}{2} = 8$$

$$\Rightarrow x = 16$$

$$\text{So, required ratio} = \frac{4}{16} = \frac{1}{4}$$

- 12.(B)** Let the number of five-rupee, two-rupee and one-rupee coins be x , y and z respectively.

$x + y + z = 300$
$5x + 2y + z = 960$
$5x + y + 2z = 920$
$y - z = 40$

$$\text{and } x + 2y = 340$$

Use the answer choices now.

If $x = 140$, $y = 100$ and $z = 60$, this satisfies all the given conditions.

- 13.(A)** Let Kamla's income, Bimla's income, Vimla's income be Rs. $7x$, Rs. $9x$ and Rs. $12x$, respectively and their expenditure be Rs. $8y$, Rs. $9y$ and Rs. $15y$ respectively.

Therefore, $4(7x - 8y) = 7x$

$$28x - 32y = 7x$$

$$28x - 7x = 32y$$

$$21x = 32y$$

$$y = \frac{21x}{32}$$

$$\text{Kamla's saving} = \frac{7x}{4}$$

$$\text{Bimla's saving} = 9x - 9y$$

$$= 9\left(x - \frac{21x}{32}\right) = 9\left(\frac{32x}{32} - \frac{21x}{32}\right)$$

$$= \frac{9 \times 11x}{32} = \frac{99x}{32}$$

$$\text{Vimla's saving} = 12x - 15y$$

$$= 12x - 15 \times \frac{21x}{32} = \frac{69x}{32}$$

Hence, the required ratio

$$= \frac{7x}{4} : \frac{99x}{32} : \frac{69x}{32} = 56 : 99 : 69$$

- 14.(A)** Suppose the wages of each man = $5x$ and wages of each woman = $4x$

$$\text{Number of men} = \frac{2}{5} \times 25 = 10$$

$$\text{Number of women} = \frac{3}{5} \times 25 = 15$$

Now, Rs. 22000 are to be divided among 10 men and 15 women.

$$\text{So, } 10 \times 5x + 15 \times 4x = 27500 \times 0.8$$

$$110x = 22000 \Rightarrow x = 200$$

$$\text{So, each woman get} = 4x$$

$$= 4 \times 200 = 800$$

- 15.(A)** Original cost of article = Rs. 6

Then, original raw material cost = Rs. 2

New cost of raw material

$$= 2 \times \frac{12}{5} = \text{Rs. } 4.80$$

Original manufacturing expenses = $(6 - 2) =$ Rs. 4

New manufacturing expenses

$$= 4 \times \frac{5}{4} = \text{Rs. } 5$$

So, new cost of article = $4.80 + 5 =$ Rs. 9.80

- 16.(D)** Let the fares of first, second and third class fares be $8k$, $6k$ and $3k$.

Reduced fares of first, second and third class are

$$8k\left(1 - \frac{1}{6}\right), 6k\left(1 - \frac{1}{12}\right) \text{ and } 3k$$

Ratio of passengers of first, second and third class $9 : 12 : 26$.

As per the question,

$$9 \times 8k \times \frac{5}{6} + 12 \times 6k \times \frac{11}{12} + 26 \times 3k = 1088$$

$$\Rightarrow 60k + 66k + 78k = 1088$$

$$\Rightarrow 204k = 1088$$

$$\Rightarrow k = 5.33$$

$\therefore$ Amount paid by first class

$$= 60k$$

$$= 60 \times 5.33 = \text{Rs. } 320$$

- 17.(A)** Let the number of boys under the age of 15 be b and the number of boys over the age of 15 be B .

$$\therefore \frac{b}{B} = \frac{5}{4} \Rightarrow b = \frac{5}{4}B$$

$$\text{Also, } \frac{b-20}{B+20} = \frac{7}{8}$$

$$\Rightarrow 8b - 160 = 7B + 140$$

$$\Rightarrow \frac{5}{4} \times 8B - 160 = 7B + 140$$

$$\Rightarrow 10B - 7B = 300$$

$$\Rightarrow B = 100$$

$$\therefore b = \frac{5}{4} \times 100 = 125$$

$\therefore$ Total number of boys

$$= 100 + 125 = 225$$

- 18.(B)** Let the number of men, women and children be m , w and c respectively.

Ratio of individual shares of man, woman and child is $6 : 5 : 4$.

Ratio of sum of men's, women's and children's share is $12 : 15 : 16$.

$$\therefore 6m : 5w : 4c = 12 : 15 : 16$$

$$\Rightarrow 6m = 12k, 5w = 15k, 4c = 16k$$

$$\Rightarrow m = 2k, w = 3k, c = 4k$$

$$\text{Also, } m + w + c = 45$$

$$\Rightarrow 2k + 3k + 4k = 45$$

$$\Rightarrow 9k = 45$$

$$\Rightarrow k = 5$$

$\therefore$ Share of each man, woman and child are 12×5 , 15×5 , $16 \times 5 = 60, 75, 80$ respectively.

- 19.(A)** As we know, $R \propto \frac{S}{T}$

$$R = K \frac{S}{T}$$

According to the question,

$$20 = K \frac{40}{10} \Rightarrow K = 5$$

Now, as we know, that one thing has to be constant to find the relation between other two things.

$$R = K \frac{S}{T}$$

$$10 = 5 \times \frac{40}{T}$$

$$\text{So, } T = 20$$

- 20.(B)** Let a be the first proportional, b be the middle proportional and c be the third or last proportional.

Here, 24 is the middle number

$$b^2 = a \times c$$

$\therefore$ Third proportional

$$= \frac{(24)^2}{16} = 36$$

- 21.(A)** Given that, $P \propto \frac{Q}{\sqrt{R}}$

$$P = \frac{KQ}{\sqrt{R}}$$

$$5 = \frac{K \times 10}{\sqrt{16}} \Rightarrow K = 2$$

$$P = 2 \times \frac{20}{\sqrt{8}} \times \frac{20}{\sqrt{2}}$$

- 22.(A)** Let the weight of stone be 5 gms.

$$V = KW^2$$

$$25000 = K5^2$$

$$K = 1000$$

Now, the stone breaks into the ratio of $2 : 3$

The weights of the two stones will be 2 gms and 3 gms.

$$V_1 = K(W_1)^2 \text{ and } V_2 = K(W_2)^2$$

$$V_1 = 1000 \times 2^2 \text{ and } V_2 = 1000 \times 3^2$$

$$V_1 + V_2 = 1000 [2^2 + 3^2]$$

$$= 1000 \times 13 = 13000$$

$$\text{So, Loss} = 25000 - 13000$$

$$= \text{Rs. } 12000$$

- 23.(D)** Radii of the two pipes are 2 cm and 8 cm.

Squares of the radii of the two pipes are 4 cm and 64 cm.

So, Rates of flow in the two pipes are in the ratio.

$$\frac{1}{4} : \frac{1}{64} = 16 : 1$$

- 24.(D)** Four years ago, let the ages of the son and the father be x and $3x$, respectively. After four years their ages will be $x + 8$ and $3x + 8$, respectively.

$$\text{Now, } x + 8 + 3x + 8 = 64$$

$$4x = 48 \Rightarrow x = 12$$

$$\text{Father's present age will be } 3x + 4$$

$$= 36 + 4 = 40$$

- 25.(D)** Let the age of daughter and son be x and y respectively.

$$\text{So, age of Father} = 3x$$

$$\text{and age of Mother} = 2y$$

Now, wife is 9 years younger to her husband.

$$\text{So, } 3x - 2y = 9$$

also, $y - x = 7$

Solving the equations we get,

$x = 23$ and $y = 30$

So, age of Mother = $2y = 2 \times 30 = 60$ years

26.(A) Ratan's age = $4x$, say

Supriya's age = $3x$, say

$$\text{So, } \frac{4x+4}{3x+4} = \frac{9}{7} \Rightarrow x = 8$$

Ratan's age = 32 years

Supriya's age = 24 years

Suppose they were married n years ago.

$$\frac{32-n}{24-n} = \frac{5}{3} \Rightarrow n = 12$$

27.(A) Let the ages of father, mother, first son, second son and daughter be f , m , S_1 , S_2 , d respectively.

$$\therefore f = 4S_2, \frac{S_1}{d} = \frac{3}{1} \Rightarrow S_1 = 3d,$$

$$m = \frac{7}{2}S_2, S_2 = \frac{2}{3}S_1$$

Given, $d = 5$

$$\therefore S_1 = 3 \times 5 = 15$$

$$\therefore S_2 = \frac{2}{3} \times 15 = 10$$

$$\therefore f = 4 \times 10 = 40$$

$$\therefore m = 3.5 \times 10 = 35$$

28.(D) Let the age of man be m , wife be w and the total number of children they have be n , also let the combined age of their children be c .

$$\text{then } m + w = 6c \dots (1)$$

Two years ago,

$$m + w - 4 = 10(c - 2n)$$

$$6c - 4 = 10c - 20n \text{ [from equ. (1)]}$$

$$4c = 20n - 4 \dots (2)$$

Also, six years hence,

$$m + w + 12 = 3(c + 6n)$$

$$6c + 12 = 3c + 18n$$

$$3c = 18n - 12 \dots (3)$$

On solving (2) and (3), we get

$$n = 3$$

29.(A) Vikram leaves n months before the end of the year.

So, Vikram stays for $(12 - n)$ months.

Mahesh joins after n months.

Mahesh remains for $(12 - n)$ months.

Profits are shared among Tarun, Vikram and Mahesh in the ratio.

$$50000 \times 12 : 60000 \times (12 - n) : 70000 \times (12 - n)$$

$$= 20 : 18 : 21 \text{ (Given)}$$

$$\text{So, } \frac{60}{6(12-n)} = \frac{20}{18} \Rightarrow n = 3$$

30.(D) In this question, we need to find the equivalent contribution of each partner by multiplying the capital with the period of investment.

The profit will get divided in the same ratio as the equivalent contribution.

$$A = 6000 \times 5 + 8000 \times 7 = 86000$$

$$B = 4000 \times 3 + 2000 \times 7 = 26000$$

The profit will get divided in the ratio 86 : 26 or 43 : 13.

31.(A)

	A : B
Ratio of Investment	5 : 6
Ratio of Time	8 : x
Ratio of Profit	5 : 9

$$\frac{P_1}{P_2} = \frac{I_1 \times T_1}{I_2 \times T_2}$$

$$\text{So, } \frac{5 \times 8}{6 \times x} = \frac{5}{9}$$

$$x = 12$$

So, B's investment was used for 12 months.

32.(B) 12.5% of profit

$$= \frac{12.5}{100} \times 44000 = 5500$$

Remaining Rs. 38500 is divided in the ratio = 75000 : 90000 = 5 : 6

$$\text{Profit of Suman} = \frac{6}{11} \times 38500$$

$$= \text{Rs. 21000}$$

Profit of Poonam

$$= \frac{5}{11} \times 38500 + 5500 = ₹ 23000$$

- 33.(B)** Let first year investment of P, Q and R, be $5x$, $7x$, and $6x$ respectively. The second year's investment will be in ratio:
 $5x + 26\% \text{ of } 5x : 7x + 20\% \text{ of } 7x : 6x + 15\% \text{ of } 6x$

$$= \frac{630}{100}x : \frac{840}{100}x : \frac{690}{100}x$$

$$= 63 : 84 : 69$$

$$= 21 : 28 : 23$$

- 34.(B)** Ratio of investment of Kavya, Shagun and Ishita is $[(5000 \times 3) + (7000 \times 9)] : [(4000 \times 1) + (3000 \times 11)] : [(7000 \times 11)]$
 $= 78000 : 37000 : 77000$
 $= 78 : 37 : 77$

So, Share of Ishita in profit

$$= \frac{77}{78+37+77} \times 4800 = ₹ 1925$$

- 35.(C)** Ratio of capital = $2 : 7 : 9$

$$\text{Ratio of time} = \frac{1}{2} : \frac{1}{7} : \frac{1}{9}$$

So, Ratio of investment

$$= 2 \times \frac{1}{2} : 7 \times \frac{1}{7} : 9 \times \frac{1}{9} = 1 : 1 : 1$$

Investment shared by each partner

$$= \frac{15300}{3} = ₹ 5100$$

- 36.(A)** Ratio of the share of investment of Exxon, Chevron and Shell

$$= \frac{2}{3} : \frac{1}{2} \times \left(1 - \frac{2}{3}\right) : \frac{1}{2} \times \left(1 - \frac{2}{3}\right)$$

$$= 4 : 1 : 1$$

Let total income of Chevron be x .

Total income increased

$$= \frac{(7-4)x}{100} = \frac{3x}{100}$$

According to the question,

$$\frac{3x}{100} = 180$$

$$x = \$ 6000 \text{ million}$$

Income of Shell = Income of Chevron = \$ 6000 million

- 37.(B)** Ratio of profits to be divided is $2160000 \times 3 : 1440000 \times 10 : 1200000 \times 12$
 $\Rightarrow 648 : 1440 : 1440$
 $\Rightarrow 9 : 20 : 20$

Profit c receives = 12000

$$\therefore 9k = 12000$$

$$k = \frac{12000}{9} = \frac{4000}{3}$$

$\therefore$ Profit received by A and B = $40k$

$$= 40 \times \frac{4000}{3} = \frac{160000}{3}$$

$$= \text{Rs. } 53333.33$$

- 38.(D)** Marks of Student $\propto$ Number of hours

$$\text{Hence, } \frac{M_1}{M_2} = \frac{H_1}{H_2}$$

$$\frac{80}{M_2} = \frac{4}{3}$$

$$M_2 = \frac{240}{4} = 60$$

- 39.(C)** $V \propto$ current

$$\frac{V_1}{V_2} = \frac{C_1}{C_2}$$

$$\frac{36}{V_2} = \frac{54}{90}$$

$$V_2 = \frac{36 \times 90}{54} = 60 \text{ volts}$$

- 40.(A)** Runs $\propto \sqrt{\text{Time}}$

$$\text{Hence, } \frac{R_1}{R_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{32}{100} = \frac{8}{\sqrt{T_2}}$$

$$\sqrt{T_2} = \frac{8 \times 100}{32}$$

$$\sqrt{T_2} = \frac{100}{4} = 25$$

$$T_2 = 625$$

Hence, to complete the century, the player has to stay for $625 - 64 = 561$ minutes or 9 hours 21 minutes.

- 41.(D)** Let the amount Arun withdraw be x .

$$\text{Amount Tarun added} \times 62.5\% = x$$

$$\text{Amount Tarun added} \times \left(\frac{1}{2} + \frac{1}{8}\right) = x$$

$$\text{Amount Tarun added} = x \times \frac{16}{10} = \frac{8}{5}x$$

$$12500 \times 5 + (12500 - x) \times 7 :$$

$$62500 + 87500 - 7x : 70000 + 50000 + 8x$$

$$= 1 : 1$$

$$10000 \times 7 + \left(10000 + \frac{8}{5}x\right) \times 5 = 1 : 1$$

$$150000 - 120000 = 8x + 7x$$

$$30000 = 15x$$

$$x = 2000$$

Hence, Tarun added Rs. 2000 after 5 months.

42.(B) $C_M = 51000 ; C_N = 33000$

After every month 2000 was withdrawn and deposited respectively

$$C_M = 51000 + 49000 + 47000 \dots + 27000$$

Total Capital for Manikchand

$$\frac{150000 - 7x}{120000 + 8x} = \frac{1}{1}$$

$$= 39 \times 13 \times 1000 = 507000$$

$$\text{For } C_N = 33000 + 35000 + 37000 \dots 57000$$

Total capital for Nanabhai

$$= \left(\frac{51+27}{2}\right) \times 13 \times 1000$$

$$= 45 \times 13 \times 1000 = 585000$$

$$P_M : P_N$$

Ratio of profit will be = 507 : 585

$$= \left(\frac{33+57}{2}\right) \times 13 \times 1000$$

$$P_N = \frac{585}{1092} \times 84000 = 45000$$

Averages

06

BASICS OF AVERAGES

Average is the mean of a set of numbers or values.

Therefore average of a set of numbers is, Average

$$= \frac{X_1 + X_2 + X_3 + \dots + X_n}{n} \text{ or in other words, Average of some observations}$$

$$= \frac{\text{Sum of all observations}}{\text{Number of all observations}}$$

Average is also called the **Arithmetic Mean**.

Also, Sum of all observations = Average $\times$ Number of observations

$$\text{Number of observations} = \frac{\text{Sum of all observations}}{\text{Average}}$$

Let us try to find the average of the numbers given below.

20, 22, 25, 30, and 32

So, according to the above formula:

$$\text{Average} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\text{Average} = \frac{20 + 22 + 25 + 30 + 32}{5}$$

$$\text{Average} = 25.8$$

Let us see another approach to find the average of the numbers that are close to each other which will also help us in Data Interpretation questions i.e. Net Deviation Method.

Net Deviation Method

Let us take the same numbers as we had taken above 20, 22, 25, 30 and 32.

As we know, average can never be less than 20 and can never be more than 32. So, let us assume any of the numbers above as our average.

Let 25 be our assumed average.

Observation	20	22	25	30	32
Deviation	-5	-3	0	+5	+7

$$\text{So, Net deviation} = (-5) + (-3) + 0 + 5 + 7 = 4$$

$$\text{Now, required average} = 25 + \frac{4}{5} = 25.8$$

So,

$$\text{Average} = \text{Assumed average} + \frac{\text{Net deviation}}{\text{Number of observations}}$$

Example 1: What will be the average of 1105, 1120, 1132, 1140 and 1144?

Solution:

Here, we will find the average by Net Deviation Method as the given numbers are close to each other.

So, assumed average be 1120.

Observation	1105	1120	1132	1140	1144
Deviation	-15	0	+12	+20	+24

$$\text{Net deviation} = -15 + 12 + 20 + 24 = 41$$

$$\text{Now, required average} = 1120 + \frac{41}{5} = 1128.2$$

Example 2: If the average of p and q is 58 and the average of q and s is 64, then what is the value of (s - p)?

Solution:

Applying, $\text{Average} = \frac{\text{Sum of all observations}}{\text{Number of all observations}}$

$$\text{Given, } \frac{p+q}{2} = 58 \Rightarrow p+q = 116 \quad \dots (1)$$

$$\text{Also, } \frac{q+s}{2} = 64 \Rightarrow q+s = 128 \quad \dots (2)$$

Subtracting Eq. (1) from Eq. (2), we get

$$\Rightarrow (q+s) - (p+q) = 128 - 116$$

$$s - p = 12$$

Example 3: The average age of 9 members of Sharma family is 32 years. What would be the average age of the same family 4 years ago?

Solution:

Present average age of family = 32 years

In question of ages the average age changes by the number of years after/before which average is asked.

So, 4 years ago, average age of family = $32 - 4$
= 28 years

Example 4: The average of 11 numbers is 40 and that of the first five is 33 and that of the last five is 30. What is the value of the 6th number?

Solution:

Average of 11 numbers = 40

So, sum of 11 numbers = $11 \times 40 = 440$

Sum of the first 5 = $5 \times 33 = 165$

Sum of the last 5 = $5 \times 30 = 150$

$\therefore$ Value of the sixth number = $440 - (165 + 150)$
= $440 - 315 = 125$

Example 5: The average marks of 20 students in a class were calculated to be 18 out of 20. Later on, the teacher realized that the marks of two students were taken as 15 and 17, respectively, instead of 12 and 15. What will be the new average of the class?

Solution:

The marks of the two students were taken as 15 and 17, that is, a total of 32.

Actual marks received = $12 + 15 = 27$ marks

The total will get decreased by 5 marks.

So, the average will get decreased by $\frac{5}{20} = \frac{1}{4} = 0.25$

New average = $18 - 0.25 = 17.75$ marks.

Note:

- If each of the given quantities is increased by a constant p , then their average is also increased by p .
- If each of the given quantities is decreased by a constant p , then their average is also decreased by p .
- If each of the given quantities is multiplied by a constant p , then their average is also multiplied by p .
- If each of the given quantities is divided by a constant p , then their average is also divided by p .

Average in Arithmetic Sequence

Whenever the given quantities form an arithmetic sequence and

- If the given sequence has odd number of terms, then the average of the sequence is the middle term of the sequence. For example, the average of 1, 2, 3, 4, 5, 6 and 7 is 4 (the middle term).
- If the given sequence has even number of terms, then the average of the sequence is the average of the middle two terms. For example, the average of 1, 2, 3, 5 and 6 is $\frac{(3+4)}{2} = 3.5$

Shortcut:

- Average of first 'n' natural numbers = $\frac{(n+1)}{2}$

- Average of first 'n' even numbers = $(n + 1)$
- Average of first 'n' odd numbers = n

Example 6: What is the average of first five even numbers?

Solution:

The first five even numbers are 2, 4, 6, 8, 10

Average = $n + 1$

So, here $n = 5$; Average = 6

Example 7: The average of five consecutive even numbers is 50. What is the largest number of these numbers?

Solution:

Let the number be $x - 4, x - 2, x, x + 2, x + 4$

$$\text{Average} = \frac{\text{Sum of all observations}}{\text{Number of all observations}}$$

$$= \frac{x - 4 + x - 2 + x + x + 2 + x + 4}{5} = 50$$

$$\Rightarrow \frac{5x}{5} = 50 \Rightarrow x = 50$$

So, the numbers are 46, 48, 50, 52, 54.

Hence, the largest number of these numbers is 54.

PERSON LEAVING OR JOINING THE GROUP

As we know that the average is nothing but a quantity that defines the term's equal distribution, as it is very clear that if all the values of a set are replaced by their average value, then also the sum of values will remain unchanged, but if we replace a single value by any value other than that value, then sum will surely change.

There are basically four cases that we will discuss.

- Whenever a value larger than the average value comes inside a set of values, then the average will increase.

- Whenever a value smaller than the average value comes inside that set, then the average will decrease.
- Whenever a value larger than the average value leaves that set, then the average will decrease.
- Whenever a value smaller than the average value leaves that set, then the average will increase.

Let us understand this concept by taking an example each of first case and fourth case.

Example 8: Average age of a class of 6 students is 10 years. When a new student admits into the class, then the average increases by 1. What is the age of that new student?

Solution:

As we know, average is equal distribution, so initially those 6 students were assigned the age as 10 years, even when the new student came, he will also be assigned the age 10 years. But after his entry the average rose by 1 year that means he got some age extra that got equally divided among all including him. So, the age assigned to him was 11 years.

New average age is 11 years i.e. increment of 1 in the age of each student.

$$\text{Required age} = 11 + (6 \times 1) = 17 \text{ years}$$

Example 9: Average weight of a class of 10 students is 13 kg. A girl left the group and the average increased to 13.5 kg. What was the weight of that girl?

Solution:

As we studied in the fourth case, that when a value smaller than the average value leaves the set then the average increases. So, the weight of that girl must have been less than 13.

After she left the group, 9 students were remaining and their average rose by 0.5 kg.

$$\text{Total increment} = 0.5 \times 9 = 4.5 \text{ kg}$$

At the time of her entry in the group, she must have provided 4.5 kg weight by taking from the other students in group.

So, her original weight = $13 - 4.5 = 8.5$ kg

Example 10: Average weight of 32 students of a class is 30.5 kg. If the weight of a teacher is also included then the average weight is increased by 500 g. What is the weight of the teacher?

Solution:

$$\text{Applying, Average} = \frac{\text{Sum of all observations}}{\text{Number of all observations}}$$

$\Rightarrow$ Sum of all observations = Average $\times$ Number of observations

Total weight of 32 students = $30.5 \times 32 = 976$ kg

Average weight of (32 students + 1 teacher)

= $(30.5 + 0.5) = 31$ kg

$\therefore$ Total weight of (32 students + 1 teacher)

= $31 \times 33 = 1023$ kg

$\therefore$ Weight of the teacher = $(1023 - 976)$ kg = 47 kg

Alternate Method:

When teacher comes, average increase by 500 gm.

Weight of the teacher = $31 + (32 \times 0.5) = 47$ kg

Example 11: A student on her first 3 tests received an average score of N points. If she exceeds her previous average score by 20 points on her fourth test, then what is the average score for the first 4 tests?

Solution:

Sum of first 3 tests = $3N$

Score in fourth test = $N + 20$

$$\text{Average of four tests} = \frac{3N + N + 20}{4} = N + 5$$

Alternate Method:

Extra 20 marks in the fourth test should be equally divided among the four tests.

$$\text{So, average in the four tests} = N + \frac{20}{4} = N + 5$$

Example 12: The average age of a group of people was calculated as 28 years. This was 2 years more than the correct average as there was an error in noting the ages of 2 persons as 32 and 36 instead of 21 and 23. How many persons were there in the group?

Solution:

Incorrect average = 28

Correct average = 26

Let the number of persons be x .

$$28x - (32 + 36) + (21 + 23) = 26x$$

$$28x - 68 + 44 = 26x$$

$$2x = 24$$

$$x = 12$$

WEIGHTED AVERAGE

When two groups of observations are combined, then we talk of the average of the entire group. For example, there are two sections A and B of a class, where the average height of students of section A is 120 cm and that of section B is 150 cm. On the basis of this information alone, we cannot find the average of the entire class, (of the two sections), since we do not have any information regarding the number of students in the two sections. Now, suppose that we are given that there are 60 students in section A and 40 students in section B, then we can calculate the average height of the entire class.

The average height of the two classes combined is

$$= \frac{\text{Total height of the entire class}}{\text{Total number of students in the entire class}}$$

$$\text{So, average} = \frac{60 \times 120 + 40 \times 150}{60 + 40} = \frac{7200 + 6000}{100} = 132$$

This average height 132 cm of the entire class is called "weighted average" of the class.

Even if there are more than two groups of observations that have to be combined, then also the weighted average can be calculated by the same method. For example, if three sections in a class have their average marks as 75, 76 and 79 respectively and their respective strengths are 30, 35 and 35, then the average mark of the entire class is given by

$$\frac{75 \times 30 + 76 \times 35 + 79 \times 35}{30 + 35 + 35} = 76.75$$

The general formula for weighted average is

$$A_{wt} = \frac{(A_1 \times w_1) + (A_2 \times w_2) + \dots + (A_n \times w_n)}{w_1 + w_2 + \dots + w_n}$$

The above step in calculating the weighted average of the class can be rewritten as

$$= \frac{60 \times 120 + 40 \times 150}{60 + 40} = \frac{20(3 \times 120 + 2 \times 150)}{20(3 + 2)}$$

$$= \frac{(3 \times 120 + 2 \times 150)}{(3 + 2)} = \frac{660}{5} = 132$$

It is clear from the above step that we would have been able to calculate the average height of the entire class even if we had not been given the number of students in the individual sections, but we only had the ratio of the number of students in the two sections (which in this case is 3 : 2).

Example 13: Three varieties of almonds costing Rs. 400/kg, Rs. 500/kg and Rs. 700/kg are mixed in the ratio 1 : 1 : 2. What is the cost/kg of the resultant mixture?

Solution:

$$\text{Cost of resultant mixture/kg} = \frac{400 \times 1 + 500 \times 1 + 700 \times 2}{1 + 1 + 2}$$

$$= \frac{400 + 500 + 1400}{4}$$

$$= \frac{2300}{4} = 575 \text{ /kg}$$

Example 14: The ratio of boys to girls in a class is 5 : 3. In an exam, the average score of the boys was 45 while the average score of the girls was 35. What is the average score of the entire class in the exam.

Solution:

Average score of the class

$$= \frac{5 \times 45 + 3 \times 35}{5 + 3} = \frac{225 + 105}{8} = 41.25$$

Average score of the class = 41.25

Example 15: In colony Sitamani, there are 12 houses with an average of four members per house, while in colony Gitamani, there are 20 houses with an average of Y members per house. If the two colonies together have an average of 3.5 members per house, then what is the value of y?

Solution:

It is given that the average number of members in the two colonies together is 3.5. Therefore,

$$\frac{12 \times 4 + 20 \times Y}{12 + 20} = 3.5$$

$$12 \times 4 + 20 \times Y = 3.5 \times 32$$

$$48 + 20Y = 112$$

$$20Y = 64$$

$$\Rightarrow Y = 3.2 \text{ members per house}$$

Practice exercise Level 1

1. Five students obtained 14, 12, 18, x and 20 marks in an exam. If the average marks of the group is 15, then what is the value of x ?
(A) 12 (B) 13
(C) 11 (D) 14
2. What is the average of the squares of the first 19 natural numbers?
(A) 124 (B) 127.5
(C) 130 (D) 133.5
3. What is the average of all numbers between 11 and 80 which are divisible by 6?
(A) 46 (B) 47
(C) 44 (D) 45
4. The average of 12 numbers is 9. If each number is multiplied by 2 and added to 3, the average of the new set of numbers will be:
(A) 9 (B) 18
(C) 21 (D) 27
5. What is the average of the first seven even multiples of 4.
(A) 16 (B) 12
(C) 20 (D) 24
6. The average marks of a group of six students decreases by 3 when the marks of the highest scorer is replaced by the marks of the lowest scorer. If the marks of the lowest scorer are 25, then what are the marks of the highest scorer.
(A) 33 (B) 43
(C) 25 (D) 40
7. In a match, average of runs scored by 5 players is 49. If the runs scored by four players are 75, 30, 62 and 21 respectively, then how many runs did the 5th player scored?
(A) 43 (B) 49
(C) 57 (D) 89
8. The average of 45 results was calculated as 27 but later it was found that while calculating, 39 was taken as 93 by mistake, then what will be the correct average?
(A) 25.8 (B) 26.8
(C) 27.2 (D) 28.2
9. The average weight of a group of nine people decreases by 3 kg when a person weighing 56 kg was replaced by a new person. What is the weight of the new person?
(A) 29 kg (B) 27 kg
(C) 47 kg (D) 38 kg
10. The average age of 36 students in a group is 14 years. When the teacher's age is included, the average age increases by one year. What is the teacher's age?
(A) 31 years (B) 36 years
(C) 51 years (D) None of these
11. A teacher gave a test in which there were three sections. The average number of questions in the three sections was 25. If the number of questions in the third section was 50% more than the first section and five more than the second section, then what was the number of questions in the second section.
(A) 32 (B) 16
(C) 20 (D) 25
12. Five years ago, the average age of a husband and wife was 35 years. Today, the average age of the husband, wife and a child is 28 years. What was the average age of the couple when the child was born?
(A) 35 years (B) 40 years
(C) 38 years (D) 36 years
13. The average age of 5 members of a family is 24 years. If the youngest member is 8 years old, then what was the average age of the family at the time of the birth of the youngest member?
(A) 16 years (B) 20 years
(C) 24 years (D) 32 years

14. The average weight of a class of 40 students is 40 kg. If the weight of the teacher is included, the average weight increases by 500 grams. What is the weight of the teacher?
 (A) 60 kg (B) 60.5 kg
 (C) 50.5 kg (D) 44 kg
15. Out of three annual examinations, each with a total of 500 marks, a student secured average marks of 45% and 55% in the first and second annual examinations. To have an overall average of 60%, how many marks does the student need to secure in the third annual examination?
 (A) 450 (B) 400
 (C) 350 (D) 300
16. Three years ago, the average age of Akhil and Bimal was 18 years, With Chetan joining them, the present average age becomes 22 years. How old is Chetan now?
 (A) 24 years (B) 27 years
 (C) 28 years (D) 32 years
17. The average age of 14 cricketers and their coach is 15 years. If the age of coach is excluded, the average reduces by 1. What is the age of coach?
 (A) 35 years (B) 32 years
 (C) 30 years (D) 29 years
18. The average of 11 results is 182. If the average of first 6 results is 199 and that of the last 6 results is 161, then what will be the 6th result?
 (A) 79 (B) 118.5
 (C) 158 (D) 237
19. The average of 11 numbers is 10.9. If the average of the first six numbers is 10.5 and that of the last six numbers is 11.4, then what is the middle number?
 (A) 11.5 (B) 11.4
 (C) 11.3 (D) 11.0
20. Having scored 98 runs in the 19th innings, a cricketer increases his average score by 4. What will be his average score after the 19th innings?
 (A) 28 (B) 26
 (C) 24 (D) 22
21. A batsman in his 12th innings makes a score of 120, and thereby increased his average by 5. The average score after 12th innings is:
 (A) 60 (B) 55
 (C) 65 (D) 70
22. In the first 40 overs of a 50 over innings, the run rate was 4.8 runs/over. What is the required run rate in the remaining 10 overs to reach the target of 241 runs?
 (A) 6.7 (B) 6.5
 (C) 6.3 (D) 4.9
23. The average of any 5 consecutive odd natural numbers is k. If two more such numbers, just next to these five numbers are added, then what will be the new average?
 (A) $\frac{2}{7}(k+1)$ (B) $2k-3$
 (C) $2k+1$ (D) $k+2$
24. Narendra was conducting an experiment in which the average of 11 observations came to be 90, while the average of first five observations was 87, and that of the last five was 84. What was the measure of the 6th observation?
 (A) 135 (B) 150
 (C) 145 (D) 135
25. Average age of father and his two sons is 27 years. Five years ago, the average age of the two sons was 12 years. What is the present age of the father?
 (A) 34 years (B) 47 years
 (C) 64 years (D) 27 years
26. The average of 8 numbers is A, and one of the numbers is 14. If 14 is replaced with 28, then what is the new average in terms of A?
 (A) $A + \frac{1}{2}$ (B) $2A + \frac{3}{2}$

- (C) $\frac{3A}{2}$ (D) $A + \frac{7}{4}$
27. The average age of a class of 30 students and a teacher reduce by 0.5 years if we exclude the teacher. If the initial average is 14 years, then what is the age of the class teacher?
(A) 29 years (B) 30 years
(C) 31 years (D) 32 years
28. The average of 30 results is 20 and the average of other 20 results is 30. What is the average of all the results?
(A) 24 (B) 25
(C) 48 (D) 50
29. The average height of 30 girls in a class of 40 is 160 cm and the average of remaining girls is 156 cm. What is the average height of the whole class?
(A) 158 cm (B) 158.5 cm
(C) 159 cm (D) 159.5 cm
30. There are 197 boys and 591 girls in a college. If the average weight of boys is 63 kg and average weight of girls is 51 kg, then what is the average weight of the students in the college?
(A) 56 kg (B) 57 kg
(C) 54 kg (D) 60 kg
- (A) 104 (B) 103.5
(C) 102.5 (D) 101
3. The average temperature on Monday to Thursday in a particular week is 35° , while that on Tuesday to Friday is 30° . If the temperature on Monday is 50% more than that on Friday, then what is the temperature on Friday?
(A) 40° (B) 35°
(C) 26.66° (D) None of these
4. If the average of a group of six distinct natural numbers is 15, what could be the maximum value of any number in this group?
(A) 85 (B) 75
(C) 80 (D) 78
5. The average number of runs given per wicket was 35 runs for a bowler who had taken a total of 45 wickets before the start of a match. If the bowler took five wickets in the match by giving away 60 runs, what is the new average number of runs per wicket for the bowler?
(A) 40 (B) 32.7
(C) 43.5 (D) None of these
6. The average weight of a group of six people is 35 kg. It increases by 3 kg when a new member joins the group. What would be the average if one of the six members weighing 45 kg leaves the group?
(A) 36.83 (B) 35.33
(C) 34.00 (D) 37.00
7. The average score of 40 students in a Mathematics test is 50. If the highest and lowest scores were excluded, the average score of the class would decrease by 1. If the difference of these 2 scores is 60, then what is the highest score?
(A) 84 (B) 95
(C) 99 (D) 115
8. In 2008. Ravi spent Rs.5000 per month on an average for the first 4 months, Rs.6500 per month on an average in the next 5 months and

Practice exercise Level 2

1. The average of 7 consecutive odd numbers is A. If next 4 and previous 3 odd numbers to these 7 odd numbers are also included, then what is the new average of these 14 consecutive odd numbers?
(A) $A + 3$ (B) $A + 4$
(C) $A + 2$ (D) $A + 1$
2. Average of 30 numbers is 106. Later it was seen that two numbers 82 and 136 were wrongly taken as 92 and 186 for calculation of average. What is the correct average?
7. The average score of 40 students in a Mathematics test is 50. If the highest and lowest scores were excluded, the average score of the class would decrease by 1. If the difference of these 2 scores is 60, then what is the highest score?
(A) 84 (B) 95
(C) 99 (D) 115
8. In 2008. Ravi spent Rs.5000 per month on an average for the first 4 months, Rs.6500 per month on an average in the next 5 months and

Rs.4500 per month on an average in the last 3 months. His savings were Rs.18,000 in the year. What is his average monthly income for given year?

- (A) Rs.6000 (B) Rs.6400
(C) Rs.6600 (D) Rs.7000

9. The average marks of 14 students was calculated as 71. But it was later found that the marks of one student had been wrongly entered as 42 instead of 56 and of another as 74 instead of 32. What is the correct average?

- (A) 67 (B) 68
(C) 69 (D) 71

10. The average age of 30 boys in a class is 15 years. One boy, aged 20 years, left the class, but two new boys came in his place whose ages differ by 5 years. If the average age of all the boys now in the class becomes 15 years, then what is the age of the younger newcomer?

- (A) 20 years (B) 15 years
(C) 10 years (D) 8 years

11. A student on his birthday distributed on an average 5 chocolates per student. If on the arrival of the teacher and the headmaster to whom the student gives 10 and 15 chocolates respectively, the average chocolate distributed per head increases to 5.5, then what is the strength of the class?

- (A) 28 (B) 30
(C) 32 (D) 36

12. A painter is paid x rupees for painting every 10 metres of a wall and y rupees for painting every extra metre. During one week, he painted 10 metres on Monday, 13 metres on Tuesday, 12 metres on Wednesday, 11 metres on Thursday and 12 metres on Friday. What is his average daily earning in rupees for the five day week?

(A) $x + \frac{8y}{5}$

(B) $\frac{(5x + 9y)}{5}$

(C) $10x + \frac{8y}{5}$

(D) $5x + 8y$

13. The average age of 11 players in a cricket team is 28 years. Out of these, the average ages of three groups of three players each are 25 years, 28 years and 30 years respectively. If in these groups, the captain and the youngest player are not included and the captain is eleven years older than the youngest player, what is the age of the captain?

- (A) 33 years (B) 34 years
(C) 35 years (D) 36 years

14. The captain of a cricket team of 11 players is 25 years old and the wicket keeper is 3 years elder to the captain. If the ages of these two are excluded, the average age of the remaining players is 1 year less than the average age of the whole team. What is the average age of the whole team?

- (A) 21.5 years (B) 22 years
(C) 22.5 years (D) 32 years

15. In a class with a certain number of students, if one new student weighing 50 kg is added, then the average weight of the class increased by 1 kg. If one more student weighing 50 kg is added, then the average weight of the class increases by 1.5 kg over the original average. What is the original average weight of the class?

- (A) 46 kg (B) 42 kg
(C) 27 kg (D) 47 kg

16. The average weight of three men A, B and C is 84 kg. D joins them and the average weight of the four becomes 80 kg. If E, whose weight is 3 kg more than that of D, replaces A, the average weight of B, C, D and E becomes 79 kg. What is the weight of A?

- (A) 65 kg (B) 70 kg

- (C) 75 kg (D) 80 kg
17. Professor Bee noticed something peculiar while entering the quiz marks of his five students into a spreadsheet after each score was entered. Professor Bee entered the marks in a random order and noticed that after each mark was entered, the average was always an integer. In ascending order, the marks of the students were 71, 76, 80, 82 and 91. What were the fourth and fifth marks that Professor Bee entered?
- (A) 71 and 82 (B) 71 and 76
(C) 71 and 80 (D) 76 and 80
18. A set of consecutive positive integers beginning with 1 is written on the blackboard. A student came along and erased one number. The average of the remaining numbers is $35\frac{7}{17}$. What was the number erased?
- (A) 7 (B) 8
(C) 9 (D) None of these
19. The average weight of a body, as calculated by the average of seven different experiments is 53.735 gms, while the average of first three is 54.005 gms, the fourth was greater than the fifth by 0.004 gms, while the average of sixth and seventh was 0.01 gms less than the average of the first three. What is the weight of the body as obtained by the fourth experiment.
- (A) 54.036 (B) 53.072
(C) 52.096 (D) 56.028
20. There were 30 members in a certain hostel. If 10 more members are admitted, the expenses of the mess was increased by Rs. 40 per month, while the average expenditure per head diminished by Rs. 2. What is the original monthly expenses?
- (A) 480 (B) 400
(C) 360 (D) 380
21. The average number of depositor's in a post office saving bank for the year 1970, 1971, 1972 and 1973 was 2506. The number of depositor's in the year 1975 was 11% higher than those in the year 1974, while the average for the 6 year's 1970 to 1975 was 3007. What is the number of depositor's in each of the year's 1974 and 1975?
- (A) 3218, 3882 (B) 5200, 4800
(C) 3800, 4218 (D) 4612, 3600
22. A shop selling gift articles is closed on Monday. The average sale per day for the remaining six days of the week is Rs. 12500. The average sale per day for Tuesday to Friday is Rs. 8960. If Sunday's sale is 3 times the sale on Saturday, then what is the sale on Sunday?
- (A) Rs. 30200 (B) Rs. 18650
(C) Rs. 12800 (D) None of these
23. Asha calculated the average weight of all the students of her class as A. She also calculated the average of the average weights of all the possible pairs of students as B. Further, she calculated the average of the average weights of all the possible triplets of students as C. Which of the following is true about the relationship among A, B and C?
- (A) $A + B = 2C$ (B) $A + 2B = C$
(C) $A = B = 3C$ (D) None of these
24. The average of marks obtained by 120 candidates was 35, If the average of the passed candidates was 39 and that of the failed candidates was 15, then what was the number of those candidates, who passed the examination?
- (A) 100 (B) 110
(C) 120 (D) 150
25. The average age of women and child workers in a factory was 15 years. The average age of all the 16 children was 8 years and the average age of women workers was 22 years. If ten

women workers were married, then what was the number of unmarried women workers?

- (A) 16 (B) 12
(C) 8 (D) 6

26. Three math classes: X, Y, and Z, take an algebra test.

The average score in class X is 83.

The average score in class Y is 76.

The average score in class Z is 85.

The average score of all students in classes X and Y together is 79.

The average score of all students in classes Y and Z together is 81.

What is the average score for all the three classes, taken together?

- (A) 81 (B) 81.5
(C) 82 (D) 84.5

27. At an exam $\frac{1}{5}$ of a class individually gained $\frac{7}{8}$ of the maximum marks, $\frac{1}{10}$ gained $\frac{3}{4}$ of the maximum, $\frac{1}{4}$ gained $\frac{4}{5}$ of the maximum, and the rest $\frac{1}{2}$ of the maximum. The average marks obtained by the whole class are 162. What are the maximum marks?

- (A) 220 (B) 260
(C) 280 (D) 240

28. During the year 1963, the back rate was as follows: 6% for 3 weeks, $5\frac{1}{2}\%$ for 4 weeks, 5% for 2 weeks, $4\frac{1}{2}\%$ for 6 weeks, 3% for 5 weeks and 4% for 32 weeks. What was the average weekly rate during the year?

- (A) 5.6% (B) 4.2%
(C) 4.6% (D) 5.2%

29. Mr. Roy buys 100 units of the Crompton Greaves shares at Rs. 10.30 per unit. He purchases another lot of 200 at Rs. 10.40 per unit. At Rs. 10.50 per unit he takes up another lot of 400 and a further lot of 300 at Rs. 10.80 per unit. He watches as the price goes down and desires to take up as many units at Rs. 10.25 per unit as would make the average cost of his holding come down to Rs. 10.50 per unit. Assuming that Mr. Roy always buys units in multiples of 100, find the number of units he purchases at the lowest price of Rs. 10.25 per unit?

- (A) 300 (B) 350
(C) 250 (D) 200

30. The number of students in section A and section B in a school are 60 and 70 respectively and their respective average weights are 35 kg and 38 kg. If one of the students from section A is shifted to section B then the average weight of both the sections decreases. What can be said about the weight of concerned student?

- (A) His weight is less than 35 kg.
(B) His weight is more than 38 kg.
(C) His weight is more than 35 kg and less than 38 kg.
(D) Cannot be determined

Solution

Practice Exercise Level 1

- 1.(C) Given that average marks for the five students is 15.

Therefore,

$$14 + 12 + 18 + x + 20 = 75$$

$$x = 75 - 64 = 11 \text{ marks}$$

2.(C) $\text{Average} = \frac{19 \times 20 \times 39}{6 \times 19} = 130$

3.(D) Required average

$$= \frac{12 + 18 + 24 + \dots + 78}{12} = 45$$

- 4.(C)** Whatever operation is performed on the observations, the same operation is performed on average too.
Hence, Average will be $9 \times 2 + 3 = 21$
- 5.(A)** First seven multiples of 4 are: 4, 8, 12, 16, 20, 24, 28
Now, the above sequence is in AP and has odd number of terms.
So, the average will be the middle number i.e. 16.
- 6.(B)** Average score of the group of six students decreases by 3.
Therefore, total must have decreased by $6 \times 3 = 18$ marks.
Marks of the lowest scorer = 25
Marks of the highest scorer = $25 + 18 = 43$ marks
- 7.(C)** Let the runs scored by 5th player be x .
 $x = 49 \times 5 - (75 + 30 + 62 + 21)$
 $x = 57$
- 8.(A)** Correct average
$$= \frac{45 \times 27 + 39 - 93}{45} = 25.8$$
- 9.(A)** Average decreases by 3 kg
i.e. Total decrease in the sum = $3 \times 9 = 27$
Weight of person replaced = 56 kg
Weight of the new person = $56 - 27 = 29$ kg
- 10.(C)** After the inclusion of teacher, the average increases by 1 year
Total increase in the sum = $1 \times 37 = 37$ years
Initial age that must be assigned to teacher = 14 year
So, teacher's age = $37 + 14 = 51$ years
- 11.(D)** Let the number of questions in the first section be x .
Number of questions in the third section = $1.5x$
Number of questions in the second section = $1.5x - 5$
Also, total number of questions = 75,
Therefore,
 $x + 1.5x + 1.5x - 5 = 75$
 $4x = 80 \Rightarrow x = 20$
Number of questions in the second section = $30 - 5 = 25$
- 12.(D)** Five years ago, the average age of the couple was 35 years.
Today, the average age of the couple will be $35 + 5 = 40$ years
Therefore, the total age of the couple = $40 \times 2 = 80$ years
Average age of the husband, wife and child = 28 years
Total age of the family = $28 \times 3 = 84$ years
Age of the child is 4 years and so the child was born 4 years ago.
Average age of the couple 4 years ago = $40 - 4 = 36$ years
- 13.(B)** Total age of the family = $5 \times 24 = 120$ years
Total age 8 years ago = $120 - (5 \times 8) = 80$ years
Number of members at the time of birth of youngest member = 4
Required average = $\frac{80}{4} = 20$ years
- 14.(B)** Increase in average = 0.5 kg
Increase in total weight = $41 \times 0.5 = 20.5$ kg
Teacher's weight = $40 + 20.5 = 60.5$ kg
- 15.(B)** Let the marks in third be x . Then,
$$\left(\frac{45\% \text{ of } 500 + 55\% \text{ of } 500 + x}{1500} \right) \times 100 = 60\%$$

$$\frac{500+x}{15} = 60 \Rightarrow x = 400$$

- 16.(A)** Let A = Akhil's age,
B = Bimal's age, C = Chetan's age
 $(A - 3) + (B - 3) = 36 \Rightarrow A + B = 42$
Also, $A + B + C = 66 \Rightarrow C = 24$
- 17.(D)** Let the coach's age be x years.
So, $\frac{15 \times 15 - x}{14} = 14$
 $225 - x = 196$
 $x = 29$
- 18.(C)** Let the 6th result be x.
 $x = 199 \times 6 + 161 \times 6 - 182 \times 11$
 $x = 158$
- 19.(A)** Middle numbers
 $= (6 \times 10.5) + (6 \times 11.4) - (11 \times 10.9)$
 $= 63 + 68.4 - 119.9$
 $= 131.4 - 119.9 = 11.5$
- 20.(B)** Let the average score of the previous 18 innings be x.
So, $18x + 98 = 19(x + 4)$
 $x = 22$
Average score after 19th innings
 $x + 4 = 26$
- 21.(C)** Let the average of first 11 innings be 'x' runs.
 $\frac{\text{Total score of 11 innings}}{11} = x$
Also, $\frac{\text{Total score of 11 innings} + 120}{12} = x + 5$
 $\frac{\text{Total score of 11 innings} + 120}{12} = \frac{\text{Total score of 11 innings}}{11} + 5$
 $11 \times \text{Total score of 11 innings} + 1320 = 12 \times \text{Total score of 11 innings} + 660$
 $\text{Total score of 11 innings} = 1320 - 660 = 660$
Hence, average of 12 innings
 $= \frac{660 + 120}{12} = 65$
- 22.(D)** Runs scored in 40 overs
 $= 40 \times 4.8 = 192$

Remaining runs to be scored
 $= 241 - 192 = 49$

$$\text{Required run rate} = \frac{49}{10} = 4.9$$

- 23.(D)** Let us find the numbers answer by using the 1, 3, 5, 7, 9.

$$\text{Average} = \frac{1+3+5+7+9}{5} = 5$$

If we include next two number i.e. 11 and 13.

Now, average will be

$$\frac{1+2+3+4+5+6+7}{7} = 7$$

Now, average is 2 more than previous average.

[Note: The question can also be solved by taking the general form of numbers, like k - 4, k - 2, k, k + 2, k + 4]

- 24.(D)** Value of 6th observation
 $= (11 \times 90) - (5 \times 87) - (5 \times 84)$
 $= 990 - 435 - 420 = 135$

- 25.(B)** Let F denote the father and S_1 and S_2 his two sons

$$F + S_1 + S_2 = 81$$

$$(S_1 - 5) + (S_2 - 5) = 24$$

$$S_1 + S_2 = 34$$

$$\text{Age of father} = 81 - 34 = 47$$

- 26.(D)** $\frac{\text{Total of 7 numbers} + 14}{8} = A$

$$\text{Total of 7 numbers} + 14 = A \times 8$$

$$\text{Now, Total of 7 numbers} + 28$$

$$= \text{Total of 7 numbers} + 14 + 14$$

Now, average

$$= \frac{(\text{Total of 7 no.} + 14) + 14}{8}$$

$$= \frac{A \times 8}{8} + \frac{14}{8} = A + \frac{7}{4}$$

- 27.(A)** $= \frac{A \times 8}{8} + \frac{14}{8} = A + \frac{7}{4}$

Total age of students + Teacher

$$= 14 \times 31 = 434 \text{ years} \dots (1)$$

$$\frac{\text{Total age of 30 students}}{30} = 13.5$$

Total age of 30 students

$$= 405 \quad \dots (2)$$

From (1) and (2),

$$\text{Age of teacher} = 434 - 405$$

$$= 29 \text{ years}$$

28.(A) Required average

$$= \frac{30 \times 20 + 20 \times 30}{30 + 20}$$

$$\text{Average} = \frac{600 + 600}{50} = \frac{1200}{50} = 24$$

29.(C) Required value

$$= \frac{30 \times 160 + 10 \times 156}{30 + 10}$$

$$= \frac{4800 + 1560}{40} = \frac{6360}{40} = 159 \text{ cm}$$

30.(C)

$$\frac{197 \times 63 + 591 \times 51}{197 + 591}$$

$$= \frac{197 \times 63 + 591 \times 51}{788}$$

$$= \frac{63 + 153}{4} = \frac{216}{4} = 54 \text{ kg}$$

Practice Exercise Level 2

1.(D) Let the initial seven numbers be $A - 6, A - 4, A - 2, A, A + 2, A + 4, A + 6$.

$$\text{New sum} = 7A + 4A + 8 + 10 + 12 + 14 + 3A - 8 - 10 - 12$$

$$\text{Average} = \frac{14A + 14}{14} = A + 1$$

2.(A) Let the correct average be x , change in sum of observations = $(92 - 82), (186 - 136)$

$$x = \frac{30 \times 106 - 10 - 50}{30}$$

$$x = 104$$

3.(A) The average temperature from Monday to Thursday = 35°

$$\text{Total temperature} = 35 \times 4$$

$$= 140^\circ \quad \dots (1)$$

$$\text{Average temperature for Tuesday to Friday} = 30^\circ$$

$$\text{Total temperature} = 120^\circ \quad \dots (2)$$

From Eqs. (1) and (2), we have

$$\text{Temperature on Monday} - \text{Temperature on Friday} = 20^\circ$$

But given that temperature on Monday is 50% more than that on Friday, therefore

$$1.5 \times (\text{Temperature on Friday}) - \text{Temperature on Friday}$$

$$= 20^\circ \Rightarrow \text{Temperature on Friday}$$

$$= 40^\circ$$

4.(B) As the average of the group of six numbers is 15, the total value will be $15 \times 6 = 90$

As we are looking at the maximum value of the sixth number, we need to keep the other five at the minimum level.

Let the five numbers be 1, 2, 3, 4 and 5, thereby making a total of 15.

The maximum possible value of the sixth number is $90 - 15 = 75$

5.(B) Total number of runs given away by the bowler before the match = $45 \times 35 = 1575$

Number of runs given away in the match = 60 runs

$$\text{New average} = \frac{1575 + 60}{50}$$

$$= 32.7 \text{ runs per wicket taken}$$

6.(A) Initial average = 35

$$\text{New average} = 35 + 3 = 38$$

Now whenever the value larger than average moves out, then average decreases.

Difference between the value and average

$$= 45 - 38 = 7$$

This 7 will be equally subtracted from all the members.

$$\text{Final average} = 38 - \left(\frac{7}{6}\right) = 36.83$$

7.(C) Total score of 40 students

$$= 40 \times 50 = 2000$$

Total score except High and Low

$$= 38 \times 49 = 1862$$

$$\text{High} + \text{Low} = 2000 - 1862 = 138$$

$$\text{High} + \text{Low} = 138$$

$$\text{High} - \text{Low} = 60$$

By solving we get High = 99

8.(D) Total expenditure

$$= 5000 \times 4 + 6500 \times 5 + 4500 \times 3$$

$$= 20000 + 32500 + 13500$$

$$= 66000$$

$$\text{Saving} = 18000$$

$$\text{Total income} = 66000 + 18000$$

$$= \text{Rs. } 84000$$

Average monthly income

$$= \frac{84000}{12} = \text{Rs. } 7000$$

9.(C) Marks obtained by 14 students

$$= 14 \times 71 = 994$$

Exact marks of 14 students

$$= 994 + [(56 - 42) + (32 - 74)]$$

$$= 994 + [14 + (-42)] = 994 + (-28)$$

$$= 994 - 28 = 966$$

$$\text{So, correct average} = \frac{966}{14} = 69$$

10.(B) Let the age of younger boy be x .

Required value age of another boy = $x + 5$

$$= [31 \times 15] - [(30 \times 15) - 20]$$

$$= x + x + 5$$

$$x = 15$$

11.(A) Suppose strength of the class = x

$$\text{So, } 5x + 10 + 15 = 5.5(x + 2)$$

$$0.5x = 14$$

$$x = 28$$

12.(A) On Monday, the payment = Rs. x

On Tuesday, the payment = Rs. $(x + 3y)$

On Wednesday, the payment = Rs. $(x + 2y)$

On Thursday, the payment = Rs. $(x + y)$

On Friday, the payment = Rs. $(x + 2y)$

So, Average daily earnings during the five day week

$$= \frac{5x + 8y}{5} = x + \frac{8y}{5}$$

13.(C) Total age of all the players

$$= 28 \times 11 = 308$$

$$\text{Total group 1} = 3 \times 25 = 75$$

$$\text{Total group 2} = 3 \times 28 = 84$$

$$\text{Total group 3} = 3 \times 30 = 90$$

Total age of the group

$$= 75 + 84 + 90 = 249$$

Total age of captain and youngest players = $308 - 249 = 59$, also their difference is 11.

So, age of Captain is 35 years and the youngest is 24 years

14.(B) Let the average age of the whole team be x .

$$\text{Total age} = 11x$$

Total age of 9 players

$$= 11x - (28 + 25)$$

Average of 9 players

$$= \frac{11x - 53}{9} = x - 1$$

$$\therefore x = 22 \text{ years}$$

15.(D) Let number of students be n and average weight w .

According to the given condition,

$$\frac{nw + 50}{n + 1} = w + 1$$

$$n + w = 49 \quad \dots (1)$$

$$\text{and } \frac{nw + 50 + 50}{n + 2} = w + 1.5$$

$$1.5n + 2w = 97 \quad \dots (2)$$

On solving Eqs. (1) and (2), we get

$$w = 47$$

16.(C) Sum of weights of A, B and C = $84 \times 3 \text{ kg}$

$$= 252 \text{ kg}$$

$$\text{Weight of D} = (4 \times 80) - 252$$

$$= 320 - 252 = 68 \text{ kg}$$

$$\text{Weight of E} = 68 + 3 = 71 \text{ kg}$$

$$A + B + C + D = 320$$

$$B + C + D + E = 79 \times 4 = 316$$

$$A - E = 4 \text{ kg}$$

$$\text{So, } A = 4 + E = 4 + 71 = 75 \text{ kg}$$

- 17.(C) The marks of the 5 students and the remainder that they leave when divided by 3 are tabulated below.

Marks	71	76	80	82	91
Remainder	2	1	2	1	1

After the first 3 marks are entered, the total has to be a multiple of 3. The remainders can only be 1, 1, 1. (2, 1, 1 or 2, 2, 1 would not produce a multiple of 3)

So, The fourth and fifth marks entered were 71 and 80

- 18.(A) Let n = total number of numbers

$$\therefore \text{Average} = \frac{n(n+1)}{2(n)} = \frac{n+1}{2}$$

Now, Let x be the number which is erased.

$\therefore$ New average

$$= 35 \frac{7}{17} = \frac{\frac{n(n+1)}{2} - x}{n-1}$$

$$= \frac{602}{17} = \frac{\frac{n(n+1)}{2} - x}{n-1}$$

$$= \frac{602 \times 4}{17 \times 4} = \frac{\frac{n(n+1)}{2} - x}{n-1}$$

$$= \frac{2408}{68} = \frac{\frac{n(n+1)}{2} - x}{n-1}$$

Hence, number of observations

$$= 68 = n - 1$$

$$\therefore n = 69$$

$$\text{Also, } 2408 = \frac{69 \times 70}{2} - x$$

$$\Rightarrow x = \frac{69 \times 70}{2} - 2408$$

$$= 69 \times 35 - 2408$$

$$\therefore x = 7$$

So, the number erased was 7.

- 19.(B) Let the seven different experiments be a , b , c , d , e , f and g .

$$a + b + c + d + e + f + g = 7 \times 53.735$$

$$= 376.145 \quad \dots (1)$$

$$a + b + c = 3 \times 54.005$$

$$= 162.015 \quad \dots (2)$$

$$d - e = 0.004 \text{ gm} \quad \dots (3)$$

$$\frac{f+g}{2} = 54.005 - 0.01$$

$$= 53.995 \quad \dots (4)$$

$$\therefore (1) - (2) + (3)$$

$$2d + f + g = 376.145 - 162.015 + 0.004$$

$$= 214.134$$

$$\Rightarrow 2d + f + g = 214.134$$

$$\Rightarrow d + \frac{f+g}{2} = 107.067 \quad \dots (5)$$

$$\therefore d = (5) - (4)$$

$$= 107.067 - 53.995 = 53.072$$

- 20.(C) Let the monthly expenses per head be x .

Then, total expenses = $30x$

As per the question,

$$\frac{30x+40}{40} = x-2$$

$$\Rightarrow 30x + 40 = 40x - 80$$

$$\Rightarrow 10x = 120$$

$$\Rightarrow x = 12$$

$\therefore$ Total monthly expenses

$$= 30 \times x = 30 \times 12 = 360$$

- 21.(C) Total number of depositors in the first four years = 2506×4

$$= 10024$$

$$\text{Number of depositors in six years} = 3007 \times 6$$

$$= 18042$$

$\therefore$ Number of depositors in the last two

$$\text{years} = 18042 - 10024$$

$$= 8018$$

Also, number of depositors in sixth year =

$$1.11 \text{ number of depositors in the fifth year}$$

$$\Rightarrow D_5 + D_6 = 8018 \text{ and } D_6 = 1.11D_5$$

$$\Rightarrow D_5 + 1.11D_5 = 8018$$

$$\Rightarrow 2.11D_5 = 8018$$

$$\Rightarrow D_5 = \frac{8018 \times 100}{211} = 3800$$

$$\therefore D_6 = 8018 - 3800 = 4218$$

- 22.(D) Total Sales for 6 days

$$6$$

$$= 12500$$

Total sales for 6 days

$$= 12500 \times 6 = 75000$$

$$\text{Now, } \frac{\text{Tue} + \text{Wed} + \text{Thu} + \text{Fri}}{4} = 8960$$

Hence, Tue + Wed + Thu + Fri

$$= 8960 \times 4 = 35840$$

$$\text{Hence, Sat} + \text{Sun} = 75000 - 35840 = 39160$$

Sunday = 3 × Saturday

$$\text{Hence, } \frac{\text{Sunday}}{3} + \text{Sunday} = 39160$$

$$\frac{4}{3} \times \text{Sales on Sunday} = 39160$$

$$\text{Sales on Sunday} = \frac{39160 \times 3}{4}$$

$$= 29370$$

- 23.(D)** Let there be three students in the class x, y and z.

$$\text{Now, } A = \frac{x+y+z}{3}$$

$$B = \frac{\frac{x+y}{2} + \frac{y+x}{2} + \frac{x+z}{2}}{3} = \frac{(x+y+z)}{3}$$

$$B = \frac{x+y+z}{3} = A$$

$$C = \frac{x+y+z}{3} = B = A$$

$$\Rightarrow A = B = C$$

- 24.(A)** Suppose the number of candidates passed = x

$$\text{So, } 39x + 15(120 - x) = 120 \times 35$$

$$24x = 120 \times 35 - 120 \times 15$$

$$= 120(35 - 15) = 120 \times 20$$

$$x = 100$$

- 25.(D)** Let unmarried women workers are x, then as per question,

$$\frac{[16 \times 8] + [22 \times (10 + x)]}{16 + 10 + x} = 15$$

$$128 + 220 + 22x = 390 + 15x$$

$$7x = 42$$

$$\text{So, } x = 6$$

- 26.(B)** Let the number of students be X, Y and Z.

$$83X + 76Y = 79(X + Y)$$

$$4X = 3Y$$

$$\text{So, } \frac{X}{Y} = \frac{3}{4}$$

$$76Y + 85Z = 81(Y + Z)$$

$$4Z = 5Y$$

$$\frac{Y}{Z} = \frac{4}{5}$$

$$\text{So, } X : Y : Z = 3 : 4 : 5$$

Average of X, Y and Z

$$= \frac{83 \times 3 + 76 \times 4 + 85 \times 5}{3 + 4 + 5} = 81.5$$

- 27.(D)** Let the total number of students be x and maximum marks be m.

$$\left[\frac{x}{5} \times \frac{7}{8}m + \frac{x}{10} \times \frac{3}{4}m + \frac{x}{4} \times \frac{4}{5}m + \frac{9x}{20} \times \frac{m}{2} \right] = 162$$

$$\Rightarrow \frac{7}{40}m + \frac{3}{40}m + \frac{8}{40}m + \frac{9}{40}m = 162$$

$$\Rightarrow m = 240$$

- 28.(B)** Average weekly rate

$$= \frac{6 \times 3 + \frac{11}{2} \times 4 + 5 \times 2 + \frac{9}{2} \times 6 + 3 \times 5 + 4 \times 32}{3 + 4 + 2 + 6 + 5 + 32}$$

$$\therefore \text{Average weekly rate} = \frac{220}{52} = 4.2\%$$

- 29.(D)** Let the number of units to be bought at Rs. 10.25 per unit be x.

Then,

$$\frac{(100 \times 10.30 + 200 \times 10.40 + 10.50 \times 400 + 10.80 \times 300 + 10.25 \times x)}{100 + 200 + 400 + 300 + x} = 10.50$$

$$\Rightarrow 1030 + 2080 + 4200 + 3240 + 10.25x$$

$$= 10.5(1000 + x)$$

$$\Rightarrow 10.5x - 10.25x = 10550 - 10500$$

$$\Rightarrow 0.25x = 50$$

$$\Rightarrow x = \frac{100 \times 50}{25} = 200 \text{ units}$$

- 30.(C) The average weight of the student is more than 35 and less than 38 for this situation to be possible.

TOPRANKPREP

Allegation, Mixtures

07

The basic concept of Alligation is based on that of Weighted Average. Alligation rule helps in finding the ratio in which two weights has to be mixed to get a mixture with desired values.

Let us take an example of weighted average and derive the formula of alligation from it.

For example, in class A, the average weight of x students is 15 kgs. In class B, the average weight of y students is 20 kgs. When both the classes were combined, then the average weight was found to be 18 kg.

Let us apply the formula of weighted average and find out the ratio of x and y :

$$A_{wt} = \frac{A_1 \times w_1 + A_2 \times w_2}{w_1 + w_2}$$

$$18 = \frac{15 \times x + 20 \times y}{x + y}$$

$$18(x + y) = 15x + 20y$$

$$18x - 15x = 20y - 18y$$

$$x(18 - 15) = y(20 - 18)$$

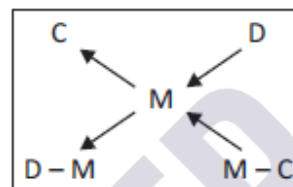
$$\frac{x}{y} = \frac{(20 - 18)}{(18 - 15)}$$

i.e. the ratio of x and y . Now, what we can see here is, 15 is the average weight of x students is the cheaper value (lesser value), 20 is the average weight of y students is the dearer value (larger value) and 18 is their combined/weighted average weight is the mean value. So, the above expression can be written as

$$\frac{x}{y} = \frac{\text{Dearer value (D)} - \text{Mean value (M)}}{\text{Mean value (M)} - \text{Cheaper value (C)}}$$

This is the Alligation formula.

Now, we can also represent the rule of Alligation in the following way.



It should be noted that wherever weighted average can be used, Alligation can also be used and wherever Alligation is used, weighted average can also be used to find the answer.

For example, if weighted average is given and the ratio of weights is to be found out, Alligation may prove to be faster and if the weights are given and the weighted average is to be found out, then Weighted Average Formula may be faster.

Note:

The most important thing in alligation is to understand about, what we get as ratios after applying rule. Suppose we apply alligation on cost of oil, which is given in Rs./litre, then we will get ratio of litres (or quantity). If we apply alligation on profit/loss percentage, then we will get ratio of cost price (as profit/loss is profit/cost price). So, we can say after applying alligation we get ratio of denominator of the unit of the quantities alligated.

Example 1: In what ratio, tea at Rs. 3.10 per kg is to be mixed with tea at Rs. 3.70 per kg, so that the mixture is worth Rs. 3.25 per kg?

Solution:

The conventional approach to solve this question is: We can say that let ' a ' kgs of type 1 and ' b ' kgs of type 2 are mixed.

$$a(3.10) + b(3.70) = 3.25a + 3.25b$$

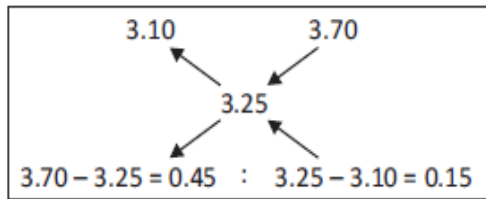
$$b(3.70 - 3.25) = a(3.25 - 3.10)$$

$$\frac{a}{b} = \frac{0.45}{0.15} = \frac{3}{1}$$

3 : 1 is the ratio of the quantity of the tea of first type to the quantity of the tea of second type i.e. for every 3 kgs of the first variety, 1 kg of the second is needed. We can also use the Rule of Alligation to solve this question as follows:

$$\frac{w_1}{w_2} = \frac{D-M}{M-C}$$

$$\text{i.e. } \frac{\text{Quantity of first type}}{\text{Quantity of second type}} = \frac{3.70 - 3.25}{3.25 - 3.10} = \frac{.45}{.15} = \frac{3}{1}$$

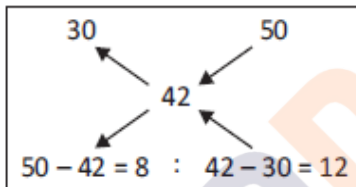


So, the ratio is 0.45 : 0.15 = 3 : 1

Example 2: In what ratio 30% alcohol solution must be mixed with 50% alcohol solution in order to get a 42% solution of alcohol?

Solution:

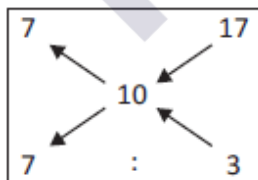
Using the rule of alligation,



So, the required ratio is 8 : 12 or 2 : 3.

Example 3: A merchant has 100 kg of sugar, part of which he sells at 7% profit and the rest at 17% profit. He gains 10% on the whole. Find how much is sold at 7% profit.

Solution:

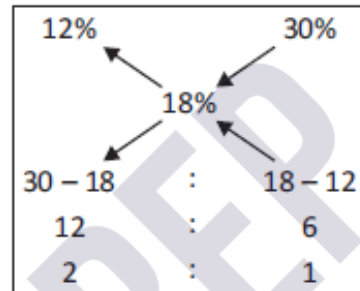


So, the ratio is 7 : 3

∴ The quantity of sugar sold at 7% profit = $\frac{7}{10}$ of 100 kg = 70 kg

Example 4: A butler stole wine from a butt of a sherry, which contained 30% spirit and he replaced what he had stolen by wine containing 12% spirit. The butt was then of 18 % strength. How much of the butt did he steal?

Solution:



By the alligation rule, we find that wine containing 30% of spirit and wine containing 12% spirit should be mixed in the ratio 1 : 2 to produce a mixture containing 18% spirit. This means that 1/3 of the butt of sherry was left; in other words the butler drew out 2/3 of the butt. Thus, 2/3 of the butt was stolen.

Example 5: A dealer mixes two varieties of apples, costing Rs. 25 per dozen and Rs. 36 per dozen and sells them at Rs. 40 per dozen, thereby gaining 25% on the transaction. Find out the ratio in which he had mixed the two varieties of apples.

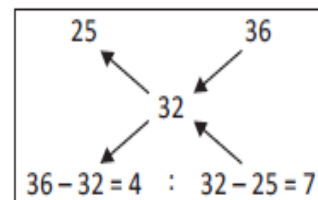
Solution:

SP of mixture = 40

Profit = 25%

So, SP = 1.25 CP

$$\text{Then, CP} = \frac{40}{1.25} = 32$$



So, the ratio is 4 : 7.

Example 6: There are some shepherds and their sheeps in a grazing field. The total number of heads

are 60 and total legs are 168 in the field, then what is the number of sheeps?

Solution:

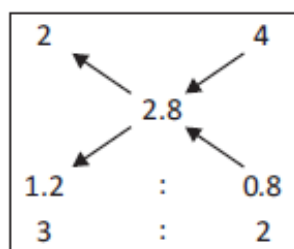
As everyone has only one head, so there is obviously a mixture of their legs.

Number of legs of a shepherd = 2

Number of legs of a sheep = 4

Average number of legs = $168/60 = 2.8$

Now, applying the rule of alligation,



Total number of creatures = 60

So, number of sheep = $60 \times 2/5 = 24$

Alternate Method:

Let the number of shepherds and sheep be x and y .

$$x + y = 60$$

$$2x + 4y = 168$$

By solving, we get, $x = 36$ and $y = 24$

So, the number of sheep = 24

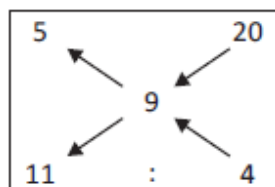
Example 7: Rs. 675 was divided among 75 boys and girls. Each boy gets Rs. 20 whereas each girl gets Rs. 5. What is the number of girls?

Solution:

Each boy's share = 20 and each girl's share = 5

$$\text{Average share} = \frac{675}{75} = 9$$

By rule of alligation,



Girls : Boys $\Rightarrow 11 : 4$

So, number of girls = $75 \times 11/15 = 55$

Alternate Method:

Let the number of boys and girls be x and y .

$$x + y = 75$$

$$20x + 5y = 675$$

By solving the two equations, we get

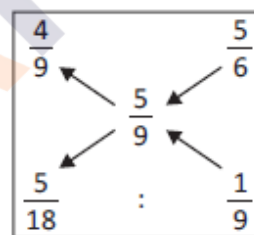
$$x = 20 \text{ and } y = 55$$

Example 8: Vessel A contains milk and water in the ratio 4 : 5 and vessel B contains milk and water in the ratio 5 : 1. In what proportion should the quantities be taken from A and B to form a mixture in which milk and water are in the ratio 5 : 4?

Solution:

In vessel A, milk is $\frac{4}{9} \left(\frac{4}{4+5} \right)$ of the total

Similarly, in vessel B, milk is $\frac{5}{6} \left(\frac{5}{5+1} \right)$ of the total.



So, the ratio is $\frac{5}{18} : \frac{1}{9} = 5 : 2$

Example 9: Three equal glasses are filled with the mixture of milk and water. The ratio of milk to water in each glass is as follows : in the first glass 2 : 3, in the second glass 3 : 4 and the third glass 4 : 5. The contents of the three glasses is emptied into a single vessel. What is the proportion of milk and water in it?

Solution:

Let the capacity of each glass be 1.

$$\text{So, the quantity of milk in the vessel} = \frac{2}{5} + \frac{3}{7} + \frac{4}{9} = \frac{401}{315}$$

$$\text{The quantity of water in the vessel} = \frac{3}{5} + \frac{4}{7} + \frac{5}{9} = \frac{544}{315}$$

$$\text{The required ratio} = \frac{401}{315} : \frac{544}{315} = 401 : 544$$

REPLACEMENT OF A PART OF A MIXTURE

- In the problem of removal/replacement, the solution is considered to be homogeneous.
- In these type of problems, generally the total volume remains constant which means the quantity which is removed is same as the quantity which is added.
- In these problems, we are asked to find out the final concentration of liquid which is replaced by another liquid after 'n' number of operations.

Let us understand it through an example,

Let there be 100 ml of wine present in a cellar, a person replaced 10% of this wine with the apple juice and performed this same operation two more times, now we are required to find the final concentration of that wine in the cellar.

So, whenever the same quantity of a liquid is removed and replaced with another liquid for 'n' number of times, then we can simply use a formula i.e.

⇒ Final concentration of solute = Initial concentration of solute ×

$$\left[1 - \frac{\text{Amount taken out each time}}{\text{Total volume of solution}} \right]^n$$

Here, 'n' is the number of times the operation is performed.

Let's us find the final concentration x in the above example,

$$x = 100 \times \left[1 - \frac{10}{100} \right]^3$$

$$x = 100 \times \frac{90}{100} \times \frac{90}{100} \times \frac{90}{100}$$

$$x = 72.9 \text{ ml}$$

Example 10: From a cask of wine containing 64 litres, 8 litres is drawn out and the cask is filled up with water. If the same process is repeated for two more times, how many litres of wine will be left in the

cask? What will be the proportion of wine to water in the resulting mixture?

Solution:

$$\text{Amount of wine left} = 64 \times \left[1 - \frac{8}{64} \right]^3 = \frac{343}{8}$$

$$\text{Amount of water} = 64 - \frac{343}{8} = \frac{169}{8}$$

$$\text{Proportion of wine to water} = \frac{343/8}{169/8} = \frac{343}{169}$$

Example 11: From a container containing milk, 12L is taken out. It is replaced with an equal quantity of concentrated wine. The process is done twice. The ratio of milk to wine in the cask now is 25 : 11. What is the volume of the container?

Solution:

Let us say, the total volume of the container is 'x' litres. After 2 such process we can say:

Amount of milk left

$$= \text{Initial amount of milk} \times \left(1 - \frac{12}{x} \right)^2$$

$$\text{So, } \left(1 - \frac{12}{x} \right)^2 = \frac{\text{amount of milk left}}{\text{initial amount of milk}} = \frac{25}{25+11} = \frac{25}{36}$$

Solving, we get x = 72 litres

Example 12: A container contains pure milk. 10% of this is removed and replaced with water. Then 25% of this is removed and replaced with water and finally 30% of this is removed and replaced with water. What is the percentage of milk in the final solution?

Solution:

Initially all is milk, so the initial percentage of milk = 100%

Percentage of milk left = Initial percentage ×

$$\left(1 - \frac{10}{100} \right) \times \left(1 - \frac{25}{100} \right) \times \left(1 - \frac{30}{100} \right)$$

$$= 100 \times \left(\frac{90}{100} \right) \times \left(\frac{75}{100} \right) \times \left(\frac{70}{100} \right) = 47.25\%$$

Practice exercise Level 1

1. A trader had 960 kg of rice. He sold a part of it at 20% profit and the rest at 8% profit, so that he made a total profit of 12%. How much rice (in kg) did he sell at 8% profit?
(A) 460 (B) 560
(C) 540 (D) 640
2. 24 kg of rice costing Rs. 20/kg is mixed with 16 kg of another rice costing Rs. x /kg to give a mixture costing Rs. 24/kg. What is the price of the second variety of rice?
(A) Rs. 16/kg (B) Rs. 28/kg
(C) Rs. 32/kg (D) Rs. 30/kg
3. In what ratio should a 70% milk solution be mixed with a 20% milk solution in order to get a 55% milk solution?
(A) 7 : 3 (B) 3 : 7
(C) 3 : 9 (D) 9 : 3
4. 5 L of a 70% milk solution is mixed with 3 L of a 40% milk solution. What is the percentage of milk in the resultant mixture?
(A) 55% (B) 60%
(C) 58.75% (D) 62%
5. What is the ratio in which the rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg?
(A) 1 : 3 (B) 2 : 3
(C) 3 : 4 (D) 4 : 5
6. A shopkeeper has 50 kg of rice. He sells a part of it at 20% profit and the rest at 40% profit. If he gain 25% on the whole, What is the quantity of each part?
(A) 39 kg, 11 kg (B) 37.5 kg, 12.5 kg
(C) 30 kg, 20 kg (D) None of these
7. How much pure alcohol can be added to 400 ml of a 15% solution to make its strength 32%?
(A) 100 ml (B) 60 ml
(C) 120 ml (D) 150 ml
8. A goldsmith has two qualities of gold, one of 22 carats and another of 36 carats purity. In what proportion should he mix both to make an ornament of 30 carats purity?
(A) 3 : 4 (B) 1 : 2
(C) 4 : 5 (D) 2 : 3
9. In what ratio must a grocer mix two tea's worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg, he may gain 10%?
(A) 3 : 2 (B) 3 : 4
(C) 3 : 5 (D) 4 : 5
10. A man bought a certain quantity of sugar for Rs. 8000. He sells $\frac{1}{4}$ th of it at 10% loss. At what percent profit should he sell the remaining stock so as to make an overall profit of 20%?
(A) 32% (B) 19%
(C) 25% (D) 30%
11. A sum of Rs. 18000 is lent out in two parts at 5% and 8% simple interest such that the simple interest on the whole sum at the end of 2 years is Rs. 1944. What is the sum that is lent out at 5% simple interest?
(A) Rs. 15600 (B) Rs. 17200
(C) Rs. 14400 (D) Rs. 15500
12. A mixture of rice is sold at Rs. 3.00 per kg. This mixture is formed by mixing the rice of Rs. 2.10 and Rs. 2.85 per kg. What is the ratio of price of cheaper to the costlier quality in the mixture if the profit of 10% on SP is being earned.
(A) 4 : 1 (B) 2 : 7
(C) 2 : 3 (D) 1 : 4
13. Some amount of Rs. 6000 was lent out at 10% per annum and the rest amount at 20% per annum and thus in 4 years the total interest from both the amounts collected was Rs. 4200. What is the amount which was lent out at 10% per annum?

- (A) Rs. 2800 (B) Rs. 3500
(C) Rs. 4500 (D) Rs. 1500
14. When 7 litres of milk at Rs. 30 per litre is mixed with 4 litres of another brand of milk, the resultant mixture costs Rs. 23 per litre. What is the cost (per litre) of the 4 litres milk brand?
(A) 9.33 (B) 6.67
(C) 10.75 (D) 11.25
15. A person buys two watches for Rs. 1000. He sells one at a loss of 5% and the other at 20% gain, and on the whole he gains Rs. 50. What is the cost price of each watch?
(A) Rs. 600, Rs. 400 (B) Rs. 750, Rs. 250
(C) Rs. 650, Rs. 350 (D) None of these
16. How many kg of tea at Rs. 62.5 per kg must be added to 100 kg of tea at Rs. 78 per kg so that a profit of 33.33% is made by selling the mixture at Rs. 94 per kg?
(A) 83.25 kg (B) 92.75 kg
(C) 93.75 kg (D) 82.25 kg
17. A man lends Rs. 36000 for two years - a part of it at an interest of 12.75% p.a. and the rest at an interest of 10.35% p.a., both at simple interest. If he receives Rs. 8460 as interest on the whole, how much money did he lend at two rates respectively?
(A) Rs. 28000, Rs. 8000
(B) Rs. 21000, Rs. 15000
(C) Rs. 24000, Rs. 12000
(D) Rs. 19000, Rs. 17000
- (C) 4 : 1 (D) 7 : 1
2. Milk and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio the liquids in both the vessels should be mixed to obtain a new mixture in vessel C containing half milk and half water?
(A) 1 : 1 (B) 1 : 3
(C) 1 : 2 (D) 7 : 5
3. Rs. 675 was divided among 75 boys and girls. Each boy gets Rs. 20 whereas a girl gets Rs. 5. What is the number of girls?
(A) 45 (B) 55
(C) 25 (D) 35
4. Nirupama mixed 40 ml of 40% sulphuric acid solution with 76 ml of 25% sulphuric acid. In what ratio must the resulting solution should now be mixed with 50% sulphuric acid to give 40% sulphuric acid again?
(A) 58 : 57 (B) 23 : 32
(C) 32 : 23 (D) 35 : 24
5. In what proportion must water be added to spirit to gain 20% by selling it at the cost price?
(A) 2 : 5 (B) 1 : 5
(C) 3 : 5 (D) 4 : 5
6. A butler stole wine from a butt of sherry containing 50% of spirit, then he replenished it by different wine containing 20% spirit. Thus there was only 30% strength (spirit) in the new mixture. How much of the original wine did he steal?
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$
7. There are some shepherds and sheep in a grazing field. The number of heads is 60 and total legs are 168 including both men and sheep. What is the number of sheep?
(A) 18 (B) 26
(C) 24 (D) 36

Practice exercise Level 2

1. A person mixed a solution containing spirit and water in the ratio 2 : 5 with another solution containing spirit and water in the ratio 2 : 1 resulting in a solution containing spirit and water in the ratio 1 : 2. In what ratio were the two solutions mixed?
(A) 2 : 5 (B) 5 : 3

8. How much water must be added to a bucket which contains 40 litres of milk at the cost price of Rs. 3.50 per litre so that the cost of milk reduces to Rs. 2 per litre?
 (A) 25 litres (B) 28 litres
 (C) 30 litres (D) 35 litres
9. Two vessels A and B contain alcohol and water in the ratios of 5 : 2 and 7 : 6, respectively. What is the ratio in which these mixtures are to be mixed to get a new mixture containing alcohol and water in the ratio of 8 : 5?
 (A) 5 : 2 (B) 7 : 9
 (C) 3 : 5 (D) 4 : 3
10. A milkman has 40 litres of milk. He mixes 8 litres of water, which is freely available, in 40 litres of pure milk. If the cost of pure milk is Rs. 22 per litre, then what is the profit of the milkman, if he sells all the mixture at cost price?
 (A) 25% (B) 20%
 (C) 33.33% (D) 9.09%
11. Three pots have the same volume. The ratio of milk and water in first, second and third pots are 3 : 2, 7 : 3 and 11 : 4, respectively. If the liquid of three pots are mixed, then what is the ratio of milk and water in the mixture?
 (A) 61 : 29 (B) 61 : 30
 (C) 61 : 31 (D) 61 : 37
12. In a municipal parking there are some two wheelers and rest are 4 wheelers. If wheels are counted, there are total 520 wheels but the incharge of the parking told me that there are only 175 vehicles. If no vehicle has a stepney, then what is the number of two wheelers?
 (A) 75 (B) 100
 (C) 90 (D) 85
13. A manufacturer has 200 litres of acid solution which has 15% acid content. How many litres of acid solution with 30% acid content may be added so that acid content in the resulting mixture will be more than 20% but less than 25%?
 (A) More than 100 litres but less than 300 litres.
 (B) More than 120 litres but less than 400 litres.
 (C) More than 100 litres but less than 400 litres.
 (D) None of the above
14. An alloy of zinc and tin contains 37% by weight of zinc. What is the weight of zinc which must be added to 400 kg of this alloy if the final percentage of zinc is to be 70%.
 (A) 550 (B) 460
 (C) 440 (D) 500
15. Three chests were full of three separate kinds of tea leaves, A, B and C. The first chest contained 16 kg of A; the second 28 kg of B; and the third, 36 kg of C. The chests were later emptied and the leaves thoroughly mixed, and the mixture was put back in the chests. How many kgs of each kind of tea would be found in the third chest?
 (A) 13.2, 7.6, 12.4 (B) 12.8, 6.2, 14.4
 (C) 14.6, 8.2, 16.8 (D) 7.2, 12.6, 16.2
16. A certain alloy contains, by weight, 5 parts of A and 3 parts of B. Another alloy contains, by weight, 6 parts of A and 7 parts of B. How many pounds of A, in its pure state, must be melted along with 20 lbs of the first alloy and 65 lbs of the second so as to produce a new alloy containing 40% by weight of B?
 (A) 21.25 lbs (B) 25.5 lbs
 (C) 26.25 lbs (D) 28.5 lbs
17. Two metals A and B are to be used for making two different alloys. There are only 253 tons of A and 161 tons of B. If the ratio by weight of A to B in the first alloy is 5 : 3 and that in the second is 4 : 7, then what weight of each alloy

must be produced so that no metal will be left over?

- (A) 400 kg, 14 kg (B) 392 kg, 22 kg
(C) 384 kg, 30 kg (D) 356 kg, 58 kg

18. Two boxes A and B were filled with beans and rice mixed in A in the ratio of 5 : 3, and in B in the ratio of 7 : 3. What quantity must be taken from each to form a mixture which shall contain 6 kg of beans and 3 kg of rice?

- (A) 4 kg, 5 kg (B) 3 kg, 6 kg
(C) 5 kg, 4 kg (D) 6 kg, 3 kg

19. Three substances X, Y and Z are mixed together, their volumes are proportional to 4, 3 and 2 respectively and the weights of equal volumes are proportional to 5, 4 and 3 respectively. What is the weight of each substance if the weight of the mixture is 114 kg?

- (A) 40 kg, 24 kg, 12 kg (B) 30 kg, 18 kg, 9 kg
(C) 50 kg, 30 kg, 15 kg (D) 60 kg, 36 kg, 18 kg

20. Three substances A, B and C are mixed together such that their ratios of volumes are 2 : 3 : 4. The ratios of their weights of equal volumes are 3 : 4 : 5 respectively. What is the weight of each substance if the weight of the mixture is 228 gm?

- (A) 28 kg, 60 kg, 140 kg (B) 18 kg, 48 kg, 96 kg
(C) 36 kg, 72 kg, 120 kg (D) 80 kg, 68 kg, 80 kg

21. Two liquids, A and B, are mixed together in the ratio 7 : 3 by volume, 1 c.c. of A weighs 1.408 gm, 1 c.c. of B weighs 1.320 gm. How many gm does 1 c.c. of the mixture weigh?

- (A) 1.382 gm (B) 1.3816 gm
(C) 1.342 gm (D) 1.3704 gm

22. A pot contains a mixture of milk and water in the ratio of 4 : 3 and another pot contains a mixture of milk and water in the ratio of 3 : 4. How many litres of the second mixture should be added to 7 litres of the first such that the milk and water in the resulting mixture is 6 : 5?

(A) $\frac{25}{3}$ l

(B) $\frac{14}{9}$ l

(C) $\frac{16}{7}$ l

(D) $\frac{21}{8}$ l

23. A certain alloy contains 3 parts by weight of A and 5 parts of B, another alloy contains 9 parts of A and 4 parts of B. If these alloys are melted and mixed together, how many kg of the second alloy must be mixed with 48 kg of the first alloy to make a mixture which contains 40% of B?

- (A) 120 kg (B) 117 kg
(C) 116 kg (D) 118 kg

24. A certain product C is made of two ingredients A and B in the proportion of 2 : 5. The price of A is three times that of B. The overall cost of C is Rs. 5.20 per kg including labor charges of 80 paise per kg. What is the cost of A and B per kg?

- (A) 8.4, 2.8 (B) 6.6, 2.2
(C) 7.2, 2.4 (D) 9.6, 3.2

25. Two lumps composed of Gold and Silver together weigh 20 kg, one lump contains gold 75% and silver 31.25 gm per kg. The other contains gold 85% and silver 30 gm per kg. The total quantity of silver in two lumps is 617.5 gm. If the two lumps are melted and formed into one, what percent of gold will it contain?

- (A) 78.4% (B) 76.5%
(C) 77.8% (D) 75.4%

26. Three vessels whose capacities are as 5 : 3 : 2 are completely filled with milk mixed with water. The ratio of milk and water in the mixture of vessels are as 3 : 2, 2 : 1 and 3 : 1 respectively. What is the percentage of milk and water in the new mixture obtained when $\frac{1}{3}$ rd of first, $\frac{1}{2}$ of second and $\frac{2}{3}$ rd of the third vessels is taken out and mixed together?

- (A) 66.67%, 33.33% (B) 63.63%, 36.37%
(C) 60%, 40% (D) 62.5%, 37.5%

27. A container contains a mixture of two liquids A and B in the ratio of 4 : 1. When 10 litres of the mixture is replaced with liquid B, the ratio becomes 2 : 3. How many litres of liquid A was present in the container initially?
 (A) 11 litres (B) 16 litres
 (C) 18 litres (D) None of these
28. A bonus of Rs. 985000 was divided among 300 employees of Timas Systems. Each male employee gets Rs. 5000 and each female employee gets Rs. 2500. What is the number of male employees in the company?
 (A) 138 (B) 175
 (C) 94 (D) 46
29. Can A contains 2 parts kerosene, 3 parts gasoline and 4 parts ethanol. Another can of equal capacity contains 5 parts kerosene, 6 parts gasoline and 7 parts ethanol. The contents of the two cans are mixed. What is the ratio of kerosene, gasoline and ethanol in the mixture?
 (A) 2 : 3 : 10 (B) 1 : 2 : 5
 (C) 3 : 4 : 5 (D) 3 : 4 : 10
30. A jar contains a mixture of a certain chemical and water, mixed in the ratio 5 : 3. If sixteen litres of the mixture is taken out and replaced with an equal quantity of water, the ratio becomes 15 : 17. What was the quantity of the chemical in the jar initially?
 (A) 56 litres (B) 48 litres
 (C) 54 litres (D) 40 litres
31. 18 litres of a milk solution in water contains 81% milk. A few litres of the mixture is removed and replaced with water. The process is repeated one more time. If the resultant mixture has 64% milk, how many litres of the mixture was replaced each time?
 (A) 1.25 (B) 2
 (C) 3 (D) 4
32. Two equal containers are filled with a mixture of 2 chemicals A and B. One of them contains 3 times as much chemical A as the other. The mixture in the two containers are then mixed and it is found that the ratio of A and B becomes 2 : 5. What is the original ratio of A : B in each of the containers?
 (A) 2 : 5, 6 : 1 (B) 3 : 4, 1 : 6
 (C) 2 : 5, 3 : 4 (D) 3 : 4, 2 : 5
33. A storekeeper bought oranges of three different grades. The first cost him Rs. 4.20 per dozen; the second cost him Rs. 27.00 for a box of 100. What he paid for a case of 150 of the third grade, if he made a profit of 25% by mixing them in the ratio 1 : 2 : 3 respectively, and selling them at Rs. 3.50 per dozen?
 (A) 24 (B) 25.5
 (C) 26.5 (D) 22
34. A container has a solution containing 50% milk. A part of this is taken out and replaced by an equal quantity of water. As a result, the percentage of milk in the container becomes 35%. What fraction of the container was replaced?
 (A) $\frac{3}{10}$ (B) $\frac{5}{10}$
 (C) $\frac{2}{5}$ (D) $\frac{4}{5}$
35. Container A is full of milk while container B is full of water. Half the volume of the first container is transferred to the second. Now half the volume of the second container is transferred back to the first. What is the ratio of milk to water in the second container, if the volume of both the containers are equal?
 (A) 1 : 2 (B) 2 : 1
 (C) 1 : 1 (D) 3 : 1
36. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio

- of A and B becomes 7 : 9. How many litres of liquid A was there in the can initially?
- (A) 10 litres (B) 20 litres
(C) 21 litres (D) 25 litres
37. A container is filled with pure alcohol. One fourth of alcohol is taken out and is replaced with water. Then from this mixture, half the amount is taken out and replaced with water. Finally, three fourth of the mixture is taken out and replaced with water. What is the concentration of alcohol in the container?
- (A) $13\frac{2}{3}\%$ (B) $9\frac{3}{8}\%$
(C) $8\frac{1}{2}\%$ (D) $14\frac{3}{8}\%$
38. In 80 litres mixture of milk and water, milk and water are in the ratio 5 : 3. If 16 litres of this mixture is replaced by 16 litres of milk and this process is repeated a total of 3 times. Then what is the ratio of milk and water in the resulting mixture?
- (A) 4 : 1 (B) 101 : 24
(C) 808 : 1571 (D) None of these
39. Two jars contain milk and water in the ratio 2 : 1 and 3 : 2 respectively. What volume should be taken out from the first jar if volumes have to be taken out from both the jars so as to fill up a 30litre jar with milk and water in the ratio 1 : 1?
- (A) 7.5 litres (B) 15 litres
(C) 22.5 litres (D) Not possible
40. A vessel is filled with liquid, 5 parts of which are water and 7 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?
- (A) $\frac{12}{7}$ (B) $\frac{1}{5}$
(C) $\frac{2}{5}$ (D) $\frac{1}{7}$
41. 8 litres are drawn from a cask filled with wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the total solution is 16 : 81. How much wine did the cask hold originally?
- (A) 72 litres (B) 48 litres
(C) 24 litres (D) 16 litres
42. From a 3 : 5 solution of milk and water, 20% is taken out and replaced by milk. How many times should this process be done to make the ratio of milk to water as 17 : 8?
- (A) Four times (B) Thrice
(C) Twice (D) Once
43. Four gallons is drawn from a cask full of petrol. It is then filled with water. Four gallons of mixture is again drawn and the cask is again filled with water. The ratio of quantity of petrol now left in the cask to that of the mixture in it is 36 : 49. How much does the cask hold?
- (A) 25 gallons (B) 28 gallons
(C) 30 gallons (D) 35 gallons
44. A litre of glycerine was poured out of a vessel filled up with pure glycerine to the brim and a litre of water was poured in. After the solution was mixed up, a litre of the mixture was poured out again and a litre of water was added. As a result of these operations, there was three times as much water in the vessel (by volume) as the remaining glycerine. How many litres of glycerine remain in the vessel as a result of the operations performed?
- (A) 0.25 litres (B) 0.50 litres
(C) 0.75 litres (D) 1.5 litres
45. There are two beakers each filled with 100 ml of alcohol and water respectively. First 10 ml of water is poured from the water in beaker 2 to the alcohol in beaker 1. Then 11 ml mixture from beaker 1 is poured back into beaker 2. Then from this new mixture in beaker 2, 10 ml

is poured back into beaker 1. Which of the following is true?

- (A) There is more water in beaker 1 than alcohol in beaker 2.
 (B) There is more alcohol in beaker 1 than water in beaker 2.
 (C) Neither (A) nor (B)
 (D) Both (A) and (B)

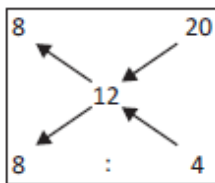
46. A cask contains 90 gallons of wine. A quantity of wine is drawn and replaced by water; 20 gallons of the mixture are again withdrawn and replaced by water with the result that the wine and water are in the ratio of 91 to 89. What is the quantity of wine first drawn off?

- (A) 30 (B) 60
 (C) 62.5 (D) 31.5

Solution

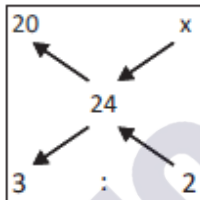
Practice Exercise Level 1

- 1.(D) Using rule of mixture and alligation.



$$\text{Rice sold at } 8\% = \frac{8}{12} \times 960 = 640 \text{ kg}$$

- 2.(D) By using rule of alligation,

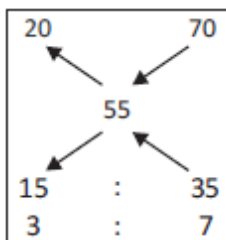


$$\text{So, } \frac{x-24}{24-20} = \frac{3}{2}$$

$$2x - 48 = 12$$

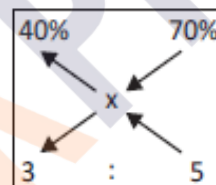
$$x = 30/\text{kg}$$

- 3.(A) Using the rule of alligation,



The required ratio is 7 : 3

- 4.(C) Using the rule of alligation,



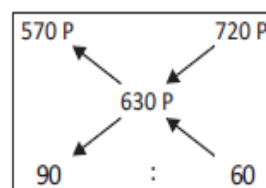
$$\text{So, } \frac{70-x}{x-40} = \frac{3}{5}$$

$$350 - 5x = 3x - 120$$

$$470 = 8x$$

$$x = 58.75\%$$

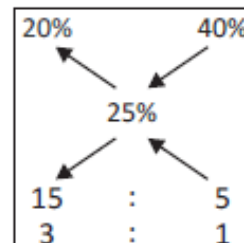
- 5.(B)



Hence, required ratio = 60 : 90

= 2 : 3

- 6.(B) According to the rule of alligation,



Quantity sold at 20% profit

$$= \frac{3}{3+1} \times 50 = 37.5 \text{ kg}$$

Quantity sold at 40% profit

$$= (50 - 37.5) = 12.5 \text{ kg}$$

7.(A) Alcohol present = $400 \times \frac{15}{100} = 60 \text{ ml}$

Now, let pure alcohol to be added be $x \text{ ml}$.

$$\therefore \text{Alcohol} = (60 + x) \text{ ml}$$

$$\text{Total solution} = (400 + x) \text{ ml}$$

Alcohol strength of this solution

$$= \left(\frac{60+x}{400+x} \right) \times 100 = 32$$

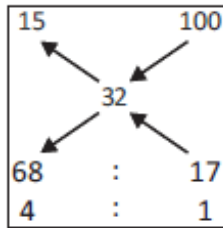
$$\Rightarrow 6000 + 100x = 12800 + 32x$$

$$\Rightarrow 68x = 6800$$

$$\Rightarrow x = 100 \text{ ml}$$

Using the rule of alligation,

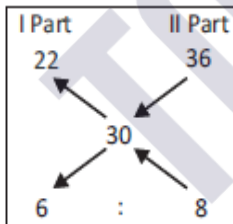
Let the alcohol to be added be 100%.



Quantity to be added

$$= \frac{400}{4} \times 1 = 100 \text{ litres}$$

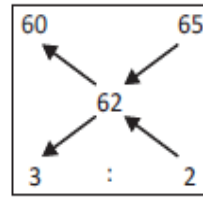
8.(A) According to the rule of alligation,



So, required ratio = I part : II part = 6 : 8 = 3 : 4

9.(A) SP of 1 kg mix = Rs. 68.20, gain = 10 %

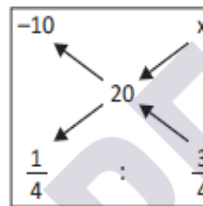
$$\text{CP of 1 kg mix} = \left(\frac{100}{110} \times 68.20 \right) = 62$$



(Cheaper tea) : (Dearer tea)

$$= 3 : 2$$

10.(D)



$$\text{So, } \frac{x-20}{20-(-10)} = \frac{1}{3}$$

$$3x - 60 = 30$$

$$3x = 90$$

$$x = 30$$

11.(A)

Let the sum lent out at 5% be x .

So, the sum lent out at 8% will be $(18000 - x)$

$$\text{Now, } 2[5\% \text{ of } x + 8\% \text{ of } (18000 - x)] = 1944$$

By solving we get $x = 15600$

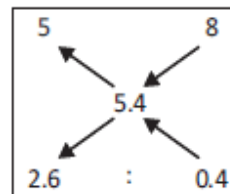
Let's first find the average rate of interest.

$$\text{SI} = \frac{P \times R \times T}{100}$$

$$1944 = \frac{18000 \times R \times 2}{100}$$

$$R = 5.4\%$$

Now, applying alligation rule,



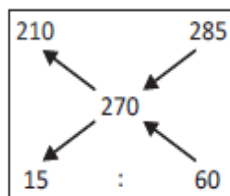
So, the ratio of interest is 26 : 4 or 13 : 2.

Now, Investment at 5%

$$= \frac{13}{15} \times 18000 = 15600$$

- 12.(D) The cost price of the mixture = Rs. 2.7
(3/1.1)

Now, applying alligation rule,



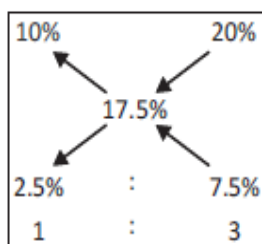
Therefore, the ratio of cheaper to costlier rice is 15 : 60, i.e., 1 : 4

- 13.(D) The total interest of one year

$$= ₹ 1050 = \left(\frac{4200}{4} \right)$$

Therefore, the average rate of interest = 17.5%

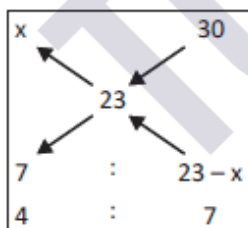
Thus, applying alligation rule



Hence, the ratio of amount which is lent at 10% per annum to the amount lent at 20% per annum is, 1 : 3.

Therefore, the amount which is lent out at 10% per annum is Rs. 1500.

- 14.(C) Using the rule of alligation,



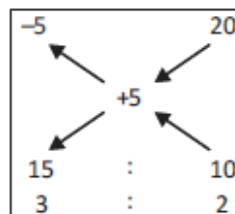
$$\therefore \frac{23-x}{7} = \frac{7}{4}$$

x = Rs. 10.75 per litre

Hence, the 4 litres milk brand costs Rs. 10.75 per litre.

- 15.(A) Overall % profit = $\frac{50}{1000} \times 100 = 5\%$

Using the rule of alligation,



CP of watch sold at 5% loss

$$= \frac{3}{3+2} \times 1000 = ₹ 600$$

CP of watch sold at 20% gain = $1000 - 600 = ₹ 400$

- 16.(C) SP of mixture = 94

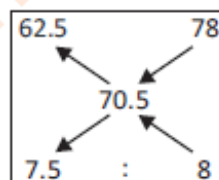
P% = 33.33%

$$\therefore \text{CP} = \text{SP} \times 0.75$$

$$= 94 \times 0.75 = 70.5$$

Let x kg be the quantity of tea to be sold at Rs. 62.5/kg.

Using the rule of alligation,



$$\therefore \frac{7.5}{8} = \frac{x}{100}$$

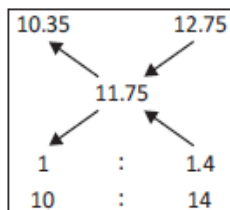
$$\Rightarrow x = \frac{7.5 \times 100}{8}$$

$$\Rightarrow x = 93.75 \text{ kg}$$

- 17.(B) Total interest % = $\frac{8460}{2 \times 36000} \times 100 = \frac{23.5}{2}\%$

= 11.75%

Using the rule of alligation,



$$5x + 7x = 36000$$

$$x = 3000$$

He lends 15000 and 21000 at the two rates of 10.35 and 12.75 respectively.

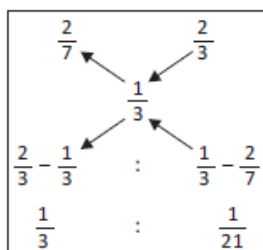
Practice Exercise Level 2

1.(D) Spirit in first solution = $\frac{2}{7}$

Spirit in second solution $\frac{2}{3}$

Spirit in mixture $\frac{1}{3}$

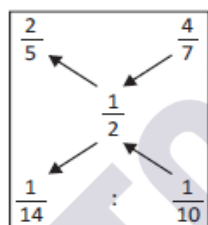
Using the rule of alligation,



$$21\left(\frac{1}{3} : \frac{1}{21}\right) = 7:1$$

2.(D) Milk in A = $\frac{4}{7}$, milk in B = $\frac{2}{5}$. Milk in mixture of A and B = $\frac{1}{2}$

Using the rule of alligation,



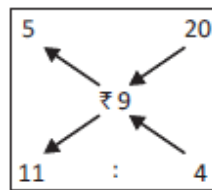
$$= \frac{1}{10} : \frac{1}{14} = 14:10 = 7:5$$

3.(B) Average money per head (boy or girl)

$$= \frac{675}{75} = 9$$

Each boy = 20; Each girl = 5

Using the rule of alligation,



$$\text{Number of girls} = \frac{11}{4+11} \times 75 = 55$$

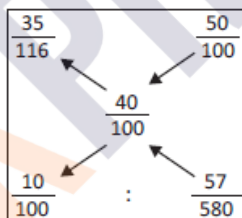
4.(A) 40% of 40 ml when mixed with 25% of 76 ml solution gives resultant sulphuric acid

By using weighted average,

$$= \frac{0.4(40) + 0.25(76)}{40 + 76} = \frac{35}{116}$$

Now, this is mixed with 50% solution to give 40%. Then the ratio would be

Using the rule of alligation,



$$\Rightarrow \frac{1}{10} : \frac{57}{580}$$

$$\Rightarrow 580\left(\frac{1}{10} : \frac{57}{580}\right)$$

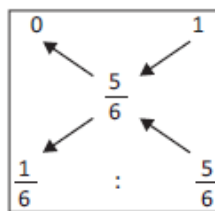
$$\Rightarrow 58 : 57$$

5.(B) Let the CP be Re. 1 per litre.

SP of 1 litre of mix = Re. 1,

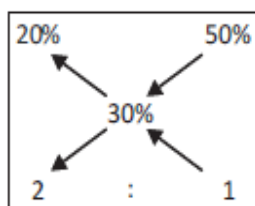
gain = 20%

$$\text{CP of 1 litre of mix} = \left(\frac{100}{120} \times 1\right) = \text{Re } \frac{5}{6}$$



$$\text{Ratio of water and spirit} = \frac{1}{6} : \frac{5}{6} = 1:5.$$

6.(B)



Now, in the final mixture out of the total 3 parts, 2 parts are of low quality wine. So, of the initial 3 parts, the butler must have stolen 2 parts. So, he has stolen $\frac{2}{3}$ rd of the original wine.

- 7.(C) Let the number of shepherds and Sheep be x and y .

$$x + y = 60$$

$$2x + 4y = 168$$

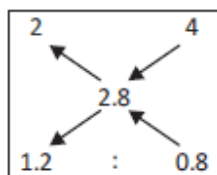
By solving we get $x = 36$ and

$$y = 24$$

So, the number of sheeps = 24

$$\text{Average legs per head} = \frac{168}{60} = 2.8$$

Using alligation rule,



Required ratio = 3 : 2

$$\therefore \text{Number of sheeps} = \frac{2}{5} \times 60 = 24$$

- 8.(C) Total Cost Price = $\left(40 \times \frac{7}{2}\right) = 140$

Reduced cost per litre = Rs. 2

$$\text{Total quantity} = \frac{140}{2} \text{ l} = 70 \text{ l}$$

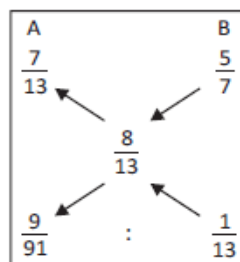
$$\text{Water to be added} = (70 - 40) \text{ l} = 30 \text{ l}$$

- 9.(B) Alcohol in 1 L mixture of A = $\frac{5}{7}$ L

$$\text{Alcohol in 1 L mixture of B} = \frac{7}{13} \text{ L}$$

$$\text{Alcohol in the mixture} = \frac{8}{13}$$

Using the rule of alligation,



So, required ratio

$$= \frac{1}{13} : \frac{9}{91} = 1 : \frac{9}{7} = 7 : 9$$

- 10.(B) When the water is freely available and all the water is sold at the price of the milk, then the water gives the profit on the cost of 40 litres of milk.

Therefore, profit percentage

$$= \frac{8}{40} \times 100 = 20\%$$

- 11.(A) Ratio of milk and water

$$= \frac{3}{5} + \frac{7}{10} + \frac{11}{15} : \frac{2}{5} + \frac{3}{10} + \frac{4}{15}$$

$$= \frac{18+21+22}{30} : \frac{12+9+8}{30}$$

$$= 61 : 29$$

- 12.(C) Let number of two and four wheelers be x and y .

$$x + y = 175$$

$$2x + 4y = 520$$

By solving, we get

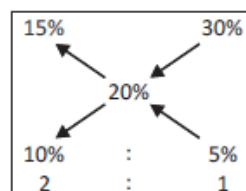
$$x = 90 \text{ and } y = 85$$

So, the number of two wheelers = 90

- 13.(C) Let the volume of the mixture be x l.

For the resulting mixture to contain 20% acid content.

Using the rule of alligation,



$$2x/3 = 200$$

$$x = 300 \text{ litres}$$

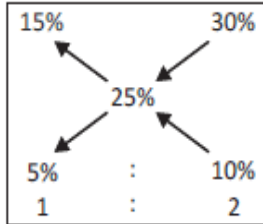
The mixture with 30% acid content to be

$$\text{added} = \frac{x}{3}$$

$$= \frac{300}{3} = 100 \text{ litres}$$

For the resulting mixture to contain 25% acid content.

Using the rule of alligation,



$$x/3 = 200, \quad x = 600$$

Mixture with 30% acid content to be added

$$= \frac{2x}{3}$$

$$= \frac{2 \times 600}{3} = 400 \text{ litres}$$

Hence, the acid solution with 30% acid content may be added so that acid content in the resulting mixture will be more than 20% but less than 25% should be more than 100 litres but less than 400 litres.

14.(C) Weight of zinc in 400 kg

$$= \frac{37}{100} \times 400 = 148 \text{ kg}$$

Let the amount to be added be x .

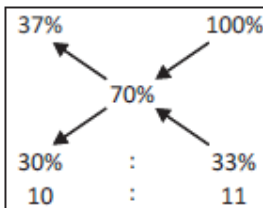
$$\therefore \frac{148+x}{400+x} = \frac{70}{100}$$

$$\Rightarrow 1480 + 10x = 2800 + 7x$$

$$\Rightarrow 3x = 2800 - 1480 = 1320$$

$$\therefore x = 440 \text{ kg}$$

Using the rule of alligation,



Hence, pure zinc to be added

$$= \frac{11}{10} \times 400 = 440 \text{ kg}$$

15.(D) Ratio of amounts of tea leaves in the three chests is 16 : 28 : 36 i.e. 4k, 7k and 9k.

$$\therefore \text{Total amount} = 20k$$

$$\therefore \text{Weight of A in third chest}$$

$$= \frac{4k}{20k} \times 36 = 7.2 \text{ kg}$$

$$\therefore \text{Weight of B in third chest}$$

$$= \frac{7k}{20k} \times 36 = 12.6 \text{ kg}$$

$$\therefore \text{Weight of C in third chest}$$

$$= \frac{9k}{20k} \times 36 = 16.2 \text{ kg}$$

16.(A) Let the weight of pure A to be added be x lbs. Then,

Alloy ₁ (A ₁) =	A	5/8
	B	3/8
Alloy ₂ (A ₂) =	A	6/13
	B	7/13

$$\therefore \text{Weight of A in 20 lbs of A}_1$$

$$= \frac{5}{8} \times 20 = 12.5 \text{ lbs}$$

$$\therefore \text{Weight of A in 65 lbs of A}_2$$

$$= \frac{6}{13} \times 65 = 30 \text{ lbs}$$

$$\therefore \text{Total weight of A in 85 lbs of mixture}$$

$$= 42.5 \text{ lbs}$$

$$\therefore \text{Weight of B in 85 lbs of mixture}$$

$$= 85 - 42.5 = 42.5 \text{ lbs}$$

As per the questions,

$$\frac{42.5+x}{85+x} = 0.6$$

$$\Rightarrow x = 21.25 \text{ lbs}$$

17.(B) Let the weight of alloy 1 and 2 to be produced be x and y kgs respectively.

Then, weight of metal A = 253 kg

$$\frac{5}{8}x + \frac{4}{11}y = 253 \quad \dots (1)$$

Weighty of metal B = 161 kg

$$\frac{3}{8}x + \frac{7}{11}y = 161 \quad \dots (2)$$

On solving (1) and (2),

$$x = 392 \text{ kg}, y = 22 \text{ kg}$$

- 18.(A)** Let the quantities taken from boxes A and B be x and y respectively.

Weight of beans required = 6 kg

$$\frac{5}{8}x + \frac{7}{10}y = 6 \quad \dots (1)$$

Weight of rice required = 3 kg

$$\frac{3}{8}x + \frac{3}{10}y = 3 \quad \dots (2)$$

On solving (1) and (2),

$$x = 4 \text{ kg}, y = 5 \text{ kg}$$

- 19.(D)** Ratio of volumes of x, y, z
= 4 : 3 : 2

Ratio of densities of x, y, z

$$= 5 : 4 : 3$$

$\therefore$ Ratio of weight of x, y, z

$$= 20 : 12 : 6 = 10 : 6 : 3$$

$$\therefore 10k + 6k + 3k = 114$$

$$\Rightarrow 19k = 114$$

$$\Rightarrow k = 6$$

$$\therefore \text{Weight of } x = 10k = 10 \times 6$$

$$= 60 \text{ kg}$$

$$\therefore \text{Weight of } y = 6k = 6 \times 6$$

$$= 36 \text{ kg}$$

$$\therefore \text{Weight of } z = 3k = 6 \times 3$$

$$= 18 \text{ kg}$$

- 20.(C)** Ratio of volumes of A, B, C
= 2 : 3 : 4

Ratio of densities of A, B, C

$$= 3 : 4 : 5$$

Ratio of weight of A, B, C

$$= 6 : 12 : 20 = 3 : 6 : 10$$

$$\therefore 3k + 6k + 10k = 228$$

$$\Rightarrow 19k = 228$$

$$\Rightarrow k = 12$$

$$\therefore \text{Weight of A} = 3k = 3 \times 12$$

$$= 36 \text{ kg}$$

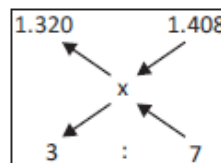
$$\therefore \text{Weight of B} = 6k = 6 \times 12$$

$$= 72 \text{ kg}$$

$$\therefore \text{Weight of C} = 10k = 12 \times 10$$

$$= 120 \text{ kg}$$

- 21.(B)** Let the weight of 1 gm of mixture be x .
Using the rule of alligation,



$$\Rightarrow \frac{1.408 - x}{x - 1.320} = \frac{3}{7}$$

$$\Rightarrow 9.856 - 7x = 3x - 3.96$$

$$\Rightarrow 10x = 13.816$$

$$\Rightarrow x = 1.3816 \text{ gm}$$

- 22.(B)** Pot 1 (P_1) has 7 l.

Then, milk in $P_1 = 4$ l

Water in $P_1 = 3$ l

Let pot 2 (P_2) has x l.

$$\text{Then, milk in } P_2 = \frac{3}{7}x$$

$$\text{Water in } P_2 = \frac{4}{7}x$$

As per the question,

$$\frac{4 + \frac{3}{7}x}{3 + \frac{4}{7}x} = \frac{6}{5}$$

$$\Rightarrow \frac{28 + 3x}{21 + 4x} = \frac{6}{5}$$

$$\Rightarrow 140 + 15x = 126 + 24x$$

$$\Rightarrow 9x = 14$$

$$\Rightarrow x = \frac{14}{9}$$

$\therefore 14/9$ l from pot 2 has to be added.

- 23.(B)**

$A_1 =$	A	3/8
	B	5/8
A_2	A	9/13
	B	4/13

Let the weight of second alloy (A_2) be x .

$\therefore$ Weight of B in A_1

$$= \frac{5}{8} \times 48 = 30 \text{ kg}$$

∴ Weight of B in A_2

$$= \frac{4}{13} \times x = \frac{4x}{13} \text{ kg}$$

Total weight of A_1 and $A_2 = 48 + x$

$$\therefore \frac{30 + \frac{4x}{13}}{48 + x} = \frac{40}{100}$$

$$\Rightarrow \frac{390 + 4x}{13(48 + x)} = \frac{2}{5}$$

$$\Rightarrow 1950 + 20x = 1248 + 26x$$

$$\Rightarrow 6x = 702$$

$$\Rightarrow x = 117 \text{ kg}$$

24.(A) Let the price of B per kg be Rs. x .

Then, the price of A per kg = Rs. $3x$

In one kg of C:

$$\frac{2}{7}(3x) + \frac{5}{7}(x) = 5.20 - 0.80$$

$$6x + 5x = 7 \times 4.4$$

$$x = \text{Rs. } 2.8$$

Hence, cost of A per kg = Rs. 8.4

and cost of B per kg = Rs. 2.8

25.(B) Let the weight of lump 1 be x kg and that of lump 2 is y kg.

$$\text{Then, } x + y = 20 \quad \dots (1)$$

$$\text{Also, } 31.25x + 30y = 617.5 \quad \dots (2)$$

On solving (1) and (2),

$$x = 14 \text{ kg, } y = 6 \text{ kg}$$

Percent gold present in mixture

$$= \frac{14 \times 0.75 + 6 \times 0.8}{20} \times 100 = 76.5\%$$

26.(A) Ratio of capacities of mixtures from 3

$$\text{vessels in the final mixture} = 5 \times \frac{1}{3} : 3 \times \frac{1}{2} : 2 \times \frac{2}{3}$$

$$= 10 : 9 : 8$$

Percentage of milk in the final mixture

$$= \frac{\left(10 \times \frac{3}{5} + 9 \times \frac{2}{3} + 8 \times \frac{3}{4}\right)}{10 + 9 + 8} \times 100$$

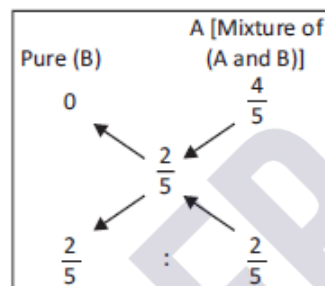
$$= 66.67\%$$

Percentage of water in the final mixture = 33.33%

27.(B) Liquid A Initially = $\frac{4}{5}$

Liquid A finally = $\frac{2}{5}$

According to the rule of alligation,



Hence, 10 litres of B is mixed to 10 litres of the mixture. Hence, total mixture was 20 litres

$$4x + x = 20$$

$$x = 4$$

Hence, Initially there was 16 litres of A.

28.(C) Let male employees be x .

Female employees be y .

$$\Rightarrow x + y = 300$$

$$5000x + 2500y = 985000$$

$$\Rightarrow 2x + y = 394$$

$$x + y = 300$$

$$\text{Hence, } x = 94$$

So, number of male employees are 94.

29.(C) Ratio of liquids in A = 2 : 3 : 4

Rate of liquids in can B = 5 : 6 : 7

Let the total amount in A and B be 90 l respectively.

$$\text{Hence, Kerosene in A} = \frac{2}{9} \times 90 = 20 \text{ l}$$

$$\text{Kerosene in B} = \frac{5}{18} \times 90 = 25 \text{ l}$$

$$\text{Gasoline in A} = \frac{3}{9} \times 90 = 30 \text{ l}$$

$$\text{Gasoline in B} = \frac{6}{18} \times 90 = 30 \text{ l}$$

$$\text{Ethanol in A} = \frac{4}{9} \times 90 = 40 \text{ l}$$

$$\text{Ethanol in B} = \frac{7}{18} \times 90 = 35 \text{ l}$$

When Mixed the ratio will be

$$= 45 : 60 : 75$$

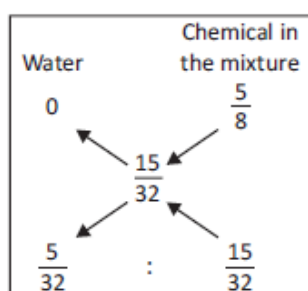
$$= 9 : 12 : 15$$

$$= 3 : 4 : 5$$

30.(D) Chemical Initially = $\frac{5}{8}$

Chemical finally = $\frac{15}{32}$

Now, using rule of alligation,



Hence, ratio of pure water to the mixture

$$= 1 : 3$$

As 16 liter of water is there

$$x = 16$$

Hence, total mixture initially was $x + 3x = 4x$

$$= 4 \times 16 = 64$$

Hence, chemical initially was

$$= \frac{5}{8} \times 64 = 40 \text{ l}$$

31.(B) Using the replacement concept,

$$\frac{64}{81} = \left(1 - \frac{x}{\text{Total Quantity}}\right)^2$$

Where x is the quantity taken out each time

$$\frac{8}{9} = \left(1 - \frac{x}{18}\right)$$

$$\frac{x}{18} = 1 - \frac{8}{9}$$

$$\frac{x}{18} = \frac{1}{9}$$

$$x = 2$$

Hence, 2 l was removed each time.

32.(B) Let the volume of each container be 7 l.

In mixture,

$$A : B = 2 : 5$$

Total volume = 14 l

$$A = \frac{2}{7} \times 14 = 4 \text{ l}; B = \frac{5}{7} \times 14 = 10 \text{ l}$$

Now, container 1 contains 3 times A as that of container 2.

$$\text{i.e. A in container 1} = 3 \text{ l}$$

$$\text{A in container 2} = 1 \text{ l}$$

$$\text{B in container 1} = 7 - 3 = 4 \text{ l}$$

$$\text{B in container 2} = 7 - 1 = 6 \text{ l}$$

$$\therefore \text{Container 1: A : B} = 3 : 4$$

$$\text{Container 2: A : B} = 1 : 6$$

33.(B) Cost of a dozen oranges of first grade = Rs. 4.20

Cost of a dozen oranges of second grade

$$= \frac{27}{100} \times 12 = 3.24$$

Cost of a dozen oranges of third grade

$$= \frac{x}{150} \times 12 = 0.08x$$

where x is the total cost of third grade orange box. Then,

$$\frac{(4.20 \times 1 + 3.24 \times 2)}{2 + 0.08x \times 3} = \frac{3.5}{1.25} \text{ (CP/dozen)}$$

$$x = \text{Rs. } 25.5$$

34.(A) Final concentration = Initial concentration

$$\left[1 - \frac{\text{Amount taken out each time}}{\text{Total volume of the solution}}\right]$$

Final concentration = 35%

Initial concentration = 50%

Total volume of the solution

$$= 100\%$$

$$\text{So, } 35 = 50 \left[1 - \frac{x}{100}\right]$$

$$35 = \frac{100 - x}{2}$$

$$x = 30\%$$

i.e. 30% quantity was replaced

$$30\% = \frac{3}{10}$$

35.(A) Let the volume of both containers be x .

$\frac{x}{2}$ amount of milk is transferred to the

second container. The second container will

have milk and water in the ratio $= \frac{x}{2} : x = 1 : 2$

The total volume of the second container = $1.5x$

Volume taken out from second container = $0.75x$

Regardless of volume reduced, ratio of milk to water in second will remain $1 : 2$.

Milk left in second container $\frac{1}{3} \times 0.75x$

Water left $= \frac{2}{3} \times 0.75x$

Ratio of milk to water in the second container = $1 : 2$.

36.(C) Initially let the can hold $7x$ litres and $5x$ litres of A and B respectively.

Quantity of A in remaining mix

$$= \left(7x - \frac{7}{12} \times 9 \right) \text{ litres} = \left(7x - \frac{21}{4} \right) \ell$$

Quantity of B in remaining mix

$$= \left(5x - \frac{5}{12} \times 9 \right) \text{ litres} = \left(5x - \frac{15}{4} \right) \ell$$

$$\therefore \frac{\left(7x - \frac{21}{4} \right)}{\left(5x - \frac{15}{4} \right) + 9} = \frac{7}{9} \Rightarrow \frac{28x - 21}{20x + 21} = \frac{7}{9}$$

$$\Rightarrow 252x - 189 = 140x + 147$$

$$\Rightarrow 112x = 336 \Rightarrow x = 3$$

$$\therefore A = 7x = 7 \times 3 = 21 \text{ l}$$

Initial concentration of liquid

$$A = \frac{7}{12}$$

Final concentration of liquid

$$B = \frac{7}{16}$$

So, according to replacement formulas

$$\frac{7}{16} = \frac{7}{12} \times \left[1 - \frac{9}{x} \right]$$

$$\frac{\frac{7}{16}}{\frac{7}{12}} = \frac{x-9}{x}$$

$$\frac{12}{16} = \frac{x-9}{x}$$

$$12x = 16x - 144$$

$$4x = 144$$

$$x = 36$$

$$\text{Quantity of A} = (7 \times 36) / 12$$

$$= 21 \text{ litres}$$

37.(B) Initial percentage of alcohol = 100%

Percentage left = Initial percentage

$$\times \left(1 - \frac{1}{4} \right) \times \left(1 - \frac{1}{2} \right) \times \left(1 - \frac{3}{4} \right) = 100 \times \left(\frac{3}{4} \right) \times \left(\frac{1}{2} \right) \times \left(\frac{1}{4} \right) = 9\frac{3}{8}\%$$

38.(B) We are working with the volume of water as it is only removed and not replaced.

Initial volume of water

$$= (3 \times 80) / 8 = 30 \text{ litres}$$

Volume of water left

$$= 30 \times \left(1 - \frac{16}{80} \right)^3 = \frac{384}{25} \text{ litres}$$

So, final volume of milk

$$= 80 - \text{final volume of water}$$

$$= 80 - \frac{384}{25} = \frac{1616}{25}$$

Hence, the ratio of milk to water in the final solution is

$$= \frac{1616}{25} : \frac{384}{25} = 101 : 24$$

39.(D) In both the jars, the concentration of milk is more than 50%. Therefore, in the third jar the concentration of milk cannot come down to 50% under any circumstances.

40.(D) Suppose the vessel initially contain 12 litres of liquid.

Let x litres of this liquid is replaced with water.

Quantity of water in new mixture

$$= \left(5 - \frac{5x}{12} + x \right) \text{ litres}$$

Quantity of syrup in new mixture

$$= \left(7 - \frac{7x}{12} \right) \text{ litres}$$

$$= \left(5 - \frac{5x}{12} + x \right) = \left(7 - \frac{7x}{12} \right)$$

$$60 + 7x = 84 - 7x$$

$$14x = 24$$

$$x = 12/7$$

So, part of the mixture replaced

$$= \left(\frac{12}{7} \times \frac{1}{12} \right) = \frac{1}{7}$$

- 41.(C)** Let the capacity of the cask be x l. The ratio of wine to total solution in cask after 4 operations:

$$= 1 \times \left(\frac{x-8}{x} \right)^4 = \frac{16}{81} = \frac{(x-8)}{x} = \frac{2}{3}$$

$$3x - 24 = 2x$$

$$x = 24 \text{ litres}$$

- 42.(B)** Let the volume of milk = $3x$ litres and volume of water = $5x$ litres

When 20% of mixture is taken out

Volume of milk left

$$= \left(3x - \frac{1}{5} \times 3x \right) = \frac{12x}{5} \text{ litres}$$

$$= 2.4x \text{ litres}$$

Volume of water left

$$= \left(5x - \frac{1}{5} \times 5x \right) = 4x \text{ litres}$$

And $1.6x$ litres of milk is added, New ratio = $1 : 1$

Similarly, proceeding we get option (b) as answer.

Let the total volume be v .

$$\text{Then, initial value of water} = \frac{5}{8}v$$

$$\text{Final volume of water} = \frac{8}{25}v$$

Say the process is repeated n times. Then,

$$\frac{8}{25}v = \frac{5}{8}v \left(1 - \frac{1}{5} \right)^n$$

On solving, $n = 3$

- 43.(B)** Let the cask hold x gallons.

So, Petrol per gallon in the mixture after 2

$$\text{operations} = \left(\frac{x-4}{x} \right)^2$$

$$\text{So, } \left(\frac{x-4}{x} \right)^2 = \frac{36}{49} = \left(\frac{6}{7} \right)^2$$

$$\frac{x-4}{x} = \frac{6}{7} \Rightarrow x = 28 \text{ gallons}$$

- 44.(B)** Let x l be the volume of vessel.

After operation I, glycerin

$$= x - 1 \text{ and water} = 1$$

After operation II,

$$\text{glycerin} = \frac{(x-1)^2}{x} \text{ and}$$

$$\text{water} = \frac{2x-1}{x}$$

According to the given condition,

$$\frac{2x-1}{x} = \frac{3(x-1)^2}{x}$$

$$\Rightarrow 2x - 1 = 3(x^2 - 2x + 1)$$

$$\Rightarrow 3x^2 - 8x + 4 = 0$$

$$\Rightarrow (3x - 2)(x - 2) = 0$$

$$\Rightarrow x = \frac{2}{3} \text{ or } 2$$

$x = \frac{2}{3}$ is not possible, since 1 litre glycerine was taken out.

$$\Rightarrow \text{So, } x = 2$$

Final quantity of glycerin in the mixture

$$= \frac{(x-1)^2}{x} = \frac{1}{2} = 0.5 \text{ litres}$$

- 45.(C)**

		Beaker 1	Beaker 2
Initial State	Alcohol	100	0
	Water	0	100
1 st exchange	Alcohol	100	0
	Water	10	90
2 nd exchange	Alcohol	90	10
	Water	9	91

Final Exchange	Alcohol	$90 + \frac{10}{101} = \frac{9100}{101}$	$10 - \frac{10}{101} = \frac{1000}{101}$
	Water	$9 + \frac{91}{101} = \frac{1000}{101}$	$91 - \frac{91}{101} = \frac{9100}{101}$

Hence, we see that the amount of water in beaker 1 is equal to the amount of alcohol in beaker 2. Hence, statements (a), (b) and (d) are false. Hence, (c) is the correct answer.

46.(D) Let the quantity of wine drawn at first be x .

∴ By replacement formula,

$$\frac{91}{180} = \left(1 - \frac{x}{90}\right) \left(1 - \frac{20}{90}\right)$$

$$\Rightarrow \frac{91}{180} = \left(\frac{90-x}{90}\right) \left(\frac{70}{90}\right)$$

$$\Rightarrow \frac{13}{2} = \frac{90-x}{9}$$

$$\Rightarrow 117 = 180 - 2x$$

$$\Rightarrow 2x = 63$$

$$\therefore x = 31.5 \text{ gallons}$$

Time & Work, Pipe & Cisterns

08

Problems on Time and Work are basically on the number of days taken to complete the work by two or more people.

But let's first look at what is work and its related parameters.

The concept of work is same as that of Time, Speed and Distance. Speed is same as rate of doing work, distance is the amount of work done and time remains as the time taken. Thus time taken to complete the work.

$$= \frac{\text{Amount of work}}{\text{Rate of doing work}}$$

When two or more persons work on the same job, their rate of doing work gets added.

Example 1: If Raja and Rohit can do a job in 10 days and 15 days independently. Then by working together, work will get completed quickly as now more amount of work will be completed per day. Let's see how to find the number of days taken to complete the work.

Solution:

Let the total work be 'W', Raja's rate of working = $w/10$ per day and that of Rohit = $w/15$ per day. Thus working simultaneously, the amount of work done by them in a single day will be = $(w/10) + (w/15)$ and thus time taken to complete 'W' work will be

$$= \frac{W}{\frac{W}{10} + \frac{W}{15}} = \frac{15 \times 10}{15 + 10} = \frac{150}{25} = 6 \text{ days}$$

LCM APPROACH

Let us understand the above problem with the help of another approach i.e. LCM Approach.

Let total work be 30 units (LCM of 10 and 15)

$$\text{Per day work of Raja} = \frac{30}{10} = 3 \text{ units}$$

$$\text{Per day work of Rohit} = \frac{30}{15} = 2 \text{ units}$$

Raja and Rohit's per day work = $3 + 2 = 5$ units

Total days they will take to complete the work

$$= \frac{30}{5} = 6 \text{ days}$$

Example 2: If A can do a piece of work in 10 days, B can do the same work in 12 days and C can do the same work in 15 days. How long will they take to finish the work, together?

Solution:

Total work = LCM (10, 12, 15) = 60 units

A's one day's work = $60/10 = 6$ units/day

B's one day's work = $60/12 = 5$ units/day

C's one day's work = $60/15 = 4$ units/day

Total time taken when they are working together

$$= \frac{60}{6 + 5 + 4} = 4 \text{ days}$$

Example 3: A and B together can together do a piece of work in 20 days and A alone can do it in 36 days. In how many days B alone can do it?

Solution:

Total work = LCM (20, 36) = 180 units

One day work of A and B together

$$= \frac{180}{20} = 9 \text{ units/day}$$

A's one day work = $180/36 = 5$ units/day

B's one day work = $9 - 5 = 4$ units

Number of days required by B to complete the work alone

Example 4: A and B can do a piece of work in 10 days. B and C can do the same work in 12 days. C and A can do the same work in 16 days. What will be the time in

which A, B and C can finish the work when they are working together? Also, find in how many days A, B and C alone can finish the work?

Solution:

Total work = LCM (10 12, 16) = 240 units

One day work of A and B together = $a + b$

$$= \frac{240}{10} = 24 \text{ units/day}$$

One day work of B and C together = $b + c$

$$= \frac{240}{12} = 20 \text{ units/day}$$

One day work of C and A together = $c + a$

$$= \frac{240}{16} = 15 \text{ units/day}$$

So, we have

$$a + b = 24 \quad \dots (1)$$

$$b + c = 20 \quad \dots (2)$$

$$c + a = 15 \quad \dots (3)$$

Adding the 3 equations we get,

$$2(a + b + c) = 59$$

$$\Rightarrow a + b + c = \frac{59}{2} \text{ units}$$

So, one day work of (A, B and C together) = $\frac{59}{2}$ units

Number of days taken by A, B and C together

$$= \frac{240}{(59/2)} = \frac{480}{59} \text{ days}$$

$$= 8\frac{8}{59} \text{ days}$$

One day work of (A + B + C) = $\frac{59}{2}$ units

One day work of (A + B) = 24 units

$$\therefore \text{One day work of C alone} = \left(\frac{59}{2} - 24 \right) \text{ units}$$

$$= \frac{59 - 48}{2} = \frac{11}{2} \text{ units}$$

$\therefore$ Number of days taken by C alone

$$= \frac{240}{(11/2)} = \frac{480}{11} = 43\frac{7}{11} \text{ days}$$

One day work of (B + C) = 20 units

$$\begin{aligned} \therefore \text{One day work of A alone} &= \left(\frac{59}{2} - 20 \right) \text{ units} \\ &= \frac{59 - 40}{2} = \frac{19}{2} \text{ units} \end{aligned}$$

$\therefore$ Number of days taken by A alone

$$= \frac{240}{(19/2)} = \frac{480}{19} = 25\frac{5}{19} \text{ days}$$

One day work of (A + C) = 15 units

$$\begin{aligned} \therefore \text{One day work of B alone} &= \left(\frac{59}{2} - 15 \right) \text{ units} \\ &= \frac{59 - 30}{2} = \frac{29}{2} \text{ units} \end{aligned}$$

$\therefore$ Number of days taken by B alone

$$= \frac{240}{(29/2)} = \frac{480}{29} = 16\frac{16}{29} \text{ days}$$

Example 5: A can do a piece of work in 45 days. After working for 5 days, A takes leave and the remaining work is completed by B in 9 days. What will be the time taken by B to complete the total work alone?

Solution:

Let total work = LCM (5, 9, 45) = 45 units

One day's work of A = $45/45 = 1$ unit

In first 5 days, A will do work = 5 units of work out of 45 (i.e. $5 \times 1 = 5$ units)

Remaining work = $45 - 5 = 40$ units.

Now, B can do the remaining work in 9 days

$\Rightarrow$ One day's work of B = $40/9$ units

So, time taken by B to complete the total work alone

$$= \frac{45}{40/9} = \frac{45}{40} \times 9 = \frac{81}{8} \text{ i.e. } 10\frac{1}{8} \text{ days}$$

Example 6: A can do a piece of work in 10 days while B can do it in 15 days. They begin together but 5 days before the completion of the work, B leaves. What will be the total number of days taken for the work to be completed?

Solution:

Let total work = LCM (10, 15) = 30 units

A's one day work = $30/10 = 3$ units

B's one day work = $30/15 = 2$ units

Let the total work lasts for X days

So, B has worked for $(X - 5)$ days

So, $3X + 2(X - 5) = 30$

$5X = 40$

$X = 8$

So, the total work lasts for 8 days.

Example 7: A, B, C are working on a job. They take 12, 15 and 18 days respectively to complete the job. If A and B start off with the job and work for 4 days together before being joined by C, when will the job be completed?

Solution:

Let total work be 180 units. (LCM of 12, 15 and 18)

A's per day work = $\frac{180}{12} = 15$ units

B's per day work = $\frac{180}{15} = 12$ units

C's per day work = $\frac{180}{18} = 10$ units

A and B did work together for 4 days.

$(A + B)$'s per day work = $15 + 12 = 27$ units

$(A + B)$'s four days work = $27 \times 4 = 108$ units

Now, remaining 72 units will be done by A, B and C together,

Also, $(A + B + C)$'s per day work = 37 units

Number of days to complete those 72 units

$= \frac{72}{37}$ or $1\frac{35}{37}$

$\therefore$ Required number of total days = $4 + 1\frac{35}{37}$

$= 4 + 1\frac{35}{37}$ days = $5\frac{35}{37}$ days

Example 8: A and B can separately do a piece of work in 20 and 15 days respectively. They worked together for 6 days, after which B was replaced by C. If the work was finished in next 4 days, then what will be the number of days in which C alone could complete the work alone?

Solution:

Let total work be 60 units. (LCM of 20 and 15)

A's per day work = $60/20 = 3$ units

B's per day work does = $60/15 = 4$ units

$(A + B)$'s per day work = 7 units

Total work done in 6 days by them = $7 \times 6 = 42$ units

Remaining work = $60 - 42 = 18$

Now, $(A + C)$ completed the remaining work in 4 days.

$(A + C)$'s per day work = $18/4 = 4.5$ units

C's per day work = $4.5 - 3 = 1.5$ units

So, total number of days required by C to complete the work alone

$= \frac{60 \times 2}{3} = 40$ days

Note:

When two or more than two people work on consecutive days rather than working together on a single day, then the work done on a single day is less as compared to the scenario of working together on a single day.

Example 9: A can do a piece of work in 6 hours and B can do it 8 hours. They both work on alternate hours, with A beginning the work. When will the work get completed?

Solution:

In the given example, let the total work be 24 units.

i.e. (LCM of 6 and 8)

A's per hour work = 4 units

B's per hour work = 3 units

In 2 h (Note that they are working alternatively and not together, they will be able to complete 7 units of work.

7 units will be completed in 2 hours.

21 units will be completed in 6 hours.

A will work in the 7th hour.

Work done by A = 4 units per hour

The remaining work = 3 units

Therefore, A will complete the remaining work in $3/4$ h.

So, total time taken is $6\frac{3}{4}$ hours $\left(\text{i.e. } 6\text{h} + \frac{3}{4}\text{h} \right)$.

Now let us calculate the time taken had B started the work, Will it be the same?

If B starts the work, we have

B's per hour work = 3 units

A's per hour work = 4 units

Total work done in 2 hours = 7 units

Therefore, 21 units will be done in 6 hours.

B will work in the 7th hour.

B is able to do 3 units in 1 h.

and remaining work = 3 units

Therefore, the work will get completed in 7 hours (i.e. 6 + 1 hours).

Example 10: A can do a work in 20 days and B can do the same work in 30 days. They are working on alternate days, with A starting the work. In how much time will the work be completed?

Solution:

Let the total work be 60 units. (LCM of 20 and 30)

A's per day work = 3 units

B's per day work = 2 units

Together they will be able to do 5 units in 2 days.

So, 60 units will be completed in 24 days.

In this example, the answer will not depend on who starts the work.

Negative Work

Negative work means that, one person doing negative work i.e. working against the person doing the positive work.

Let us understand the concept using an example.

Example 11: P and Q are working on a job. P is the builder and Q is the demolisher. P takes 10 days to construct a wall completely. Q takes 20 days to demolish it completely. How much time do they take to build the wall completely for the first time?

a) If they work simultaneously?

b) If they work on alternate days and P starts the job?

Solution:

a) Let total work be 20 units. (LCM of 10 and 20)

P's per day work = 2 units

Q's per day work = -1 unit

(P + Q)'s per day work = 2 + (-1) = 1 unit

Number of days they will take when working together

$$= \frac{20}{1} = 20 \text{ days}$$

b) As they work on alternate days starting with P.

Total work done by them in 2 days = 1 unit

They will complete 18 units i.e. (20 - 2) in 36 days.

Next day, A will come and make 2 units and work will be completed for the first time.

So, total number of days required = 36 + 1
= 37 days

PIPE AND CISTERNS

Concept of Pipes and Cisterns is equivalent to that of Time and Work. Everything we have learnt till now in time and work can be extended to the pipes and cisterns. Here

- Cistern to be filled is the Total Work to be done.
- Inlet Pipe is the Pipe which fills the tank/cistern or does positive work.
- Outlet Pipe is the Pipe which empties the tank/cistern or does negative work.

Example 12: An inlet pipe A can fill the tank in 10 h and inlet pipe B can fill the same tank in 12 h, if these pipes are opened together then in how many hours the tank gets filled?

Solution:

Let the capacity of the tank be LCM of (10 and 12) = 60 units

Inlet pipe A fills 60/10 = 6 units/hour

Inlet pipe B fills 60/12 = 5 units/hour

So, working together they fill = (6 + 5) units/hour
= 11 units/hour

Hence, 60 units of the tank will be filled in 60/11 = 5(5/11) hours.

Example 13: An inlet pipe alone can fill the whole tank in 10 hours. While an outlet pipe can empty the whole tank in 25 hours. When both the pipes are open, in how much time the tank will be filled?

Solution:

Let the volume of the tank be LCM of (10 and 25)
= 50 units

Hence, rate of inlet pipe $50/10 = 5$ units/hour

Rate of outlet pipe $50/25 = -2$ units/hour

(as outlet pipe does negative work)

Now, when they are opened together, these two pipes fill at a rate of $(5 - 2) = 3$ units/hour

So, now 50 units of the tank will be filled in

$$\frac{50}{3} \text{ hours} = 16\frac{2}{3} \text{ hours}$$

Example 14: A cistern is filled by an inlet valve in 5 hours. However because of a leak, it takes 30 minutes more to fill up. In how much time will the leak empty the filled cistern independently?

Solution:

Inlet pipe fills the tank in 5 hours

Inlet and outlet pipe open simultaneously fill the tank in $5 + 0.5 = 5.5$ hours.

Let the capacity of the tank be 5.5 (LCM of 5 and 5.5)

Inlet Pipe fills at $\frac{5.5}{5} = 1.1$ units/hour

Inlet pipe fills + outlet pipe fills

$$= \frac{5.5}{5.5} = 1 \text{ unit/hour}$$

So, rate of outlet pipe = $1 - 1.1$

= -0.1 units/hour

Hence, working independently outlet pipe will empty

the tank in $\frac{5.5}{0.1} = 55$ hours

PROBLEMS BASED ON WAGES

The questions on wages are based on a simple rule that wage earned by a person is proportional to the work done by him/her in one day.

A can do a work W in a days.

$$\therefore \text{A's one day work} = \frac{W}{a}$$

B can do a work W in b days.

$$\therefore \text{B's one day work} = \frac{W}{b}$$

C can do a work W in c days.

$$\therefore \text{C's one day work} = \frac{W}{c}$$

$$\text{So, Wage A : Wage B : Wage C} = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$$

Example 15: A can do a piece of work in 20 days, B can do the same work in 25 days and C can do the same work in 30 days. If they work together and earn a total of ₹ 1850, what are the individual shares of A, B and C?

Solution:

$$\text{Ratio of wages} = \frac{1}{20} : \frac{1}{25} : \frac{1}{30} = \frac{1}{4} : \frac{1}{5} : \frac{1}{6} = 15 : 12 : 10$$

$$\text{Wage of A} = \frac{15}{37} \times 1850 = ₹ 750$$

$$\text{Wage of B} = \frac{12}{37} \times 1850 = ₹ 600$$

$$\text{Wage of C} = \frac{10}{37} \times 1850 = ₹ 500$$

Example 16: A can do a piece of work in 10 days and B can do the same work in 12 days. They finished the work with the help of C in 4 days. If they earned a total of ₹ 4875, then what is the share of C?

Solution:

Let total work = LCM (10, 12, 4) = 60 units

One day work of A = $60/10 = 6$ units

One day work of B = $60/12 = 5$ units

One day work of all three of them = $60/4 = 15$ units

$\Rightarrow$ One day work of C = $15 - (6 + 5) = 4$ units

So, $W_A : W_B : W_C = 6 : 5 : 4$

$$\therefore \text{Share of C} = \frac{4}{15} \times 4875 = ₹ 1300$$

UNITARY METHOD

Till now, we have only seen the cases where individuals are working but now we will see when people with same efficiency are working in a group.

We can say that, efficiency to do a work

$$= \frac{\text{Work done}}{\text{Time taken}}$$

$$E = \frac{W}{T}$$

$$E \times T = W$$

Now, when number of people work together their efficiency gets added hence, if 'n' people work together, their efficiency becomes nE (E is efficiency of single person).

$$\therefore nE \times T = W$$

Now, if people work for some hours, and not the whole day, then the efficiency equation changes as

$$n \times E \times D \times H = W$$

Where D is the number of days and H is hours per day the group works.

- When work done by two different groups is then

$$\frac{W_1}{W_2} = \frac{n_1 E_1 D_1 H_1}{n_2 E_2 D_2 H_2} \text{ is different.}$$

$$\text{Where, } W_1 = n_1 E_1 D_1 H_1$$

$$\text{and } W_2 = n_2 E_2 D_2 H_2$$

- If $W_1 = W_2$

$$\text{then, } n_1 E_1 D_1 H_1 = n_2 E_2 D_2 H_2$$

This equation is known as the basic work equation and is useful in solving group efficiency problems.

Example 17: If 20 men working for 6 hours in a day can complete a work in 35 days, then in how many days can 15 men working for 7 hours in a day complete the same work?

Solution:

Using the above formula,

$$20 \times 6 \times 35 = 15 \times 7 \times Y$$

$$\Rightarrow Y = 40 \text{ days}$$

Example 18: If 15 men working 8 hours a day with an efficiency of n, can do a work in 40 days. In how many days can 25 men working for 6 hours a day with an efficiency of $\frac{n}{2}$ complete a work 1.5 times the earlier work?

Solution:

$$\text{Let } 15 \times 8 \times n \times 40 = W \quad \dots (1)$$

$$\text{and } 25 \times 6 \times \frac{n}{2} \times Z = 1.5W \quad \dots (2)$$

where Z is the number of days taken by 25 men.

Equating (1) and (2), we get

$$25 \times 6 \times \frac{n}{2} \times Z = 1.5 \times 15 \times 8 \times n \times 40$$

$$\Rightarrow Z = 96 \text{ days}$$

Example 19: Twenty men can do a work in 40 days working 6 hours every day, while 30 women can do the same job in 50 days working 4 hours every day. Find the ratio of the efficiency of a man to the efficiency of a woman.

Solution:

As we know,

$$n_1 \times E_1 \times D_1 \times H_1 = n_2 \times E_2 \times D_2 \times H_2$$

$$20 \times E_1 \times 40 \times 6 = 30 \times E_2 \times 50 \times 4$$

$$40 E_1 = 50 E_2$$

$$\therefore \frac{E_1}{E_2} = \frac{5}{4}$$

Example 20: A man works twice as fast as a woman. A woman works twice as fast as a boy. 16 men can complete a job in 12 days, how many days would be required by 32 women and 64 boys together to complete the same job?

Solution:

32 women are equivalent to 16 men.

64 boys are equivalent to 32 women and 32 women is equivalent to 16 men.

So, in the second case, we have 32 men.

Let the number of days taken to complete the job be x.

As per the equation of Unitary Method,

$$M_1 D_1 = M_2 D_2$$

$$16 \times 12 = 32 \times x$$

$$x = 6 \text{ days}$$

Example 21: Three men can complete a piece of work in 6 days. Two days after they start, 3 more men joined them. How many days will they take to complete the remaining work?

Solution:

We just have to equate the total work.

So, according to the equation of Unitary Method,

$$M_1 D_1 = M_2 D_2$$

$$3 \times 6 = 3 \times 2 + (6 \times x) \Rightarrow 18 = 6 + 6x$$

Where x is the number of days required to complete the remaining work.

$$\text{or } 12 = 6x$$

$$\therefore x = 2 \text{ days}$$

Note:

The above concept of Unitary Method can also be used for solving questions based on food available and garrison.

Example 22: A group of 60 men has food for 28 days. After 8 days, a reinforcement arrives, leaving the number of days the food would last to 15 days. What was the strength of the reinforcement?

Solution:

Let total food = $60 \times 28 = 1680$ units

Remaining food (after 8 days)

$$= 60 \times 20 = 1200 \text{ units}$$

Suppose x is the number of men who joined.

Now, this food is consumed by $(x + 60)$ men in 15 days.

$$\Rightarrow (x + 60) \times 15 = 1200$$

$$\Rightarrow x = 20 \text{ men}$$

So, 20 more men joined in.

Example 23: A certain number of persons can do a piece of work in 300 days. If there were 15 more persons, it could be finished in 30 days less. How many persons were there at the start?

Solution:

Let the original number of persons = x

So, by using $M_1 D_1 = M_2 D_2$

We get,

$$x \times 300 = (x + 15) \times 270$$

$$\Rightarrow 10x = 9x + 135$$

$$\Rightarrow x = 135 \text{ persons.}$$

Example 24: If 6 men or 8 women can build a wall in 86 days, how long will 14 men and 10 women take to build the same wall?

Solution:

$$6 \text{ men} = 8 \text{ women}$$

$$\therefore 1 \text{ men} = \frac{8}{6} = \frac{4}{3} \text{ women}$$

$$\therefore 14 \text{ men} = \frac{56}{3} \text{ women}$$

$$14 \text{ men} + 10 \text{ women} = \frac{56}{3} \text{ women} + 10 \text{ women}$$

$$= \frac{86}{3} \text{ women}$$

$$\text{Now, } M_1 = 8, M_2 = \frac{86}{3}, D_1 = 86, D_2 = ?$$

According to the formula,

$$M_1 D_1 = M_2 D_2$$

$$\Rightarrow 8 \times 86 = \frac{86}{3} \times D_2$$

$$\therefore D_2 = \frac{8 \times 86 \times 3}{86} = 24 \text{ days}$$

Example 25: 3 men and 7 women can complete a work in 10 days. Whereas 4 men and 6 women can complete the same work in 8 days. Then, 5 men and 10 women will do the same work in how many days?

Solution:

Let total Work = LCM (10 and 8) = 40 units

Also, let the per day work done by 1 man and 1 woman be a and b respectively.

$$\text{One day work of 3 men and 7 women} = 3a + 7b$$

$$= \frac{40}{10} = 4 \text{ units}$$

$$\text{One day work of 4 men and 6 women} = 4a + 6b$$

$$= \frac{40}{8} = 5 \text{ units}$$

$$\text{So, } 3a + 7b = 4 \text{ and } 4a + 6b = 5$$

On solving the above equations, we get

$$a = \frac{11}{10} \text{ and } b = \frac{1}{10}$$

$$\text{One day work of 5 M and 10 W} = 5 \times \frac{11}{10} + 10 \times \frac{1}{10}$$

$$= \frac{55 + 10}{10} = \frac{65}{10} = \frac{13}{2}$$

$$\text{Required number of days} = \frac{40}{13/2} = \frac{80}{13} = 6 \frac{2}{13} \text{ days}$$

Practice exercise level 1

1. A and B together can dig a trench in 12 days, which A alone can dig in 30 days. In how many days B alone can dig it?
(A) 30 days (B) 20 days
(C) 40 days (D) 10 days
2. A, B and C can finish a job working alone in 72, 24 and 36 days respectively. In how many days they can finish the job if they worked together?
(A) 12 (B) 9
(C) 15 (D) 18
3. A can do a piece of work in 25 days, which B alone can finish in 20 days. Both together work for 5 days and then A leaves. How many days will B take to finish the remaining work?
(A) 21 days (B) 31 days
(C) 11 days (D) 41 days
4. A and B together can finish a work in 30 days. They worked for it for 20 days and then B left the work. The remaining work was done by A alone in 20 days more. In how many days can A alone finish the work?
(A) 48 days (B) 50 days
(C) 54 days (D) 60 days
5. Mahesh and Umesh can complete a work in 10 days and 15 days respectively. Umesh starts the work and after 5 days Mahesh also joins him. In how many days, the total work will be completed?
(A) 7 days (B) 9 days
(C) 11 days (D) None of these
6. Anil is twice as good as workman as Bimal and together they finish a piece of work in 9 days. In how many days will Anil alone finish the work?
(A) 12.5 (B) 10.5
(C) 11.5 (D) 13.5
7. Ram and Shyam together can finish a job in 8 days. Ram can do the same job on his own in 12 days. How long will Shyam take to do the job by himself?
(A) 16 days (B) 20 days
(C) 24 days (D) 30 days
8. Kamal can do a work in 15 days. Bimal is 50% more efficient than Kamal. What is the number of days, Bimal will take to do the same piece of work?
(A) 10 days (B) $10\frac{1}{2}$ days
(C) 12 days (D) 14 days
9. P, Q and R together can complete a work in 36 days. If the ratio of efficiency of P, Q and R is 3 : 2 : 1 respectively, then in how many days R alone can complete the same work?
(A) 108 (B) 216
(C) 72 (D) 144
10. A man, a woman and a boy together can complete a piece of work in 3 days. If a man alone can do it in 6 days and a boy alone in 18 days, how long will a woman take to complete the work?
(A) 9 days (B) 21 days
(C) 24 days (D) 27 days
11. A and B can do a work in 60 days, B and C in 90 days and A and C in 120 days. In how many days can A do the work alone?
(A) 72 days (B) 150 days
(C) 100 days (D) 144 days
12. A can do $\frac{3}{4}$ th of a work in 36 days. Working at half his normal efficiency, in how many days can A finish the work alone?
(A) 48 days (B) 72 days
(C) 96 days (D) None of these
13. A can do a work in 10 days while B can do the work alone in 30 days. If they work on alternate days in how many days can the work get completed?

- (A) 15 and $\frac{1}{3}$ days (B) 15 days
(C) 14 and $\frac{2}{3}$ days (D) Can't say
14. A and B can do a piece of work in 25 days and 30 days respectively. They start the work together but after some days, A leaves the job. B alone does the remaining work in 8 days. How many days after the work started did A leave the job?
(A) 12 days (B) 8 days
(C) 10 days (D) 16 days
15. Ramesh can finish a job in 20 days. He worked for 10 days alone and completed the remaining job working with Dinesh, in 2 days. How many days would both Dinesh and Ramesh together takes to complete the entire job?
(A) 4 (B) 5
(C) 10 (D) 12
16. A can do $\frac{1}{10}$ th piece of work in a day. B can do 25% of the same work in a day and A, B, C together can do 50% of the same work in a day. How much work can be done by C in a day?
(A) 15% (B) 10%
(C) 25% (D) None of these
17. Avinash and Emraan can finish a job working alone in 18 and 24 days, respectively. They started together but Avinash left 3 days before the completion of the work. What is the total time taken to finish the work?
(A) 9 days (B) 12 days
(C) 15 days (D) 10 days
18. A can build a wall in 30 days, which B alone can build in 40 days. If they build it together and get a payment of Rs. 700, what is B's share?
(A) Rs. 300 (B) Rs. 100
(C) Rs. 200 (D) Rs. 500
19. Raman can do a work in 5 days, Jatin can do the same work in 7 days and Sachin can do the same work in 9 days. If they do the same work together and they are paid Rs. 2860, then what is the share of Raman?
(A) Rs. 1260 (B) Rs. 700
(C) Rs. 900 (D) Rs. 870
20. P alone can complete the work in 5 days, Q alone can do same work in 6 days and R alone can do the same work in 12 days. They jointly complete the work and earn Rs. 5400. What is the share of R?
(A) 1000 (B) 1200
(C) 1500 (D) 1800
21. A pump can fill a tank with water in 2 hours. Because of a leak in the tank, it takes $2\frac{1}{3}$ hours to fill the tank. The leak can empty the filled tank in:
(A) 5 hours (B) 7 hours
(C) 8 hours (D) 14 hours
22. A cistern has two pipes. One can fill it with water in 8 hours and the other can empty it in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when $\frac{3}{4}$ th of the cistern is already full of water?
(A) 9 hours (B) 6 hours
(C) 10 hours (D) 8 hours
23. Two inlet pipes can fill a cistern in 10 and 12 hours respectively and an outlet pipe can empty 80 gallons of water per hour. All the three pipes working together can fill the empty cistern in 20 hours. What is the capacity of the tank?
(A) 360 gallons (B) 300 gallons
(C) 600 gallons (D) 900 gallons
24. Two pipes A and B can fill an empty tank in 10 hours and 15 hours respectively. Pipe C alone can empty the completely filled tank in 12 hours. First both pipes A and B are opened and

- after 5 hours pipe C is also opened. What is the total time in which the tank will be filled?
- (A) 7 hours (B) 5 hours
(C) 9 hours (D) 8 hours
25. A pipe fills a tank in 30 minutes but it takes 45 minutes due to a leak. In how much time can the leak alone empty the full tank?
- (A) 60 minutes (B) 45 minutes
(C) 15 minutes (D) 90 minutes
26. A pipe can fill a tank alone in 12 hours but it takes 3 hours more due to a leak. In how much time can the leak alone empty half the tank?
- (A) 30 hours (B) 60 hours
(C) 75 hours (D) 90 hours
27. Two men undertake to do a piece of work for Rs. 200. One alone could do it in 6 days, the other in 8 days. With the assistance of a boy they finish it in 3 days. What is the share of the boy?
- (A) Rs. 25 (B) Rs. 100
(C) Rs. 75 (D) Rs. 50
28. Wages of 10 women for 5 days is Rs. 1250. The daily wage of a man is twice that of a woman. How many men must work for 8 days to earn Rs. 1600?
- (A) 5 men (B) 8 men
(C) 4 men (D) 6 men
29. A pipe can fill a tank in 15 hours. Due to a leak in the bottom, it is filled in 20 hours. If the tank is full, how much time will the leak take to empty it?
- (A) $17\frac{1}{7}$ hours (B) 60 hours
(C) 90 hours (D) 20 hours
30. A pipe can fill a tank in 'x' hours and another can empty it in 'y' hours. What is the time they will take together to fill it? (assume $y > x$)
- (A) $(x - y)$ hours (B) $(y - x)$ hours
(C) $\frac{xy}{(x - y)}$ hours (D) $\frac{xy}{(y - x)}$ hours
31. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning and after 10-hour, C is closed. What is the total time taken to fill the tank?
- (A) 12 hours (B) $13\frac{1}{2}$ hours
(C) 16 hours (D) 18 hours
32. A tap can fill an empty tank in 12 hours and a leakage can empty half the tank in 10 hours. If the tap and the leakage are working simultaneously, how long would it take for the empty tank to fill to half its capacity?
- (A) 45 hours (B) 15 hours
(C) 12 hours (D) 30 hours
33. Tap T1 can fill a water tank in 25 minutes, Tap T2 can fill the same tank in 40 minutes and Tap T3 can empty the tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up?
- (A) $3\frac{2}{13}$ (B) $15\frac{5}{13}$
(C) $8\frac{2}{13}$ (D) $31\frac{11}{19}$
34. Pipe P1 can fill an empty tank in six hours and pipe P2 can empty the full tank in two more hours. If both the pipes are opened simultaneously, then how long would it take to fill an empty tank?
- (A) 12 hours (B) 24 hours
(C) 18 hours (D) 30 hours
35. A water tank is $\frac{2}{5}$ th full. Pipe A can fill this tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
- (A) 150 min. to full (B) 120 min. to empty
(C) 150 min. to empty (D) 120 min. to full
36. If 20 men working 8 hours per day can complete a piece of work in 21 days. How

many hours per day must 48 men work to complete the same job in 7 days?

- (A) 12 (B) 20
(C) 10 (D) 15

37. 12 men or 18 women can reap a field in 14 days. What is the number of days that 8 men and 16 women will take to reap it?

- (A) 5 days (B) 7 days
(C) 8 days (D) 9 days

38. If 3 men or 4 women can construct a wall in 43 days, then what is the number of days that 7 men and 5 women take to construct it?

- (A) 12 days (B) 18 days
(C) 24 days (D) 30 days

39. 3 men or 5 women can finish a piece of work in 12 days. How long will 6 men and 5 women take to finish the work?

- (A) 4 days (B) 10 days
(C) 15 days (D) 20 days

40. 8 men can finish a piece of work in 40 days. If 2 more men join with them, then the work will be completed in:

- (A) 30 days (B) 32 days
(C) 36 days (D) 25 days

41. If 20 men and 36 women can do a job in 36 days, in how many days can 45 men and 81 women do the same job?

- (A) 36 days (B) 16 days
(C) 81 days (D) Can't say

42. Mohan can mow his lawn in x hours. After 2 hours it begins to rain. What is the unmowed part of the lawn?

- (A) $\frac{2}{x}$ (B) $\frac{2-x}{2}$
(C) $\frac{x}{2}$ (D) $\frac{x-2}{x}$

43. If 24 men can do a piece of work in 15 days working 8 hours/day, how many men will be required to do the same work in 10 days working 6 hours/day?

- (A) 32 (B) 48
(C) 60 (D) 30

44. If 24 carpenters can make 12 stools in 8 days working for 6 hours every day, how many stools can 12 carpenters make in 6 days working for 8 hours every day?

- (A) 24 (B) 6
(C) 12 (D) 10

Practice exercise Level 2

1. A is thrice as good a workman as B and is therefore able to finish a piece of work in 60 days less than B. In how much time can they finish it working together?

- (A) 23 days (B) $22\frac{1}{2}$ days
(C) 24 days (D) 25 days

2. A and B together can complete a work in 3 days. They start together. But, after 2 days, B left the work. If the work is completed after 2 more days. B alone could do the work in:

- (A) 5 days (B) 6 days
(C) 9 days (D) 10 days

3. A man and a boy working together can complete a work in 24 days. If for the last 6 days man alone does the work, then it is completed in 26 days. How long will the boy take to complete the work alone?

- (A) 72 days (B) 20 days
(C) 24 days (D) 36 days

4. In a barrack of soldiers, there was stock of food for 190 days for 4000 soldiers. After 30 days, 800 soldiers left the barrack. For how many days shall the left over food last for the remaining soldiers?

- (A) 175 days (B) 200 days
(C) 225 days (D) 250 days

5. A and B can finish a job working alone in 18 and 24 days, respectively. They started

- together but A left 3 days before the completion of the work. What is the total time taken to finish the work?
- (A) 9 days (B) 15 days
(C) 12 days (D) 10 days
6. The efficiency of A is thrice of B and half of C. If B alone completes a work in 45 days, in how many days can A along with C complete the entire work?
- (A) 7.5 days (B) 5 days
(C) 15 days (D) 30 days
7. A takes twice as much time as B and thrice as much time as C to finish a work. If all of them can do the work together in two days, in how many days can B do the work alone?
- (A) 4 days (B) 6 days
(C) 8 days (D) 12 days
8. A can do a certain work in the same time in which B and C can do it together. If A and B can do it in 10 days and C alone in 50 days, then B alone could do it in?
- (A) 15 days (B) 20 days
(C) 25 days (D) 30 days
9. A and B together can do half as much work as C and D together. Also, the efficiency of A is $\frac{1}{3}$ of B. If all 4 of them working together can finish a work in 32 days, in how many days can A alone do the work?
- (A) 320 days (B) 480 days
(C) 384 days (D) 420 days
10. Ram completes 60% of a task in 15 days and then takes the help of Rahim and Rachel. Rahim is 50% as efficient as Ram and Rachel is 50% as efficient as Rahim. In how many more days will they complete the work?
- (A) $13\frac{1}{3}$ (B) $5\frac{5}{7}$
(C) $8\frac{1}{3}$ (D) $7\frac{5}{7}$
11. A can complete a job in 9 days. B in 10 days and C in 15 days. B and C start the work and are forced to leave after 2 days. What is the time taken to complete the remaining work by A alone?
- (A) 13 days (B) 10 days
(C) 9 days (D) 6 days
12. A can do a piece of work in 12 days, B can do the same work in 8 days, and C can do the same job in $\frac{4}{5}$ th time required by both A and B. A and B work together for 3 days, then C completes the job. How many complete days did C work?
- (A) 8 (B) 6
(C) 3 (D) None of these
13. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete a work, how much time shall B take to do it?
- (A) 40 days (B) 35 days
(C) 30 days (D) None of these
14. If factory A turns out x cars an hour and factory B turns out y cars every 2 hours, the number of cars which both factories turn out in 8 hours _____.
- (A) $8(x + y)$ (B) $8x + \frac{y}{2}$
(C) $16(x + y)$ (D) $(2x + y)4$
15. Three pipes A, B and C are connected to a tank. A and B together can fill the tank in 12 hours, B and C together in 8 hours and C and A together in 6 hours. In how much time (in hours) will each pipe fill the tank separately?
- (A) 16, 48, $9\frac{3}{5}$ (B) 16, 24, $9\frac{3}{5}$
(C) 8, 48, $9\frac{3}{5}$ (D) 16, 48, $8\frac{4}{5}$
16. Two pipes can separately fill a tank in 10 hours and 15 hours respectively. Both the pipes are opened to fill the tank but when the tank is $\frac{1}{6}$

full a leak develops in the tank through which $\frac{1}{6}$ of the water supplied by both the pipes leak out. What is the total time (in hours) taken to fill the tank?

- (A) 8 (B) 5
(C) 6 (D) 7

17. Devashish can do a piece of work in 12 days and Rahul can do the same work in 18 days. If Devashish works for three days and then quits, in how many days will the remaining work be completed by Rahul alone?

- (A) $13\frac{1}{2}$ days (B) 9 days
(C) $13\frac{1}{3}$ days (D) 12 days

18. J, K and L can finish a work in 10, 12 and 15 days respectively. They start the work together but, if K stops after 2 days, how long would it take for J and L to finish the remaining work?

- (A) 2 days (B) 3 days
(C) $2\frac{8}{11}$ days (D) $2\frac{3}{11}$ days

19. Brijesh can do a job in 6 hours, Brijesh and Charu can do it in 4 hours and Abhay, Brijesh and Charu in $2\frac{2}{3}$ hours. In how many hours can Abhay and Brijesh do it?

- (A) $3\frac{6}{7}$ hours (B) $2\frac{2}{7}$ hours
(C) $2\frac{3}{7}$ hours (D) $3\frac{3}{7}$ hours

20. Ankush can do a piece of work in 10 days, Prateek in 12 days and Ranveer in 15 days. If they work on alternate days starting with Ankush, then Prateek and then Ranveer till the work gets finished. What is the number of days in which the work gets completed?

- (A) 14 days (B) 12 days
(C) 10 days (D) 20 days

21. Deepak and Lalit together can complete a work in 6 days. They started together but after 3

days, Lalit left the work. If the work is completed after 4 more days, Lalit alone could do the work in how many days?

- (A) 20 days (B) 24 days
(C) 28 days (D) 32 days

22. Gautam and Ishan together can do a piece of work in $48/5$ days, Ishan and Anubhav together can do it in 16 days, and Gautam, Ishan and Anubhav together in 8 days. How long would Gautam and Anubhav together take to do it? In what time would Ishan do it alone?

- (A) 12 days, 24 days (B) 8 days, 16 days
(C) 8 days, 12 days (D) 12 days, 16 days

23. A, B and C can do a piece of work in 40, 8 and 16 days respectively. They all begin together. A work continuously till it finished. C takes leave for 1 day and B leaves the work 2 days before its completion, in what time is the work finished?

- (A) $6\frac{3}{17}$ days (B) 6 days
(C) $5\frac{3}{17}$ days (D) 5 days

24. Aamir can do as much work in 5 days as Binoy can do in 8 days and Binoy can do as much in 4 days as Chaman in 9 days. In what time will Chaman do a piece of work which Aamir can do in 2.5 days?

- (A) 10 days (B) 8 days
(C) 12 days (D) 9 days

25. Anjali, Bharti and Cheena can do a piece of work in 12, 18 and 24 days respectively; they work at it together; Anjali stops the work after 4 days and Bharti called off 2 days before the work is finished. In what time was the work finished?

- (A) 16 days (B) 8 days
(C) 7 days (D) 11 days

26. Jitesh and Hitesh are engaged to do a piece of work in 12 days, but after 8 days they have to

engage Ritesh to help so that the work is just finished in time. The ratio of efficiency of Jitesh, Hitesh and Ritesh is in the ratio of 5 : 4 : 3. How long will they take if they (Jitesh, Hitesh, and Ritesh) start to do the work together?

- (A) 10 days (B) 11 days
(C) 9 days (D) 13 days

27. Mohan can do a job in 10 days, Sohan in 12 days and Ramesh in 15 days. If Mohan is helped by Sohan and Ramesh every 3rd day. How long will it take for them to complete the job?

- (A) 5 days (B) 6 days
(C) 7 days (D) 8 days

Directions (28-30) Study the given information and answer the questions.

Sumesh and Yatin can build a wall each in 10 and 5 days respectively. But on a particularly difficult terrain the work is such that due to fatigue every subsequent day the efficiency of a worker falls by 10%.

28. If Sumesh is given a task of building one such wall in the difficult terrain, then in how many days will he finish the work?

- (A) 19 days (B) 23 days
(C) 29 days (D) Never

29. If Yatin is given the task of building one such wall in the difficult terrain, then in how many days will he finish the work?

- (A) 12 days (B) 9 days
(C) 7 days (D) Never

30. If both Sumesh and Yatin work together to finish two such walls then, in how many days will the work finish?

- (A) 4 days (B) 5 days
(C) 6 days (D) 7 days

31. Tarachand can build a wall in 15 days and Bhimesh can build the same wall in 30 days.

Find how much time will it take to complete the wall if another person Baalveer joins, who can destroy the same wall in 45 days, if they work together all the time?

- (A) $11\frac{6}{7}$ days (B) $\frac{90}{11}$ days
(C) $12\frac{1}{7}$ days (D) $12\frac{6}{7}$ days

32. X can build a wall in 25 days and Y can demolish the same wall in 80 days and Z can build the wall in 60 days. If they work on consecutive days one after another starting from X on the first day. Then in how many days the work will be completed?

- (A) 67 days (B) $67\frac{34}{48}$ days
(C) $66\frac{34}{48}$ days (D) $68\frac{34}{48}$ days

33. Raj alone can do a work in 12 days, Nicole in 16 days and Sarah in 20 days. They work in such a manner that Raj and Nicole work on the first day, Nicole and Sarah work on the second day, Raj and Sarah work on the third day, Raj and Nicole work on the fourth day and so on. In how many days will the work get finished?

- (A) 4 and $\frac{17}{27}$ days (B) 8 and $\frac{1}{8}$ days
(C) 9 days (D) 7 and $\frac{17}{27}$ days

34. K, L and M can do a work in 12, 15 and 30 days, respectively. L and M started the work together but worked at half of their respective efficiency. After 3 days, K joined them and all three of them worked at their normal efficiencies. In how much time will the work get completed?

- (A) 7 and $\frac{7}{11}$ days (B) $\frac{51}{11}$ days
(C) $\frac{84}{11}$ days (D) None of these

35. Manmohan and Kuldeep can finish a piece of work in $20\frac{2}{3}$ days and Neeraj and Kuldeep can finish the same work in 15 days. Manmohan and Kuldeep worked at it for 4 days, then Manmohan left. Then Kuldeep and Neeraj worked for the next 2 days after which Neeraj left. Kuldeep completed the remaining work in $16\frac{2}{3}$ days. What are the individual working capacities of each?
- (A) 8, 16 and 44 days
(B) 12, 26 and 72 days
(C) 10, 20 and 60 days
(D) 10, 24 and 66 days
36. Two pipes A and B can fill a cistern in 6 minutes and 7 minutes respectively. Both the pipes are opened alternatively for 1 minute each starting with the faster pipe. In what time (in minutes) will they fill the cistern?
- (A) 5 (B) $5\frac{2}{3}$
(C) $6\frac{3}{7}$ (D) $1\frac{1}{4}$
37. A, B and C completed a piece of work costing Rs. 1800. A worked for 6 days, B for 4 days and C for 9 days. If their daily wages are in the ratio 5 : 6 : 4, how much amount will be received by A?
- (A) Rs. 800 (B) Rs. 600
(C) Rs. 900 (D) Rs. 750
38. Two pipes A and B can separately fill a tank in 30 and 20 minutes respectively and a waste pipe C can carry off 6 litres/minute. If all the three pipes are opened, then it is emptied in 60 minutes. How many litres does the cistern hold?
- (A) 10 litres (B) 30 litres
(C) 60 litres (D) 45 litres
39. One filling pipe A is 3 times faster than second filling pipe B. If B can fill a cistern in 16 hours, then at what time cistern will be full if both fill pipes are opened together.
- (A) 5 hours (B) 4 hours
(C) 3 hours (D) Data inadequate
40. A cistern has two pipes. One can fill it with water in 8 hours and the other can empty it in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when of the cistern is already half full of water?
- (A) 7 hr (B) 6 hr
(C) 6 hr 40 min. (D) None of these
41. A tap having diameter 'd' can empty a tank in 40 minutes. How long another tap having diameter '2d' take to empty the same tank?
- (A) 5 min. (B) 20 min.
(C) 10 min. (D) 40 min.
42. There are two pipes which fill a tank in 25 and 20 hours, respectively. Initially the tank is empty when both the pipes are opened. There is a mark at $\frac{3}{4}$ th of the top level of the tank. When the water reaches this level, a leak at the bottom of the tank is opened and the tank gets empty in the next 15 hours. In how much time can the leak working alone empty the full tank?
- (A) 15 hours (B) 12.5 hours
(C) 22 hours (D) 7.14 hours
43. Two pipes X and Y can fill a cistern in 12 and 16 minutes respectively. Both filling pipes are opened together, but 4 minutes before the cistern is full, pipe X is closed. How much total time will the cistern take to fill?
- (A) $12\frac{1}{7}$ minutes (B) $9\frac{1}{7}$ minutes
(C) $9\frac{2}{3}$ minutes (D) $9\frac{1}{3}$ minutes
44. Two pipes can fill a cistern in 28 and 32 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom of the cistern, it takes 64

minutes extra for the cistern to be filled up. When the cistern is full, in what time will the leak empty it?

- (A) 224 hours (B) 226 hours
(C) 228 hours (D) 230 hours

45. A cistern is emptied by two pipes and filled by a third. If the first two can empty the cistern in 2 hours and 3 hours respectively each pipe working alone and the third can fill it in 4 hours, how much time will it take to empty the cistern $\frac{4}{5}$ th full when all three are open?

- (A) $1\frac{9}{35}$ hours (B) $1\frac{7}{35}$ hours
(C) $1\frac{3}{35}$ hours (D) $1\frac{13}{35}$ hours

46. There are 12 pipes connected to a tank. Some of them are inlet pipes and the others are outlet pipes. Each of the inlet pipe can fill the tank alone in 8 hours and each of the outlet pipe can drain the tank completely in 12 hours. If all the pipes are kept open, an empty tank gets filled in 24 hours. How many of the 12 pipes are filling pipes?

- (A) 5 (B) 6
(C) 7 (D) 8

47. P1, P2 and P3 are pipes attached to a cistern. P1 and P2 can fill it in 20 and 30 minutes, respectively, while P3 can empty it in 15 minutes. If P1, P2 and P3 be kept open successively for 1 minute each, how soon will the cistern be filled?

- (A) 158 minutes (B) 152 minutes
(C) 167 minutes (D) 169 minutes

48. 9 children can complete a piece of work in 360 days; 18 men can complete the same piece of work in 72 days and 12 women can complete it in 162 days. In how many days can 4 men, 12 women and 10 children together complete the piece of work?

- (A) 68 days (B) 81 days

- (C) 96 days (D) 124 days

49. Four examiners can examine a certain number of answer papers in 10 days by working for 5 hours a day. For how many hours in a day would 2 examiners have to work in order to examine twice the number of answer papers in 20 days?

- (A) $14\frac{1}{2}$ hours (B) 8 hours
(C) $8\frac{1}{2}$ hours (D) 10 hours

50. 10 women can complete a piece of work in 8 days and 10 children take 12 days to complete it. How many days will 6 women and 3 children together take to complete the work?

- (A) 9 days (B) 12 days
(C) 7 days (D) None of these

51. Ten persons can clean 10 floors by 10 mops in 10 days. In how many days can 8 persons clean 8 floors by 8 mops?

- (A) 10 days (B) 12 days
(C) 8 days (D) None of these

52. 72 workmen are employed to finish a certain work in 60 days. But it is found that in 30 days only $\frac{1}{4}$ work is done. How many more men must be taken to finish the work in time?

- (A) 128 (B) 144
(C) 168 (D) None of these

53. A group of men decided to do a job in 16 days. But since 8 men dropped out every day, the job got completed at the end of the 24th day. How many men were there at the beginning?

- (A) 312 (B) 186
(C) 276 (D) None of these

54. A contractor undertakes to build a house in 186 days. He employs 186 workers for the same. However, after 93 days, he finds that the work is only 30% complete. How many more workers need to be employed to complete the work in time?

- (A) 248 (B) 186

- (C) 434 (D) None of these
55. A camp of 4950 men had provision for 48 days at the rate of 1275 gram per head. At end of 12 days, a reinforcement arrives and it was found that the provision will last 25 days more at the rate of 1100 gram per head. What is the strength of the reinforcement?
- (A) 8262 (B) 3312
(C) 6600 (D) 7816
56. 15 men can complete a work in 210 days. They started the work but at the end of 10 days, 15 additional men, with double efficiency, were inducted. How many days in all did they take to finish the work?
- (A) $72\frac{1}{2}$ days (B) $84\frac{3}{4}$ days
(C) $76\frac{2}{3}$ days (D) 70 days
57. 400 students were supposed to go to a camp for 100 days. After 20 days, 40% of the students left the camp. If the camp had food for all of them for 100 days, for how many more days will the food last?
- (A) 133.33 days (B) 58.33 days
(C) 33.33 days (D) 65 days
58. A contractor undertook to finish a job in 45 days and employed 40 men to do the job. After 30 days, 50% of the work had been completed. If he wants to employ additional women where one man is as efficient as three women so that the work is completed on time, how many women needs to be employed?
- (A) 60 (B) 100
(C) 90 (D) 120
59. A man, a woman and a boy can complete a job in 3, 4 and 12 days, respectively. How many boys must assist one man and one woman to complete the work in one fourth of a day?
- (A) 1 (B) 19
(C) 4 (D) 41
60. A 10 floor building is build by 2 men, 3 women and 4 boys in 10 days. If a man, a women and a boy work in the ratio 5 : 4 : 2. What is the time (in days) that 6 men, 4 women and 7 boys take to build a 16 floor building?
- (A) 5 (B) 6
(C) 8 (D) 7
61. Two candles of the same length are lighted at the same time. The first is consumed in 6 hours and the second in 4 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the length of the second?
- (A) 1 hour (B) 2 hours
(C) 3 hours (D) None of these
62. If 12 men or 15 women or 18 boys can do a piece of work in 15 days of 8 hours each; find how many men assisted by 5 women and 6 boys will finish the same work in 16 days of 9 hours each.
- (A) 3 (B) 2
(C) 6 (D) 4
63. The work done by a woman in 16 hours is equal to the work done by a man in 12 hours and by a boy in 24 hours. If working 12 hours per day 18 men can complete a work in 12 days then in how many days can 24 men, 24 women and 24 boys together finish the same work, working 12 hours per day?
- (A) $4\frac{1}{2}$ days (B) $3\frac{2}{3}$ days
(C) 4 days (D) $4\frac{3}{2}$ days
64. A contract is to be completed in 230 days and 585 men were set to work, each working 15 hours a day. After 165 days, $\frac{5}{9}$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 20 hours a day?

- (A) 614 (B) 891
(C) 426 (D) None of these

65. Five men undertook to complete a project in 48 days, working 8 hours a day. One stopped working at the end of 12 days and a second at the end of 15 days. The others then agreed to

work, 9 hours a day. By what % to the nearest unit must they increase their rate of working to finish within the specified time?

- (A) 30% (B) 40%
(C) 48% (D) 51%

Solution

Practice Exercise Level 1

- 1.(B) Let total work be 60 units. (LCM of 12 and 30)
(A + B)'s per day work = 5 units
A's per day work = 2 units
B's per day work = 3 units
Required days = $\frac{60}{5} = 12$ days

- 2.(A) Let the total work be 72 units (LCM of 72, 24 and 36).
A, B and C's one day work is 1, 3 and 2 units respectively.
Required number of days
= $\frac{72}{6} = 12$ days

- 3.(C) Let total work be 100 units. (LCM of 25 and 20)
A's per day work = 4 units
B's per day work = 5 units
Total work done in 5 days by A and B
= $(4 + 5) \times 5 = 45$ units
Required days = $\frac{100 - 45}{5} = 11$ days

- 4.(D) Let the total work be 60 units. [LCM of 30 and 20]
A and B do = $\frac{60}{30} = 2$ units/day
As they worked for 20 days, they did $20 \times 2 = 40$ units of work.
Work left is $60 - 40 = 20$ units

Thus, 20 units is finished by A in 20 days.

Hence, A does = $\frac{20}{20} = 1$ unit/day

So, A alone can finish the work in

$$\frac{60}{1} = 60 \text{ days}$$

- 5.(B) Let total work be 30 units. (LCM of 10 and 15)

Mahesh's per day work = 3 units
Umesh's per day work = 2 units
Umesh's 5 days work = $2 \times 5 = 10$ units

Remaining work = 20 units
(Mahesh + Umesh)'s per day work = 5 units

$$\text{Required days} = 5 + \frac{20}{5} = 9 \text{ days}$$

- 6.(D) Let the efficiencies of Anil and Bimal be 100 and 50 respectively.

Let x be the number of days taken by Anil alone to finish the work.

$$150 \times 9 = 100 \times x$$

$$x = 13.5 \text{ days}$$

- 7.(C) Let total work be 24 units. (LCM of 12 and 8)
(Ram + Shyam)'s per day work = 3 units

Ram's per day work = 2 units

Shyam's per day work = 1 unit

Required days = 24 days

- 8.(A)** As we know, efficiency is inversely proportional to time.
Efficiency ratio of Kamal and Bimal = 2 : 3
Time ratio = 3 : 2
Number of days of Bimal
 $= 15 \times \frac{2}{3} = 10$ days
- 9.(B)** Let the work done by P, Q and R is 3 unit, 2 unit and 1 unit respectively.
Total work = $36 \times (3 + 2 + 1)$
 $= 216$ units
Time taken by R alone
 $= \frac{216}{1} = 216$ days
- 10.(A)** Let total work be 18 units. (LCM of 3, 6 and 18)
Man's per day work = 3 units
Boy's per day work = 1 unit
(Man + Woman + Boy)'s per day work = 6 units
Woman's per day work = 2 units
Required days = $\frac{18}{2} = 9$ days
- 11.(D)** Let the total work be 360 units. (LCM of 60, 90 and 120 per day work)
(A + B)'s per day work = 6 units
(B + C)'s per day work = 4 units
(C + A)'s per day work = 3 units
(A + B + C) $\times 1$ day = 6.5 units
But, (B + C) $\times 1$ day = 4 units
Therefore, A = $6.5 - 4 = 2.5$ units/day
Therefore, A can do the work alone in
 $\frac{360}{2.5} = 360 \times \frac{2}{5} = 72 \times 2$
 $= 144$ days
- 12.(C)** If A can do $\frac{3}{4}$ th of the work in 36 days, he can do the complete work in $36 \times \frac{4}{3}$ days i.e. 48 days.
Working at half his normal efficiency, he can complete the work in $48 \times 2 = 96$ days
- 13.(D)** The answer in this question cannot be determined as it is not mentioned who started the work.
- 14.(C)** Let the total work be 150 units. (LCM of 25 and 30)
A's per day work = 6 units
B's per day work = 5 units
Work done by B in 8 days = $5 \times 8 = 40$ units
Then, $(150 - 40) = 110$ units of work is done by A and B together.
Time taken by A and B = $\frac{110}{5+6}$
 $= 10$ days
 $\therefore$ A leaves the work after 10 days.
- 15.(A)** Ramesh alone finished $\frac{1}{2}$ of the work in 10 days.
Remaining $\frac{1}{2}$ of the job was finished by Ramesh and Dinesh together in 2 days.
Therefore, they both together can finish the complete job in 4 days.
- 16.(A)** Work done by A, B and C in 1 day together = 50%
Work done by A in one day
 $= \frac{1}{10} = 10\%$
Work done by B in one day = 25%
Work done C in one day
 $= (50\% - 10\% - 25\%) = 15\%$
- 17.(B)** Let the total work be 72 units (LCM of 18 and 24).
Total work = 72 units
Avinash's one day work = 4 units/day
Emran's one day work = 3 units/day
Let y be the number of days they work together.
 $y \times 4 + 3 \times 3 = 72$ units
 $7y = 63$ or $y = 9$ days.
Total time taken = 12 days
- 18.(A)** Let total work be 120 units. (LCM of 30 and 40)

A's per day work = 4 units

B's per day work = 3 units

Efficiency ratio = 4 : 3

Share of B = $\frac{3}{7} \times 700 = 300$

- 19.(A)** Let the total work be 315 units (LCM of 5, 7 and 9). Raman's, Jatin's and Sachin's each day work is 63, 45 and 35 units.

Hence, ratio of share

= 63 : 45 : 35

Raman's share = $\frac{63}{143} \times 2860$

= Rs. 1260

- 20.(A)** Let the total work be 60 unit's. (LCM of 5, 6 and 12)

P, Q and R's one day capacities are 12 units, 10 units and 5 units respectively.

Here, R's share:

= $\frac{5}{27} \times 5400 = 1000$

P : Q : R = $\frac{1}{5} : \frac{1}{6} : \frac{1}{12}$

P : Q : R = 12 : 10 : 5

So, share of R

= $\frac{5400}{27} \times 5 = 1000$

- 21.(D)** Let the total capacity be 2 litres.

Pump's per hour work = 1 litre

(Pump + Leak)'s per hour work

= $\frac{2 \times 3}{7} = \frac{6}{7}$

Leak's per hour work = $1 - \frac{6}{7} = \frac{1}{7}$

Required time = $\frac{2}{1/7} = 14$ hours

- 22.(C)** Let the total capacity be 40 litres. (LCM of 8 and 5)

Inlet pipe's per hour work = 5 litres

Outlet pipe's per hour work = 8 litres

(Inlet + Outlet)'s per hour work

= (5 - 8) = -3 litres

Now, tank to be emptied

= $40 \times \frac{3}{4} = 30$ litres

Required time = $\frac{30}{3} = 10$ hours

- 23.(C)** Let the capacity of the tank be 60 gallons. (LCM of 10, 12 and 20)

Capacities of inlet pipes = 6 gallons/hour and 5 gallons/hour

Net capacity of all the pipes

= 3 gallons/hour

∴ Capacity of outlet pipe = 6 + 5 - 3 = 8 gallons/hour

Now, 8 unit is equivalent to 80 units.

Hence, capacity of the tank

= $\frac{80}{8} \times 60 = 600$ gallons

- 24.(A)** Let the total capacity of tank is LCM (10, 15, 12) = 60 units

A's one hour work = 6 units

B's one hour work = 4 units

C's one hour work = 5 units

(A + B)'s 5 hour work = (6 + 4) × 5 = 50 units

Remaining 10 units filled in

$\frac{10}{6+4-5} = 2$ hours

So, total time = 5 + 2 = 7 hours.

- 25.(D)** Let the total work be 90 units. (LCM of 30 and 45)

Pipe's per minute work = 3 units

(Pipe + Leak)'s per minute work = 2 units

Leak's per minute work = 1 unit

Required time = $\frac{90}{1} = 90$ minutes

- 26.(A)** Let the total work be 60 units. (LCM of 12 and 15)

Pipe's per hour work = 5 units

(Pipe + Leak)'s per hour work

= 4 units

Leak will empty 1 unit per hour.

Therefore, the leak will empty half the tank i.e. 30 litres in 30 hours.

- 27.(A)** Let the total work be 24 units. (LCM of 6, 8 and 3)

One day work of man 1 = 4 units

One day work of man 2 = 3 units

One day work of 2 men and boy together = 8 units

One day work of boy = $8 - (4 + 3) = 1$ unit

$$\text{Share of boy} = \frac{1}{1+4+3} \times 200 = ₹ 25$$

- 28.(C)** Wage of 1 woman for 1 day

$$= \frac{1250}{5 \times 10} = ₹ 25$$

Wage of 1 man for 1 day

$$= \text{Rs. } 50 \quad (25 \times 2)$$

Number of men required to earn Rs. 1600 in

$$8 \text{ days} = \frac{1600}{8 \times 50} = 4 \text{ men}$$

- 29.(B)** Let the total capacity be 60 units. (LCM of 15 and 20)

Pipe's per hour work = 4 units

(Pipe + Leak)'s per hour work = 3 units

Leak's per hour work = -1 unit

$$\text{Required hours} = \frac{60}{1} = 60 \text{ hours}$$

- 30.(D)** Let total work be xy . (LCM of x and y)

$$\text{Inlet pipe per hour work} = \frac{xy}{x} = y$$

$$\text{Outlet pipe per hour work} = \frac{xy}{y} = -x$$

$$\text{Required hours} = \frac{xy}{y-x}$$

- 31.(A)** Part filled in 10 hours

$$= 10 \times \left(\frac{1}{15} + \frac{1}{10} - \frac{1}{25} \right) = \frac{23}{30}$$

$$\text{Remaining part} = 1 - \frac{23}{30} = \frac{7}{30}$$

Part filled by A and B in 1 hour

$$= \left(\frac{1}{15} + \frac{1}{20} \right) = \frac{4+3}{60} = \frac{7}{60}$$

$$\therefore \frac{7}{30} \text{ part will be filled by them in}$$

$$\left(\frac{60}{7} \times \frac{7}{30} \right) = 2 \text{ hours}$$

$$\therefore \text{Total time required} = 12 \text{ days}$$

- 32.(B)** Total capacity = LCM (10, 12) = 60 l

$$\text{Tank filled by tap in 1 hour} = \frac{60}{12} = 5 \text{ l/hour}$$

Leakage emptying the tank in 1 hour

$$= \frac{60}{10 \times 2} = 3 \text{ l/hour}$$

So, in 1 hour tank is filled by 2 litres.

To fill the tank to half of its capacity time

$$\text{required} = \frac{30}{2} = 15 \text{ hours}$$

- 33.(D)** Total capacity = LCM (25, 40, 30) = 600

Tap T_1 can fill in 1 minute

$$= \frac{600}{25} = 24 \text{ l}$$

Tap T_2 can fill in 1 minute

$$= \frac{600}{40} = 15 \text{ l}$$

Tap T_3 can empty in 1 minute

$$= \frac{600}{30} = 20 \text{ l}$$

$$\text{Total} = 24 + 15 - 20 = 19 \text{ litres}$$

In 1 min. 19 litres are filled.

$\therefore$ Tank will be filled in

$$= \frac{600}{19} = 31 \frac{11}{19} \text{ minutes}$$

- 34.(B)** Capacity = LCM (6, 8) = 24 l

Pipe P1 filling in 1 hour = 4 l

Pipe P2 emptying it in 1 hour

$$\frac{24}{6}$$

So, in 1 hour tank is filled by 1 litre.

It will take 24 hours to fill an empty tank.

- 35.(C)** Time taken by pipe A to fill $\frac{3}{5}$ th of the tank = 10 minutes

Time taken by pipe A to fill an empty tank

$$= \frac{10 \times 5}{3} = \frac{50}{3} \text{ min.}$$

Time taken by pipe B to empty $2/5^{\text{th}}$ of the tank $= \frac{6 \times 5}{2} = 15 \text{ min.}$

Let the capacity of the tank be 150 litres.

[LCM of $(50/3)$ and 15]

Pipe A per minute work = 9 l/min.

Pipe B per minute work

= -10 l/min.

Time taken by both A and B together to empty the whole tank

= $[150/(10 - 9)] = 150 \text{ minutes}$

36.(C) Using the chain rule,

$$M_1 D_1 T_1 = M_2 D_2 T_2$$

$$20 \times 21 \times 8 = 48 \times 7 \times T$$

$$T = \frac{20}{48} \times \frac{21}{7} \times 8$$

T = 10 hours/day

37.(D) 12 men = 18 women

$$1 \text{ man} = \frac{3}{2} \text{ women}$$

$$8M + 16W = \left(8 \times \frac{3}{2} + 16 \right) W$$

= 28 women

Now, 18 women can reap the field in 14 days.

$\therefore$ 28 women can reap it in

$$\frac{14 \times 18}{28} = 9 \text{ days}$$

38.(A) 3 men = 4 women

$$1 \text{ man} = \frac{4}{3} \text{ women}$$

$$7M + 5W = \left(7 \times \frac{4}{3} + 5 \right) W = \frac{43}{3} W$$

$\therefore \frac{43}{3}$ women can construct it in

$$= \frac{43 \times 4 \times 3}{43} = 12 \text{ days}$$

39.(A) 3 men = 5 women

$$\text{Men} = \frac{5}{3} \text{ Women}$$

So, 6 men = 10 women

$\Rightarrow (6 \text{ men} + 5 \text{ women})$

= 15 women

Let the required number of days be x.

$$5w \times 12 = 15w \times x$$

$$x = 4 \text{ days}$$

40.(B) As we know,

$$M_1 D_1 = M_2 D_2$$

$$\text{So, } 8 \times 40 = (8 + 2) \times x$$

$$x = 32 \text{ days}$$

41.(B) Given that 20 men and 36 women can do the job in 36 days.

We need to find the number of days required for 45 men and 81 women to do the job.

20/45 is equivalent to 4/9.

Similarly, 36/81 also is equivalent to 4/9.

If men increase from 4 to 9 or in proportion to 4 : 9, the days will get reduced in the ratio 9 : 4.

Number of days required

$$= 36 \times \frac{4}{9} = 4 \times 4 = 16 \text{ days}$$

42.(D) Mohan mows the whole lawn in x hours.

$\therefore$ Mohan mows, in 2 hours, $2/x$ of the lawn.

$$\therefore \text{Unmowed part} = 1 - \frac{2}{x}$$

$$= \frac{x-2}{x} \text{ part}$$

43.(B) Let the number of men required to do the work be Y.

Then

$$24 \text{ men} \times 15 \text{ days} \times 8 \text{ h} = Y \text{ men} \times 10 \text{ days} \times 6 \text{ h}$$

Solving, we get Y = 48 men.

44.(B) We need to equate the work done in both the cases,

Therefore

$$24 \times 8 \text{ days} \times 6 \text{ h} = 12 \text{ stools}$$

$$\dots (1)$$

$$12 \times 6 \text{ days} \times 8 \text{ h} = x \text{ stools}$$

$$\dots (2)$$

Dividing equation (1) by equation (2), we get

$$\frac{24 \times 6 \times 8}{12 \times 6 \times 8} = \frac{12}{x} \Rightarrow x = 6 \text{ stools}$$

Practice Exercise Level 2

- 1.(B) Ratio of work done by A and B in the same time.

Ratio of time taken by A and B

$$= 1 : 3$$

Let B takes x days to finish work, A takes $(x - 60)$ days.

$$= \frac{x-60}{x} = \frac{1}{3}$$

On solving, $x = 90$ days

Time taken by B to finish the work = 90 days

Time taken by A to finish the work = 30 days

Let total work be 90 units. (LCM of 30 and 90)

B's per day work = 1 unit

A's per day work = 3 units

(A + B)'s per day work = 4 units

$$\text{Required time} = \frac{90}{4} = 22\frac{1}{2} \text{ days}$$

- 2.(B) Let the total work be 3 units.

(A + B)'s per day work = 1 unit

(A + B)'s 2 days work = 1×2

= 2 units

Remaining work = 1 unit

A's efficiency = $\frac{1}{2}$

$$\text{B's efficiency} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Required days} = \frac{3}{1/2} = 3 \times 2 = 6$$

- 3.(A) Let the total work be 24 units.

They worked together for $(26 - 6) = 20$ days

(Man + Boy)'s per day work

= 1 unit

20 days work = $20 \times 1 = 20$ units

Remaining units = 4 units

$$\text{Man's efficiency} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Boy's efficiency} = 1 - \frac{2}{3} = \frac{1}{3}$$

Required days = $24 \times 3 = 72$ days

- 4.(B) 4000 soldiers can have the available food for 190 days.

So, by Unitary Method

$$400 \times 190 = 4000 \times 30 + (4000 - 800) \times x$$

$$4000 \times 190 - 4000 \times 30 = 3200x$$

$$4000 \times (190 - 30) = 3200x$$

$$4000 \times 160 = 3200x$$

$$x = 200 \text{ days}$$

- 5.(C) Let the total work be 72 units. (LCM of 18 and 24)

A's per day work = 4 units

B's per day work = 3 units

Let B worked for x days till the work got completed and A worked for $(x - 3)$ days.

$$4(x - 3) + 3x = 72$$

$$7x = 84 \Rightarrow x = 12$$

- 6.(B) If B completes the work in 45 days, A will do it in 15 days. Since the efficiency of A is half of C, the time taken by C will be 7.5 days.

Let the total work be 15 units. (LCM of 15 and 7.5)

A's per day work = 1 unit

C's per day work = 2 units

(A + C)'s per day work = 3 units

The work can be done in

$$= \frac{15}{3} = 5 \text{ days}$$

- 7.(B) Ratio of time of A and B

$$= 2 : 1$$

$$\text{Ratio of time A and C} = 3 : 1$$

$$\text{Ratio of time of A, B and C}$$

$$= 6 : 3 : 2$$

$$\text{Ratio of efficiencies of A, B and C} = 1 : 2 : 3$$

$$\text{Then total work} = 6 \times 2 = 12 \text{ units}$$

Time taken by B to finish the work alone =

$$\frac{12}{2} = 6 \text{ days}$$

- 8.(C)** Let the total work be 50 units. (LCM of 10 and 50)

(A + B)'s per day work = 5 units

C's per day work = 1 unit

(A + B + C)'s per day work = 6 units

If A can do the same work as B and C together then A will be able to do 3 units/day.

B and C will also be able to do 3 units/day out of which C can do 1 unit/day. Therefore, B can do 2 units/day.

Thus, B can do the work alone in 25 days $\left(\frac{50}{2}\right)$.

- 9.(C)** Let the total work be 32 units such that all of them together can do 1 unit/day.

Let x be the work done by A and B together and so the work done by C and D together will be 2x.

$$x + 2x = 1 \text{ unit}$$

$$\Rightarrow x = \frac{1}{3} \text{ units}$$

A and B together can do $\frac{1}{3}$ unit/day.

Let y be the work done by A and so the work done by B will be 3y.

$$\text{Then } y + 3y = \frac{1}{3} \text{ units}$$

$$\text{Then } y + 3y = \frac{1}{3} \text{ units}$$

$$y = \frac{1}{12} \text{ units}$$

A is able to do $\frac{1}{12}$ units/day and so A can finish the work alone in $= 12 \times 32 = 384$ days

- 10.(B)** Ram completes 60% of the task in 15 days. Therefore, Ram will complete the remaining 40% of the task in $\frac{15}{60} \times 40 = 10$ days
- Rahim will complete the task alone in 20 days while Rahel will do so in 40 days.

Let total work be 40 units. (LCM of 20 and 40)

Ram's per day work = 4 units

Rahim's per day work = 2 units

Rachel's per day work = 1 unit

Total 7 units in 1 day and so 40 units will get completed in

$$\frac{40}{7} \text{ days} = 5 \text{ and } \frac{5}{7} \text{ days.}$$

- 11.(D)** Let the total work be 90 units. (LCM of 9, 10 and 15)

A's per day work = 10 units

B's per day work = 9 units

C's per day work = 6 units

Work done by B and C in 2 days

$$= (6 + 9) \times 2 = 30 \text{ units}$$

Work remaining = 60 units

Time taken by A to complete the remaining

$$\text{work} = \frac{60}{10} = 6 \text{ days}$$

- 12.(B)** Time taken by C to complete the work alone $= \frac{4}{5} \times (12 + 8) = 16$ days

Let the total work be 48 units. (LCM of 12, 8 and 16)

A's per day work = 4 units

B's per day work = 6 units

C's per day work = 3 units

Work done by A and B = 3 (4 + 6) = 30 units

Time required by C

$$= \frac{(48 - 30)}{3} = \left(\frac{18}{3}\right) = 6 \text{ days}$$

- 13.(C)** Suppose B takes x days to do the work.

$$\therefore \text{A takes } \left(2 \times \frac{3}{4}x\right) \text{ i.e. } \frac{3x}{2} \text{ days to do it.}$$

Now, (A + B)'s 1 day's work $\frac{1}{18}$.

$$\therefore \frac{1}{x} + \frac{2}{3x} = \frac{1}{18} \text{ or } x = 30$$

- 14.(D)** Factory A turns out x cars in one hour. Factory B turns out y/2 cars in one hour.

In one hour both the factories A and B can turn out $\left(x + \frac{y}{2}\right)$ cars.

$\therefore$ In 8 hours both factories turn out $\left(x + \frac{y}{2}\right)$ cars i.e. $4(2x + y)$ cars.

- 15.(A)** Let the capacity of tank be 24 units. (LCM of 12, 8 and 6)

One hour work of Pipes A and B = 2 units

One hour work of Pipes B and C = 3 units

One hour work of Pipes C and A = 4 units

One hour work of pipes (A + B + C)

$$= \frac{2+3+4}{2} = 4.5 \text{ units}$$

Time required by A to fill the tank

$$= \frac{24}{1.5} = 16 \text{ hours}$$

Time required by B to fill the tank

$$= \frac{24}{0.5} = 48 \text{ hours}$$

Time required by C to fill the tank

$$= \frac{24}{2.5} = 9\frac{3}{5} \text{ hours}$$

- 16.(D)** Let the capacity of the tank be 30 units.

Pipe one's work of 1 hour

= 3 units

Pipe two's work of 1 hour = 2 units

Water leaked out in 1 hour

$$= \frac{1}{6} \times 30 \times \frac{1}{6} = \frac{5}{6} \text{ units}$$

Time required to fill the tank

$$= \frac{5}{5} + \frac{25}{25/6} \left(3 + 2 - \frac{5}{6}\right) = 7 \text{ hours}$$

- 17.(A)** Let total work = LCM (18, 12) = 36

$$\text{Devashish's 1 day work} = \frac{36}{12}$$

= 3 units/day

3 day work = $3 \times 3 = 9$ units

$$\text{Rahul's 1 day work} = \frac{36}{18}$$

= 2 units/day

Remaining work = $36 - 9 = 27$

Rahul will complete this work in

$$= \frac{27}{2} = 13\frac{1}{2} \text{ days}$$

- 18.(B)** Let total work = LCM (10, 12, 15) = 60

$$\text{J's 1 day work} = \frac{60}{10} = 6 \text{ units/day}$$

$$\text{K's 1 day work} = \frac{60}{12} = 5 \text{ units/day}$$

$$\text{L's 1 day work} = \frac{60}{15} = 4 \text{ units/day}$$

Total = 15 units

In 2 days, the work completed

= 30 units

J and L finish the remaining work in

$$= \frac{30}{10} = 3 \text{ days}$$

- 19.(D)** Total work = LCM (6, 4)

= 12 units

$$\text{B's 1 hour work} = \frac{12}{6} = 2 \text{ units}$$

$$\text{B's and C's 1 hour work} = \frac{12}{4}$$

= 3 units

C's 1 hour work = $3 - 2 = 1$ unit

(A + B + C) 1 hour work

$$= \frac{12}{8/3} = \frac{12 \times 3}{8} = \frac{9}{2}$$

$$\text{A + B (1 hour)} = \frac{9}{2} - 1 = \frac{7}{2}$$

A and B will do the work in

$$= \frac{12}{7/2}$$

$$= \frac{12 \times 2}{7} = 3\frac{3}{7} \text{ hours}$$

- 20.(B)** Let total job = LCM (10, 12, 15) = 60 units

Per day work of Ankush, Prateek and

Ranveer is $6\left(\frac{60}{10}\right)$, $5\left(\frac{60}{12}\right)$ and $4\left(\frac{60}{15}\right)$ units

respectively.

So, in 3 days they will complete $6 + 5 + 4 = 15$ units of work.

So, the work gets completed in

$$= \frac{60}{15} \times 3 \text{ days}$$

i.e. $4 \times 3 \text{ days} = 12 \text{ days}$

- 21.(B)** Total work = LCM (3, 6, 4)
= 12 units

Together Deepak and Lalit

$$= \frac{12}{6} = 2 \text{ units/day}$$

In 3 days work done = 3×2

= 6 units

Deepak's one day work

$$= \frac{12-6}{4} = 1.5 \text{ units/day}$$

Lalit's one day work = $2 - 1.5$

= 0.5 unit/day

Time taken by Lalit alone

$$= \frac{12-6}{4} = 1.5 \text{ units/day}$$

- 22.(A)** Let total work = 96 units

$$(G+I) = \frac{96}{48/5} = 10 \text{ units/day}$$

$$(I+A) = \frac{96}{16} = 6 \text{ units/day}$$

Gautam + Ishan + Aubhav

= 12 units/day

So, Gautam = 6 units/day

Ishan = 4 units/day

Anubhav = 2 units/day

$$(G+A) = \frac{96}{6+2} = 12 \text{ days}$$

$$\text{Ishan alone} = \frac{96}{4} = 24 \text{ days}$$

- 23.(A)** Let the work be finished in x days.

Then A's x days work + B's $(x-2)$ days work

+ C's $(x-1)$ days work

LCM (40, 8, 16 = 80)

A's one day work = 2 units

B's one day work = 10 units

C's one day work = 5 units

$$\therefore 2x + 10(x-2) + 5(x-1) = 80$$

$$\Rightarrow 17x = 105$$

$$\therefore x = \frac{105}{17} = 6\frac{3}{17} \text{ days}$$

- 24.(D)** Let's compare the work done per day

$$\Rightarrow A = \frac{8}{5}B, B = \frac{9}{4}C$$

$$\Rightarrow A = \frac{8}{5} \times \frac{9}{4}C; A = \frac{18}{5}C$$

$\Rightarrow$ If A does a work in 5 days, C required 18 days.

$\Rightarrow$ Hence, C required 9 days to the work which A does in 2.5 days.

- 25.(B)** Let total work be LCM (12, 18, 24) = 72 units

$$\text{Anjali's one day work} = \frac{72}{12} = 6 \text{ units/day}$$

$$\text{Bharti's one day work} = \frac{72}{18} = 4 \text{ units/day}$$

$$\text{Cheena's one day work} = \frac{72}{24} = 3 \text{ units/day}$$

Let the total work lasts for x days

$$\text{So, } 6 \times 4 + 4 \times (x-2) + 3x = 72$$

$$24 + 4x - 8 + 3x = 72$$

$$7x = 56$$

$$x = \frac{56}{7} = 8 \text{ days}$$

- 26.(A)** Suppose they take X days to do the work together.

Jitesh, Hitesh and Ritesh can do 5K, 4K and 3K of work in one day respectively. The work that can be completed by Jitesh and Hitesh in 8 days and Jitesh, Hitesh and Ritesh in rest of 4 days is

$$8(5K + 4K) + 4(5K + 4K + 3K)$$

$$= 120K$$

Also, the work can be completed by $X(5K + 4K + 3K) = 120K$

$$\Rightarrow 12KX = 120K. \text{ Thus, } X = 10$$

$\Rightarrow$ They take 10 days to do the work together.

- 27.(C)** Total work = LCM (10, 12, 15) = 60

Per day work of Mohan, Sohan and Ramesh is 6, 5 and 4 units respectively.

Day	Number of days
1	6
2	6
3	15
4	6
5	6
6	15
7	6
Total	60

So, the number of days = 7 days

- 28.(D)** Let us assume that building one wall = 40 units work.

So, Sumesh does = 4 units/day, and Yatin does = 8 units/day

Since, efficiency is falling by 10%, so it is a case of geometric progression.

$$4, \frac{18}{5}, \frac{81}{25}, \dots$$

where, common ratio = $\frac{9}{10}$

Now, sum of an infinite G.P.

$$= \frac{a}{1-r}$$

$$= \frac{4}{1-\frac{9}{10}} = \frac{4 \times 10}{10-9} = 40$$

Thus, it is clear that Sumesh will take infinite time, so he will never finish the work.

- 29.(C)** Yatin does = 8 units/day
Let n = Total number of days.

$$\therefore 40 = 8 + \frac{9}{10} \times 8 + \left(\frac{9}{10}\right)^2 \times 8 + \dots +$$

$$\left(\frac{9}{10}\right)^{n-1} \times 8$$

[$\odot$ Efficiency is falling by 10% each days]

$$5 = 1 + \frac{9}{10} + \left(\frac{9}{10}\right)^2 + \dots + \left(\frac{9}{10}\right)^{n-1}$$

$$\therefore 5 = \frac{1 \times \left(1 - \left(\frac{9}{10}\right)^n\right)}{1 - \frac{9}{10}}$$

$$\Rightarrow 5 = \left(1 - \left(\frac{9}{10}\right)^n\right) 10$$

$$\Rightarrow \left(\frac{9}{10}\right)^n = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

$$\text{For } n = 7; \left(\frac{9}{10}\right)^7 = 0.478297$$

$$\text{For } n = 6; \left(\frac{9}{10}\right)^6 = 0.531441$$

So, required number of days = 7 days (approx.)

- 30.(A)** Given total work = 40 units

Also, total unit of work done by both per day = $8 + 4 = 12$ units

Now, since efficiency is falling by 10% each days.

Therefore,

$$40 = 12 + \frac{9}{10} \times 12 + \left(\frac{9}{10}\right)^2 \times 12 + \dots +$$

$$\left(\frac{9}{10}\right)^{n-1} \times 12$$

$$\frac{40}{12} = 1 + \left(\frac{9}{10}\right) + \left(\frac{9}{10}\right)^2 + \dots + \left(\frac{9}{10}\right)^{n-1}$$

$$\Rightarrow \frac{1 - \left(\frac{9}{10}\right)^n}{1 - \frac{9}{10}} = \frac{10}{3}$$

$$\left(\frac{9}{10}\right)^n = 1 - \frac{10}{3} \times \frac{1}{10} = \frac{2}{3} = 0.6667$$

$\therefore n = 4$ days.

- 31.(D)** LCM of (15, 30, 45) = 90

Tarachand's one day work

$$= \frac{90}{15} = 6 \text{ units/day}$$

Bhimsen's one day work = $90/30 = 3$ units/day

Baalveer's one day work

$$= \frac{90}{45} = 2 \text{ units/day}$$

(Tarachand + Bhimsen + Baalveer) = $(6 + 3 + 2) = 11$ units/day

$$\text{Total time} = \frac{90}{7} = 12\frac{6}{7} \text{ days}$$

32.(C) LCM of (25, 80, 60)

= 1200 unit work.

$$X \text{ one day work} = \frac{1200}{25}$$

= 48 units/day

$$Y \text{ one day work} = -\frac{1200}{80}$$

= -15 units/day

$$Z \text{ one day work} = \frac{1200}{60}$$

= 20 units/day

First 3 day work = (48 + 20 - 15) = 53

Now, on 66th day = 53 × 22 = 1166 units be completed.

Work left = 1200 - 1166 = 34 units

On 67th day the work will be completed by X

$$= \frac{34}{48}$$

$$= 66\frac{34}{48} \text{ days}$$

33.(D) Let the total work be 240 units (LCM of 12, 16, 20).

Raj work in one day

= 20 units/day

Nicole work in one day

= 15 units/day

Sarah work in one day

= 12 unit/day

First day: Raj + Nicole = 35 units

Second day: Nicole + Sarah

= 27 units

Third day: Raj + Sarah = 32 units

Work done in 3 days = 94 units

Work done in 6 days = 188 units

Work done on the 7th day

= 35 units

Remaining work = 17 units

$$\text{Additional time required} = \frac{17}{27} \text{ days}$$

$$\text{Total time} = 7 \text{ and } \frac{17}{27} \text{ days}$$

34.(A) Let the total work be 60 units (LCM of 12, 15, 30).

K work in one day = 5 units/day

L work in one day = 4 units/day

M work in one day = 2 unit/day

For 3 days L and M together worked at half of their normal efficiency.

$$\text{Work done} = (4 + 2) \times \frac{3}{2} = 9 \text{ units}$$

Remaining units = 60 - 9 = 51 units

After 3 days, all three of them worked at their normal efficiency.

$$\text{Time taken} = \frac{51}{11} \text{ days}$$

The work will get completed in

$$3 + 4\frac{7}{11} = 7\frac{7}{11} \text{ days}$$

35.(C) Let the total work be 240 units.

a = work by Manmohan in one day

b = work by Kuldeep in one day

c = work by Neeraj in one day

$$a + b = \frac{240}{20/3} = 36 \text{ units}$$

$$b + c = \frac{240}{15} = 16 \text{ units}$$

Manmohan and Kuldeep worked for 4 days

= 4 × 36 = 144 units

Kuldeep and Neeraj worked for 2 days

= 2 × 16 = 32 units

Remaining work = 240 - (144 + 32)

= 64 units

Kuldeep can do 64 units in $\frac{16}{3}$ days, then

Kuldeep's one day work

$$= \frac{64}{16/3} = 12 \text{ units / day}$$

Days by Kuldeep to complete the whole

$$\text{work} = \frac{240}{12} = 20 \text{ units}$$

So, a = 36 - 12 = 24 units

Days by Manmohan to complete the whole

$$\text{work} = \frac{240}{24} = 10 \text{ days}$$

and Neeraj = $16 - 12 = 4$ units

Days by Neeraj to complete the whole work

$$= \frac{240}{4} = 60 \text{ days}$$

- 36.(C)** Let the total capacity be 42 litres. (LCM of 6 and 7)

A's per minute work = 7 litres

B's per minute work = 6 litres

Total tank to be filled in 2 minutes = 13 litres

Total tank to be filled in 6 minutes = 39 litres

Pipe A will work for 7th minute.

Remaining work = 3 litres

So, A will take = $\frac{3}{7}$ hours

$$\text{Total time} = \left(6 + \frac{3}{7}\right) \text{ or } 6\frac{3}{7} \text{ hours}$$

- 37.(B)** Ratio of wages of A, B and C = $(6 \times 5) : (4 \times 6) : (9 \times 4)$
 $= 30 : 24 : 36$
 $= 5 : 4 : 6$

$$\text{A's share} = \left(1800 \times \frac{5}{15}\right) = ₹ 600$$

- 38.(C)** Let the total capacity of the tank be 60 litres. (LCM of 30 and 20)

A's per minute work = 2 litres

B's per minute work = 3 litres

To empty this tank of 60 litres, we need (A + B + C)'s per minute work to be (–1) litres.

So, C's per minute work will be (–6) litres.

C will empty the whole tank alone in $\frac{60}{6}$

= 10 minutes

C carries 6 litres/minute.

So, total capacity of tank = $6 \times 10 = 60$ litres

- 39.(B)** Efficiency ratio of A and B is 3 : 1.
 Time ratio of A and B will be 1 : 3.

Now, A's time = x ; B's time = 3x

Here, $3x = 16$ hours ;

$$x = \frac{16}{3} \text{ hours}$$

Let total capacity be 16 litres.

A's per hour work = 3 litres

B's per hour work = 1 litre

(A + B)'s per hour work = 4 litres

$$\text{Required time} = \frac{16}{4} = 4 \text{ hours}$$

- 40.(C)** Let capacity = 40 units

$$\text{Inlet pipe work/min} = \frac{40}{8}$$

= 5 units/min.

$$\text{Outlet pipe work/min} = \frac{40}{5}$$

= 8 units/min.

Half fill means 20 units is there

Net work/min = $8 - 5$

= 3 units/min.

$$\text{Required time} = \frac{20}{3} = 6\frac{2}{3}$$

or 6 hours 40 minutes.

- 41.(C)** If diameter becomes double so the radius also becomes double.

Now, if the radius becomes double then the area becomes four times

So, the required time

$$= \frac{1}{4} \times 40 = 10 \text{ minutes}$$

- 42.(D)** Let the total work be 100 units.

Total work = 100 units

P1 in one hour fill 4 units/h.

P2 in one hour fill 5 units/h.

When the tank is $\frac{3}{4}$ th full, the total units of water in the tank

$$= \frac{3}{4} \times 100 = 75 \text{ units}$$

In 15 h, 75 units of water is emptied and so the rate of water being emptied is 5 units/h. This is inspite of 9 units getting filled per hour.

Therefore the rate at which the leak empties the tank = $5 + 9$

= 14 units/h.

Therefore, the leak alone can empty 100 units in $\frac{100}{14}$, that is 7.14 h.

- 43.(B)** Total capacity = LCM of (12 and 16) = 48 units

$$X = \frac{48}{12} = 4 \text{ units/min.}$$

$$Y = \frac{48}{16} = 3 \text{ units/min.}$$

Let the total time taken to fill the cistern be x. Then

$$4(x - 4) + 3x = 48$$

$$7x = 64$$

$$x = \frac{64}{7} \Rightarrow x = 9\frac{1}{7} \text{ min.}$$

- 44.(A)** Capacity of tank = LCM (28, 32) = 224 units

Pipe 1 in one hour can fill

$$= \frac{224}{28} = 8 \text{ units}$$

Pipe 2 in one hour can fill

$$= \frac{224}{32} = 7 \text{ units}$$

Time without leak

$$= \frac{224}{8+7} = \frac{224}{15} \text{ hours}$$

Actual time taken

$$= \left(\frac{224}{15} \times 60 \right) + 64 = 960 \text{ minutes}$$

$$\text{So, } \left(\frac{960}{60} \right) \times (8 + 7 - x) = 224 \text{ units}$$

$$15 - x = 14$$

$$x = 1 \text{ unit/hour}$$

So, leak takes out 1 unit/hour and hence it will empty tank in 224 hours.

- 45.(D)** Let capacity = LCM of (2, 3 and 4) = 12 units

Per hour work by first pipe

$$= \frac{12}{2} = 6 \text{ units/hour}$$

Per hour work by second pipe

$$= \frac{12}{3} = 4 \text{ units/hour}$$

Per hour work by third pipe

$$= \frac{12}{4} = 3 \text{ units/hour}$$

In one hour $(-6 - 4 + 3) = -7$ units

Work will be depleted

So, the outlet pipes will take

$$= \frac{12}{7} \times \frac{4}{5} = \frac{48}{35} = 1\frac{13}{35} \text{ hours}$$

- 46.(A)** Let tank capacity = LCM of (8 and 10) = 24

Per hour work of inlet and outlet pipe is

$$3\left(\frac{24}{8}\right) \text{ and } 2\left(\frac{24}{12}\right) \text{ units respectively}$$

Let the number of inlet and outlet pipes be a and b respectively.

$$\text{So, } a + b = 12$$

$$3a - 2b = 1$$

$$b = 7$$

$$\Rightarrow a = 5$$

- 47.(C)** Total capacity = LCM of (20, 30 and 15) = 60 units

P1 in 1 min. can fill = 3 units/min.

P2 in 1 min. can fill = 2 units/min.

P3 in 1 min. can fill = -4 units/min.

When opened alternatively they will do $3 + 2 - 4 = 1$ unit of work in 3 minutes.

So, 55-unit work will be done in 165 minutes.

In 166th, P1 will do 3 units and in 167th, P2 will do 2 units to completely fill the tank.

- 48.(B)** As we know,

$$n_1 \times E_1 \times D_1 = n_2 \times E_2 \times D_2 = n_3 \times E_3 \times D_3$$

$$9 \times E_1 \times 360 = 18 \times E_2 \times 72 = 12 \times E_3 \times 162$$

$$5E_1 = 2E_2 = 3E_3$$

$$E_1 : E_2 : E_3 = 6 : 15 : 10$$

Now, by Unitary Method

$$12 \times 10 \times 162 = [(4 \times 15) + (12 \times 10) + (10 \times 6)] \times x$$

$$x = 81 \text{ days}$$

- 49.(D)** By using Unitary Method,

$$\frac{4 \times 10 \times 5}{W_1} = \frac{2 \times 20 \times x}{2W_1}$$

$$\text{So, } 4 \times 10 \times 5 \times 2 = 2 \times 20 \times x$$

$$x = 10 \text{ hours}$$

50.(D) $10W \Rightarrow 8 \text{ days}$

$$\text{So, } 6W \Rightarrow \frac{8 \times 10}{6} = \frac{40}{3} \text{ days}$$

$$10C \Rightarrow 12 \text{ days}$$

$$\text{So, } 3C \Rightarrow \frac{12 \times 10}{3} = 40 \text{ days}$$

Let total work = 40 units

$$\left[\text{LCM of } \left(\frac{40}{3}, 40 \right) \right]$$

$$1 \text{ day work of } 6W = \frac{40}{40/3} = 3 \text{ units}$$

$$1 \text{ day work of } 3C = \frac{40}{40} = 1$$

Together they can finish the work in $\frac{40}{4}$ i.e.

10 days.

51.(A) To clean 10 floors, we need to have 10×10
= 100 man-days.

So, to clean 8 floors, we need to have 80 man-days.

$$\text{So, } 80 = D \times 8.$$

$$\text{So, } D = 10 \text{ days}$$

$$\text{Using } \frac{W_1}{M_1 \times D_1} = \frac{W_2}{M_2 \times D_2}$$

$$\frac{10}{10 \times 10} = \frac{8}{8 \times D_2}$$

$$\Rightarrow D_2 = 10 \text{ days}$$

52.(B) Total work = W

Men required after 30 days = M

$$\text{So, by using } \frac{W_1}{M_1 \times D_1} = \frac{W_2}{M_2 \times D_2}$$

$$\frac{\frac{1}{4}W}{72 \times 30} = \frac{\frac{3}{4}W}{M \times 30}$$

$$M = 216$$

$$\text{More men required} = 216 - 72 \\ = 144$$

53.(C) Let x be the total number of men at the start.

$$16x = x + (x - 8) + (x - 16) + \dots + (x - 184)$$

$$\Rightarrow 16x = 24x - 8(1 + 2 + 3 + \dots + 23)$$

$$\Rightarrow 8x = 8 \times \frac{23 \times 24}{2}$$

$$\Rightarrow x = 276 \text{ men}$$

54.(A) By using

$$\frac{W_1}{M_1 \times D_1} = \frac{W_2}{M_2 \times D_2}$$

$$\frac{3W}{186 \times 93} = \frac{7W}{M \times 93}$$

$$\Rightarrow 3 \times M \times 93 = 186 \times 7 \times 93$$

$$\Rightarrow M = 434$$

$$\text{More number of men} = 434 - 186 = 248$$

55.(B) Let the strength of reinforcement be R.

$$\text{Then, } 4950 \times 1275 \times 36 = R \times 1100 \times 25$$

$$R = 8262$$

$$\text{Reinforcement} = 8262 - 4950$$

$$= 3312$$

56.(C) Total work = 15×210

$$15 \times 210 = (15 \times 10) + (15 + 30)d$$

(New 15 men have double the efficiency)

$$\therefore d = \frac{200}{9} \text{ days}$$

$$= 66\frac{2}{3} \text{ days}$$

$\therefore$ Total days taken will be

$$10 + 66\frac{2}{3} = 76\frac{2}{3} \text{ days}$$

57.(A) From the problem:

$$\text{Using } M_1 D_1 = M_2 D_2$$

$$400 \text{ students} \times 80 \text{ days}$$

$$= 240 \text{ students} \times D_2 \text{ days}$$

$$D_2 = \frac{400}{3} = 133.33 \text{ days}$$

58.(D) After 30 days, 50% of the work was completed.

Therefore, at the same efficiency, the work will take another 30 days to get completed. This has to be completed in the remaining

15 days. So, $40 \text{ men} \times 30 \text{ days} = X \text{ men} \times 15 \text{ days} \Rightarrow X = 80$

So, total of 80 men are required, that is, the contractor will need 40 more men.

Now, 1 man = 3 women, so, the contractor will need 120 more women.

59.(D) Let the total work be 12 units.

Total work = 12 units

Man work in one day

= 4 units/day

Woman work in one day

= 3 units/day

Boy work in one day = 1 units/day

Therefore, 1 man is equivalent to 4 boys and 1 woman to 3 boys.

Let x boys be able to finish the work in $1/4$ of a day.

Therefore,

$1 \text{ boy} \times 12 \text{ days} = x \text{ boys} \times 1/4 \text{ day} \Rightarrow x = 48 \text{ boys}$

1 man and 1 woman will be together equivalent to 7 boys.

Additional number of boys required
= $48 - 7 = 41 \text{ boys}$

60.(C) We have 4 men = 5 women

$1 \text{ man} = \frac{5}{4} \text{ women}$

$2 \text{ women} = 4 \text{ boys}$

$1 \text{ woman} = 2 \text{ boys}$

$\frac{5}{4} \text{ women} = 2 \times \frac{5}{4} \text{ boys} = \frac{5}{2} \text{ boys}$

or $1 \text{ man} = \frac{5}{4} \text{ women} = \frac{5}{2} \text{ boys}$

Now, $2M + 3W + 4B = 2 \times \frac{5}{2} B + 3 \times 2B + 4B$

= 15 Boys

Or 15 Boys builds in 10 days (10 floors)

$6M + 4W + 7B = (6 \times \frac{5}{2} + 4 \times 2 + 7)B$

= 30 Boys

30 Boys will build 16 floors in

$$10 \times \frac{15}{30} \times \frac{16}{10} = 8 \text{ days}$$

61.(C) Let length of Candle

= LCM of (6 and 4) = 12 units

Burning/hour for first Candle

$$= \frac{12}{6} = 2 \text{ units/hour}$$

Burning/hour for second Candle

$$= \frac{12}{4} = 3 \text{ units/hour}$$

Let after x hours the length of first is twice the length of second.

$$\therefore 12 - 2x = 2(12 - 3x)$$

$$\Rightarrow 12 - 2x = 24 - 6x$$

$$\Rightarrow 4x = 12$$

$$\therefore x = 3 \text{ hours}$$

62.(B) Given that 12 men = 18 boys, 15 women = 18 boys

Hence, 1 Man = 1.5 Boys,

$$1 \text{ woman} = \frac{6}{5} \text{ boys}$$

Now, $5W + 6B = 12B$

Required answer is calculated as follows:

Total number of Boys required

$$18 \times \left[\left(\frac{15}{16} \right) \times \left(\frac{8}{9} \right) \right] = 15$$

Boys already present = 12

Hence, 3 boys more required. But 3 boys = 2 Men

So, total number of men = 2

63.(C) $16w = 12m = 24b$

$$\Rightarrow 4w = 3m = 6b$$

Total work by men = $12 \times 18 \times 12m = 2592m$ units

$$\text{So, } (24m \times 12) + (24w \times 12) + (24b \times 12)$$

$$= 288m + 216m + 144m = 648m$$

$$\text{Days required} = \frac{2592m}{648m} = 4 \text{ days}$$

64.(D) By using

$$\frac{W_1}{M_1 \times D_1 \times H_1} = \frac{W_2}{M_2 \times D_2 \times H_2}$$

$$\frac{\frac{5}{9}W}{165 \times 585 \times 15} = \frac{\frac{4}{9}W}{M \times 65 \times 20}$$

$$5 \times M \times 65 \times 20 = 4 \times 165 \times 585 \times 15$$

$$M = \frac{4 \times 165 \times 585 \times 15}{20 \times 5 \times 65} = 891$$

$$\text{More men required} = 891 - 585 = 306 \text{ men}$$

65.(D) Total work = $5 \times 48 \times 8$
= 1920 units

$$\text{Work in first 12 days} = 5 \times 12 \times 8 = 480 \text{ units}$$

$$\text{Work in next 3 days} = 4 \times 3 \times 8$$

$$= 96 \text{ units}$$

$$\text{Remaining work} = 1920 - 480 - 96$$

$$= 1344 \text{ units}$$

The remaining work is to be done by 3 people in 33 days.

$$\text{Hours/day} = \frac{1344}{3 \times 33} = 13.57$$

They agreed for 9 hours but hence to work 13.57 hours/day. Hence, percentage increase

$$= \left(\frac{13.57 - 9}{9} \right) \times 100 = 51\%$$

Time, Speed & Distance

09

Speed is nothing but the rate at which distance is covered per unit time and thus the basic relation between Time, Speed and Distance is

$$\text{Speed} = \frac{\text{Distance Covered}}{\text{Time Taken}}$$

Also, Distance = Speed $\times$ Time

$$\text{and Time} = \frac{\text{Distance}}{\text{Speed}}$$

The unit of speed in most of the cases will be either metre/second or kilometer/hour

$$1 \text{ km/hr} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{5}{18} \text{ m/s}$$

So, the method for conversion from km/hr to m/sec and vice versa is as given below:

$$\text{km/hr} \xrightarrow[18/5]{5/18} \text{m/s}$$

We should also remember these basic conversions to save time.

Every 18 km/hr corresponds to 5 m/s.

- 18 km/hr = 5 m/s
- 36 km/hr = 10 m/s
- 90 km/hr = 25 m/s

Proportionality concept in Time, Speed and Distance

1. If Time is same, then Speed is directly proportional to the distance covered. i.e. ($S \propto D$) and hence

$$\frac{S_1}{S_2} = \frac{D_1}{D_2}$$

Thus, if speed doubles, then the distance covered will also doubles.

2. If Speed is same, then distance is directly proportional to time. i.e. $D \propto T$ and hence

$$\frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Thus, if the distance at the same speed doubles, then the time taken to cover it will also double.

3. If the distance travelled is same, then Speed is inversely proportional to time. i.e. $S \propto \frac{1}{T}$ and hence.

$$\frac{S_1}{S_2} = \frac{T_2}{T_1}$$

Thus if Speed doubles, then the time taken will be halved to cover the same distance.

Example 1: Walking at $\frac{5}{6}$ th of the usual speed, a boy is late by 10 minutes for his school. What is the usual time to reach the school?

Solution:

Here, the distance is constant whenever distance is same, then

$$\frac{S_1}{S_2} = \frac{T_2}{T_1}$$

Let the initial speed be s and initial time be t minutes. then, the new speed be $(\frac{5}{6})s$ and the new time be $(t + 10)$ minutes.

$$\frac{s}{(\frac{5}{6})s} = \frac{t+10}{t}$$

$$\frac{6}{5} = \frac{t+10}{t}$$

$$t = 50 \text{ minutes}$$

So, the usual time is 50 minutes.

Alternate Method:

As speed became $\frac{5}{6}$ of usual speed i.e. s .

Then time will become $\frac{6}{5}$ of the usual time i.e. t .

So, new time will be $(6/5)t$.

$$\frac{6}{5}t = t + \frac{1}{5}t$$

Extra time taken by the boy $(1/5)t$ i.e. equal to 10 minutes.

$$\text{So, } t = 10 \times 5$$

$$t = 50 \text{ minutes}$$

Example 2: If I travel at 30 km/hr, then I reach my office in time. But if I travel at 40 km/hr, I reach my office 5 minutes early. What is the usual time I take to reach office and what is the distance from my home to the office?

Solution:

Here, the distance is constant, so

$$\frac{S_1}{S_2} = \frac{T_2}{T_1}$$

Initial speed = 30 km/hr

New speed = 40 km/hr

Initial time = t minutes

New time = $(t - 5)$ minutes

$$\text{So, } \frac{30}{40} = \frac{t-5}{t}$$

$$30t = 40t - 200$$

$$t = 20 \text{ minutes}$$

So, usual/initial time = 20 minutes

Now, Distance = Speed $\times$ Time

Initial speed = 30 km/hr

Initial time = 20 minutes

$$\text{Required distance} = 30 \times \frac{20}{60} = 10 \text{ km}$$

Alternate Method:

Let the distance from home to the office be x .

$$\text{Then, } \frac{x}{30} - \frac{x}{40} = \frac{5}{60} \Rightarrow \frac{4x - 3x}{120} = \frac{5}{60} \Rightarrow x = 10 \text{ km}$$

$$\text{Time taken} = \frac{\text{Total Distance}}{\text{Usual Speed}} = \frac{10}{30} = \frac{1}{3} \text{ hr} = 20 \text{ minutes}$$

Example 3: Mr. Raj Kishore leaves his house at a particular time to go to his office. He reaches office 10 minutes late when he travels at the speed of 20 km/hr and when he travels at 30 km/hr, he reaches

his office 5 minutes early. At what speed should he travel to reach office on time?

Solution:

Here, the distance is constant, then

$$\frac{S_1}{S_2} = \frac{T_2}{T_1}$$

Let the usual time be x minutes.

Speed₁ = 20 km/hr

Time₁ = $x + 10$

Speed₂ = 30 km/hr

Time₂ = $x - 5$

$$\text{So, } \frac{20}{30} = \frac{x-5}{x+10}$$

$$20x + 200 = 30x - 150$$

$$x = 35 \text{ minutes}$$

Usual time = 35 minutes

Total time taken at Speed₁ = $(35 + 10) = 45$ minutes

$$\text{Total distance} = 20 \times \frac{45}{60} = 15 \text{ km}$$

Now, to reach office on time, usual speed

$$= \frac{\text{Distance}}{\text{Usual time}} = \frac{15}{35/60} = 25.71 \text{ km/hr}$$

Example 4: Kamal leaves his house daily at a particular time to go to his office. He reaches the office 10 minutes late when he travels at 40 kmph and 5 minutes early when he travels at 60 kmph. At what speed should he travel to reach the office on time?

Solution:

Here, the distance travelled in both the cases is same.

$$\text{So, } \frac{S_1}{S_2} = \frac{T_2}{T_1}$$

Let ' t ' be the time taken when he reaches on time.

$$\text{Therefore, } \frac{40}{60} = \frac{t-5}{t+10} \Rightarrow t = 35 \text{ minutes}$$

Thus he should take 35 minutes to reach office.

So, when he travels at 40 km/h, he reaches in 45 minutes. (1)

and when he travels at 60 km/h, he reaches in 30 minutes. (2)

i.e., we can find distance from any of the two cases.

Distance = Speed $\times$ Time

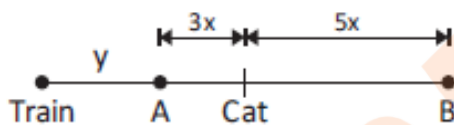
$$\text{Distance} = 60 \text{ kmph} \times \left(\frac{30}{60} \right) \text{ hours} = 30 \text{ km}$$

So, the speed he should travel to reach office on time

$$S = \frac{D}{T} \Rightarrow S = \frac{30}{35/60} = \frac{1800}{35} = \frac{360}{7} \text{ km/hr}$$

Example 5: A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is $\frac{3}{8}$ of the distance AB measured from the entrance A. When the train whistles the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat exactly at the exit. The speed of the train is greater than the speed of the cat by what order?

Solution:



Let the speed of train and cat be S_T and S_C respectively. Time taken by both the train and the cat to reach point A is same.

$$\text{So, } \frac{S_T}{S_C} = \frac{y}{3x} \quad \dots (1)$$

Also the time taken by both the train and the cat to reach the point B is same.

$$\text{So, } \frac{S_T}{S_C} = \frac{y + 8x}{5x} \quad \dots (2)$$

So, we can equate $\frac{S_T}{S_C}$ from the two cases.

$$\frac{y}{3x} = \frac{y + 8x}{5x}$$

$$y = 12x$$

Putting the value of $y = 12x$ in (1), we get

$$\frac{S_T}{S_C} = \frac{12x}{3x} = \frac{4}{1}$$

CONCEPT OF AVERAGE SPEED

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Basically there are three different cases where we can be called in to find the average speed.

$$1. \text{ Average Speed} = \frac{d_1 + d_2 + d_3 + \dots + d_n}{t_1 + t_2 + t_3 + \dots + t_n}$$

Where $d_1, d_2, d_3, \dots, d_n$ are different distances travelled for time $t_1, t_2, t_3, \dots, t_n$ respectively.

$$2. \text{ Average Speed} = \frac{S_1 t_1 + S_2 t_2 + S_3 t_3 + \dots + S_n t_n}{t_1 + t_2 + t_3 + \dots + t_n}$$

Where $S_1, S_2, S_3, \dots, S_n$ are the speeds travelled for time $t_1, t_2, t_3, \dots, t_n$ respectively.

$$3. \text{ Average Speed} = \frac{d_1 + d_2 + d_3 + \dots + d_n}{\frac{d_1}{S_1} + \frac{d_2}{S_2} + \dots + \frac{d_n}{S_n}}$$

Where $S_1, S_2, S_3, \dots, S_n$ are speeds at which one travels the distance $d_1, d_2, d_3, \dots, d_n$ respectively.

When two equal distances are covered

If a man goes from A to B to C such that distance AB is same as distance BC, then also the same method will be employed. In general, if in two parts of the journey, where the distances are same, then the average speed can also be find out by the harmonic mean of the two speed values. e.g., if speeds in two parts of the journey are u and v , then average speed

$$= \frac{2x}{\frac{x}{u} + \frac{x}{v}} = \frac{2uv}{u + v}$$

. Therefore, the only requirement is that distances in two parts of the journey should be same. Note that what is the actual distance is immaterial. So, basically if distances travelled for two parts are same, then average speed is the harmonic mean of the speeds of the individual parts.

Example 6: If a man walks from A to B at a speed of 10 km/hr and from B to A at the speed of 15 km/hr, then what is the average speed of the man?

Solution:

Let the distance from A to B = x km. Then, total distance = $2x$ km

Time taken from A to B = $x/10$ hrs $\frac{x}{10}$ hrs and time for

return journey = $\frac{x}{15}$. Therefore, total time =

$$\frac{x}{10} + \frac{x}{15} = \frac{x}{6}$$

$$\therefore \text{Average speed} = \frac{2x}{x/6} = 12 \text{ km/hr}$$

Alternate Method:

$$\text{Average speed} = \frac{2 \times 10 \times 15}{10 + 15} = 12 \text{ km/hr}$$

When two distances covered in same time

If a man moves from A to B to C such that time from A to B is same as that from B to C, say t , then distances travelled in the two parts of the journey are

$$ut \text{ and } vt. \text{ Therefore, average speed} = \frac{ut + vt}{2t} = \frac{u + v}{2}$$

(which is the average of the speeds).

If time travelled is same for each of the journey, then average speed is the arithmetic mean of the speeds in the two parts.

Example 7: If a man travels at 40 km/hr for the first 2 hours of his journey, then 30 km/hr for the next 3 hours and then the last 5 hours at 50 km/hr. What is the average speed of the man for the entire journey?

Solution:

By applying the second case of Average Speed,

$$\text{Average Speed} = \frac{40 \times 2 + 30 \times 3 + 50 \times 5}{2 + 3 + 5}$$

$$= \frac{80 + 90 + 250}{10} = \frac{420}{10} = 42 \text{ km/hr.}$$

Example 8: If a man travels $1/2$ of the total distance at the speed of 40 km/hr, $1/3$ of the total distance at the speed of 50 km/hr. and rest of the distance at 60 km/hr, then what is the average speed of the entire journey?

Solution:

Let the total distance is $6D$, then the average speed

$$= \frac{6D}{\frac{3D}{40} + \frac{2D}{50} + \frac{D}{60}} = \frac{6D \times 600}{45D + 24D + 10D}$$

$$= \frac{3600}{79} = 45.57 \text{ km/hr}$$

Alternate Method:

Let the total distance be 600 km [LCM of speeds].

$$\text{Now, Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{600}{\frac{600 \times (1/2)}{40} + \frac{600 \times (1/3)}{50} + \frac{600 \times (1/6)}{60}}$$

$$= \frac{600}{\frac{15}{2} + 4 + \frac{5}{3}}$$

$$= \frac{600 \times 6}{45 + 24 + 10} = \frac{3600}{79} = 45.57 \text{ km/hr}$$

CONCEPT OF RELATIVE SPEED

The word 'relative' means one with respect to another.

Relative Speed means the speed of an object A with respect to another object B, which may be stationary, moving in the same direction as A or in the opposite direction as A.

Case I: When one object is stationary and the other is moving.

Consider a boy standing on a platform and a train passing by. Here, the boy is stationary, while the train is moving. The relative speed of the train and the boy will be the speed of the train. As only train in covering the distance and the distance covered by boy with respect to or relative to train is 0.

Hence Relative speed of a stationary object with respect to a moving object = Speed of the moving object.

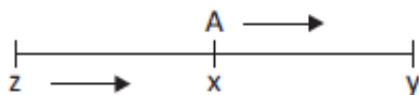
Case II: When the two objects are moving in the opposite direction.

Consider two boys, A and B, standing at two opposite ends of a ground. Now, if they start walking towards each other in a straight line, they would meet sooner than had one of them been stationary and their relative speed will be the sum of their speeds.

Relative speed of two objects moving in opposite direction with respect to each other = Sum of their speeds.

Case III: When the two objects are moving in the same direction.

Consider a boy 'A' walking from x to y. Now, if another boy 'B' walks from a point z which is behind x, in the same direction as A at a speed greater than A's, they would meet later than they would have, had A been stationary at x. Their relative speed is the difference of their speeds.



Relative speed of two objects moving in the same direction with respect to each other = Difference of their speeds.

Example 9: Two persons A and B separated by a distance of 160 km. They started simultaneously walking towards each other at the speed of 20 km/hr and 30 km/hr. After how much time will they meet?

Solution:

Here both A and B are moving in the opposite direction. So, Relative speed = $20 + 30 = 50$ km/hr

$$\text{Hence, Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{160}{50} = 3\frac{1}{5} \text{ hr}$$

= 3 hr and 12 min.

Example 10: A thief steals a car from a parking area at 1 PM and drives it away at 50 km/hr. The theft is discovered at 3 p.m. on the same day and the owner sets off in another car at 80 km/hr towards the thief to catch him up. At what time, will he catch up with the thief?

Solution:

For the time from 1 p.m. to 3 p.m. only the thief is running. So, the thief will travel a distance of 100 km. Now, at 3 p.m. the owner starts chasing the thief in the same direction, so the relative speed is 30 km/hr (i.e. $80 - 50$). So, the owner has to cover the distance of 100 km at the relative speed of 30 km/hr. So, the

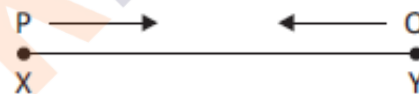
$$\text{time taken} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{100}{30} = 3\frac{1}{3} \text{ hr}$$

or 3 hr 20 min.

The owner will catch up the thief in 3 hours 20 minutes after 3 p.m. i.e. at 6:20 p.m. on the same day.

Example 11: P and Q start from X and Y respectively at 9 a.m. and travel towards Y and X respectively at uniform speeds. If P reaches Y at 1 p.m. and Q reaches X at 3 p.m. on the same day, then at what time do they meet?

Solution:



Let the distance XY be D. Thus, P takes 4 hours (i.e. 9 a.m. to 1 p.m.) and Q takes 6 hours (i.e. 9 a.m. to 3

p.m.) to travel this distance. Thus, their speed is $\frac{D}{4}$

and $\frac{D}{6}$ respectively. So, again seeing the initial position they are separated by distance D and they

move towards each other at speed of $\frac{D}{4}$ and $\frac{D}{6}$.

$$= \frac{\frac{D}{4} + \frac{D}{6}}{\frac{D}{4} + \frac{D}{6}} = \frac{6 \times 4}{6 + 4}$$

Time required for meeting $\frac{6 \times 4}{6 + 4} = 2.4$ hours i.e. 144 minutes. So, they will meet at 11:24 a.m.

Example 12: A thief escapes jail and runs at the speed of 40 km/hr. After 3 hours, the police realizes the escape and run towards the thief at the speed of 60 km/hr with its dog which runs at the speed of 100 km/hr. The dog runs up to the thief and when it reaches the thief, it runs back and forth till the police

catch the thief. What is the total distance travelled by the dog? Also find the distance travelled by the dog in the forward direction?

Solution:

The dog runs till the police will catch the thief.

$$t = \frac{40 \times 3}{60 - 40} = 6 \text{ hr}$$

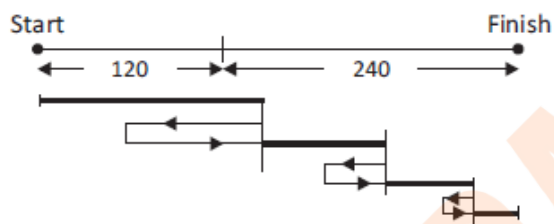
The police will catch the thief in

In these 6 hours, the dog would have run $100 \times 6 = 600 \text{ km}$

In 6 hours the policeman will cover $6 \times 60 = 360 \text{ km}$

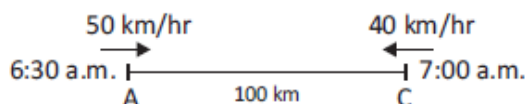
Now, the dog will definitely cover 360 km in the forward direction (shaded in the diagram given) and the remaining distance of 240 km will be equally divided among forward and backward direction. So, the total distance travelled by the dog in forward

direction will be $360 + \left(\frac{240}{2}\right) = 480 \text{ km}$

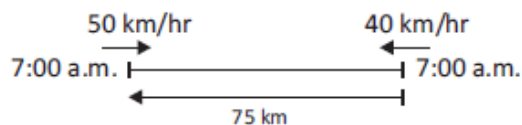


Example 13: Navjivan Express from Ahmedabad to Chennai leaves Ahmedabad at 6:30 a.m. and travels at 50 kmph towards Baroda situated 100 km away. At 7:00 a.m. Howrah-Ahmedabad Express leaves Baroda towards Ahmedabad and travels at 40 kmph. At 7:30 a.m., Mr. Shah, the traffic controller at Baroda realizes that both the trains are running on the same track. How much time does he have to avert a head-on collision between the two trains?

Solution:



From 6:30 to 7:00 i.e. for half hour, the train from Ahmedabad will move for half hour and will cover a distance of $50 \times 1/2$ i.e. 25 km. Now, past 7:00 a.m. both the trains will move.



Now, both the trains are moving in the opposite direction. So, they will collide after 50 minutes

$\left(\text{i.e. } \frac{75}{90} \times 60\right)$. So, they will collide 50 minutes after

7:00 a.m. i.e. at 7:50 a.m. Mr. Shah came to know at 7:30 that both the trains are on the same track, so he has 20 minutes to avoid the collision between the two trains.

Example 14: A military convoy 10 km long had to drive 24 km i.e., each member of the convoy travels 24 km only. One of the military policemen on a motorcycle started at the back of the convoy as it moved off, rode to the front of convoy, returned to the back of the convoy just as the convoy ground to a halt. Assume that the convoy's speed and motorcyclist's speed were constant, how far did the motorcyclists ride?

Solution:

Let the speed of the convoy be C and speed of policeman be M.

$$\therefore \frac{24}{C} = \frac{10}{M-C} + \frac{10}{M+C}$$

Understand this equation carefully, $\frac{24}{C}$ is the available time with policeman to go ahead of the convoy and comeback i.e., while going ahead of the convoy he takes $10/M+C$ time and while coming back he takes $10/M+C$.

On solving the above the equation we have $6M^2 - 5MC - 6C^2 = 0$ or $(2M - 3C)(3M + 2C) = 0$,

$$\therefore \frac{M}{C} = \frac{3}{2} \text{ or } \frac{M}{C} = -\frac{2}{3}$$

A negative relation between the speed is not possible.

$$\therefore \frac{M}{C} = \frac{3}{2}$$

The convoy covers 24 kms then the policeman will

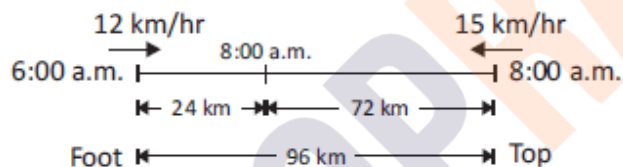
$$\text{cover } \frac{3}{2} \times 24 = 36 \text{ kms}$$

Example 15: A man starts to move from the foot of the hill to reach the top of the hill. He starts the journey at 6:00 a.m. and reaches the top at 2:00 p.m. the same day, with a constant speed of 12 km/hr. The next day, he starts at 8:00 a.m. from the top of the hill and reaches the foot of the hill the same day, moving at a constant speed of 15 km/hr. Is there a place along the path at which the man was there at the same time on these two days? Why/why not?

Solution:

The situation is equivalent to the case if two persons had started the same day, one at 6 : 00 a.m. from the foot of the hill and other at 8 : 00 a.m. from the top of the hill, with their respective speeds. Surely, they will meet somewhere along the path and that would be the place which will satisfy the criteria asked in the question.

Distance = $12 \times 8 = 96$ km as the person starting from the foot who is travelling at the speed of 12 kmph covers the distance in 8 hours.



Now, Time = $\frac{72}{12+15} = \frac{72}{27} = 2\frac{2}{3} = 2 \text{ h } 40 \text{ min.}$ So, the time at which they will meet is 10 : 40 a.m.

Time taken after meeting

Consider that A and B start from Points P and Q simultaneously towards each other. They meet on the way at some point of time. They continue to travel towards their respective destinations, Q and P (either with a stop at the point of meeting or without a stop). If, from then onwards, A takes a time of t_A and B takes a time of t_B to reach Q and P respectively

and their speeds of travel are S_A and S_B respectively, then,

$$\frac{S_A}{S_B} = \sqrt{\frac{t_B}{t_A}}$$

Example 16: A man starts from B to K, another from K to B at the same time. After passing each other they complete their journeys in 25 and 36 hours, respectively. What is the speed of the second man if the speed of the first is 12 km/hr?

Solution:

$$\frac{1^{\text{st}} \text{ man's speed}}{2^{\text{nd}} \text{ man's speed}} = \sqrt{\frac{t_2}{t_1}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$$

$$\therefore \frac{12}{2^{\text{nd}} \text{ man's speed}} = \frac{6}{5}$$

So, 2nd man's speed = 10 km/hr

TRAINS

- Time taken by a train 'x' metre long to pass a stationary object of negligible length with respect to train when the train is moving at a speed of 'a' m/s is given by t (in seconds) = $\frac{x}{a}$
- Time taken by a train 'x' metre long to cross a platform/tunnel of length 'y' metre, when the train is moving at a speed of 'a' m/s is given by t (in seconds) = $\frac{x+y}{a}$
- If two trains of length 'x' and 'y' metre move in the same direction at the speed of 'a' and 'b' m/s respectively, then the time taken to cross each other from the time they meet

$$= \frac{\text{Sum of their length}}{\text{Relative speed}} \text{ i.e., } \frac{x+y}{a-b} \text{ if } a > b \text{ or else, } \frac{x+y}{b-a}$$
- If two trains of length 'x' and 'y' metre, move in the opposite direction at the speed of 'a' and 'b' m/s respectively, then the time taken to cross each other from the time they meet

$$= \frac{\text{Sum of their length}}{\text{Relative speed}} = \frac{x + y}{a + b}$$

Note:

Please check for the units in questions for speed and time and convert accordingly.

Example 17: Consider a train running at the speed of 72 km/hr. Near the track, a person is standing. The length of train is 200 metres.

- In how much time the train will cross this man?
- If it was another train of length 100 metres (not currently moving), what would be the time taken?
- If the train in (b) was moving in the same direction as the first train at the speed of 54 km/hr, then what would be the time taken? If the trains are moving in the opposite direction, then what is the time taken?

Solution:

First of all, convert all the km/hr to m/s, because the lengths of the trains are in metre.

$$72 \text{ km/hr} = 20 \text{ m/s and } 54 \text{ km/hr} = 15 \text{ m/s}$$

$$\text{a) Time taken} = \frac{\text{Length of the Train}}{\text{Speed}} = \frac{200}{20} = 10 \text{ sec.}$$

$$\begin{aligned} \text{b) Time taken} &= \frac{\text{Sum of lengths of the two trains}}{\text{Speed of the moving train}} \\ &= \frac{200 + 100}{20} = 15 \text{ sec.} \end{aligned}$$

c) For Same Direction

$$\begin{aligned} \text{Time taken} &= \frac{\text{Sum of lengths of the two trains}}{\text{Relative Speed}} \\ &= \frac{200 + 100}{20 - 15} = \frac{300}{5} = 60 \text{ sec.} \end{aligned}$$

For opposite direction,

$$\begin{aligned} \text{Time taken} &= \frac{\text{Sum of lengths of the two trains}}{\text{Relative Speed}} \\ &= \frac{200 + 100}{20 + 15} = \frac{300}{35} = 8.57 \text{ sec.} \end{aligned}$$

Example 18: Train XYZ running at the faster speed of 48 km/hr crosses a man sitting in another running train in 10 seconds. If the length of both the trains is same i.e. 200 metre and they are travelling in

opposite directions, then what will be the speed of the other train?

Solution:

Like in the first case of trains, whenever a train crosses a stationary object then it crosses a distance equal to its own length.

So, here train will focus on that stationary man and it will cover the distance equal to its own length.

$$\text{Time} = \frac{\text{Length of train crossing the passenger}}{\text{Speed of faster train} + \text{Speed of slower train}}$$

$$10 = \frac{200}{(48 + x) \times \frac{5}{18}}$$

$$10 = \frac{200 \times 18}{(48 + x) \times 5}$$

$$x = 24 \text{ km/hr}$$

CONCEPT OF BOATS AND STREAMS

The water in a stream, usually, keeps flowing at a certain speed, in a particular direction. This speed is called the speed of the current or the speed of the stream.

A boat develops speed because of its engine power. The speed at which it travels when there is no current is called Speed of boat in still water.

A boat can travel in the direction of the current as well as against the current (as long as speed of boat in still water is greater than the current).

When the boat moves in the direction of the current it is said to be along the stream/current or downstream.

Let speed of stream be x m/s and speed of boat in still water be y m/s.

Let speed of downstream be u m/s, then

$$u = x + y$$

When the boat moves in a direction opposite to that of the current, it is said to be against the stream/current or upstream.

Let the speed of upstream be v m/s, then

$$v = y - x$$

If the downstream and upstream speeds of a boat are given, then

$$\frac{1}{2}(u + v) = \text{Speed of boat in still water, and}$$

$$\frac{1}{2}(u - v) = \text{Speed of stream (or current).}$$

A swimmer, swimming downstream or upstream, is a case similar to that of a boat.

A sail boat, sailing against the wind or in the direction of wind, is also a case of similar nature. Hence, the above principles can also be applied to these cases.

Example 19: A man rows 27 kms with the stream and 15 kms against the stream taking 4 hours each time. What is his rate per hour in still water and the rate at which the stream flows?

Solution:

Speed with the stream (downstream)

$$= \frac{27}{4} = 6\frac{3}{4} \text{ km/hr}$$

and Speed against the stream (upstream)

$$= \frac{15}{4} = 3\frac{3}{4} \text{ km/hr}$$

Speed of the man in still water

$$= \frac{1}{2} \left(6\frac{3}{4} + 3\frac{3}{4} \right) = 5\frac{1}{4} \text{ km/hr}$$

$$\text{Speed of the stream} = \frac{1}{2} \left(6\frac{3}{4} - 3\frac{3}{4} \right) = 1.5 \text{ km/hr}$$

Example 20: A person can row at $7\frac{1}{2}$ km/hr in still water. It takes him twice as long to row up a distance as to row down the same distance. What is the speed of the stream?

Solution:

Since the time taken are in the ratio 2 : 1, the speeds will be in the ratio 1 : 2. Let the upstream and downstream speed are x and $2x$ respectively.

$$\text{So, } \frac{x+2x}{2} = 7.5, x = 5 \text{ km/hr}$$

Downstream speed = 10 km/hr and

Upstream speed = 5 km/hr

$$\text{Speed of stream} = \frac{1}{2}(10 - 5) = 2.5 \text{ km/hr}$$

Example 21: If a boat travels (in a running stream) a certain distance in 5 hours and journeys back in 8 hours, then what is the time, the boat will take if the stream was not running?

Solution:

Let x be the distance. Then, downstream speed = $\frac{x}{5}$

and upstream speed = $\frac{x}{8}$. Note that there is no need

to mention that 5 hours refers to downstream. As while travelling downstream time required is less as compared to travelling upstream. The boat in upstream travels the same distance as downstream.

If ' t ' be the time taken by the boat in still water, then

$$\frac{x}{t} = \left(\frac{1}{2} \right) \left[\frac{x}{5} + \frac{x}{8} \right] = \frac{13x}{80}$$

Therefore, $t = \frac{80}{13}$ hours, which is actually the harmonic mean of 5 and 8 hours.

Example 22: It takes 4 hours less to row down a 12 mile stream than it takes to row up. For this 24 mile round-trip, if I double my rowing speed, I would take half an hour less to row downstream than to row upstream. What is the speed of the stream in miles/hr?

Solution:

Let the speed of boat in still water is y miles/hr and the speed of the stream is x miles/hr.

$$\frac{12}{y-x} - \frac{12}{y+x} = 4$$

$$\text{i.e., } \frac{24x}{y^2 - x^2} = 4 \quad \dots (1)$$

$$\text{And } \frac{12}{2y-x} - \frac{12}{2y+x} = \frac{1}{2}$$

$$\text{i.e., } \frac{24x}{4y^2 - x^2} = \frac{1}{2} \quad \dots (2)$$

Dividing (1) and (2) we get,

$$\frac{4y^2 - x^2}{y^2 - x^2} = 8,$$

$$\therefore 4y^2 - x^2 = 8y^2 - 8x^2$$

$$\therefore 4y^2 = 7x^2$$

Putting this in (2) we get,

$$\frac{24x}{7x^2 - x^2} = \frac{1}{2},$$

$$\therefore \frac{24x}{6x^2} = \frac{1}{2}$$

$$\therefore x = 8 \text{ miles/hr}$$

CIRCULAR MOTION

In circular motion, persons move in a circle instead of a straight line. When two or more persons are running around a circular track (starting at the same time and from the same point), then following types of questions are asked:

1. When will they meet for the first time anywhere on the circular track?
2. When will they meet for the first time at the starting point?

If A and B having started at the same time and at the same point are running around a circular track, of length 'x' metre with speeds 'a' m/s and 'b' m/s respectively, then they will meet for the first time after an interval (from the start) of

(i) $\frac{x}{a+b}$ if they are running in opposite direction.

(ii) $\frac{x}{|a-b|}$ if they are running in the same direction.

Time to meet at the starting point for the first time will be the LCM of the individual times to complete

one round; i.e. LCM of $\frac{x}{a}$ and $\frac{x}{b}$.

This value is independent of the directions in which they are running.

Example 23: Two persons A and B are running on a circular track of length of 1000 m with a speed of 10 m/s and 15 m/s respectively. If they start simultaneously from point P in the same direction, then after how much time will they meet for the first time?

Solution:

Time for first meeting

$$= \frac{1000}{15-10} = 200 \text{ seconds}$$

Example 24: In the above question, if they are running in the opposite direction then after how much time will they meet for the first time?

Solution:

Time for first meeting

$$= \frac{1000}{15+10} = 40 \text{ seconds}$$

Example 25: A can cover a circular track in 20 minutes and B can cover the same circular track in 36 minutes. After how much time will both A and B will meet at the starting if they start from the same starting point simultaneously? Also find the time after which they will meet anywhere on the track for the first time if they are moving in the same direction?

Solution:

For the meeting at the starting point we have to find LCM of the time taken by each A and B to cover the circular track.

$$\text{Required time} = \text{LCM}(20, 36) = 180 \text{ minutes}$$

So, after 180 minutes both A and B will meet for the first time at the starting point.

Now, to find the time of meeting anywhere on the circular track. Let the circumference of the track be D. Then, time after which they will meet for the first time

$$\begin{aligned} &= \frac{\text{Circumference}}{\text{Relative Speed}} \\ &= \frac{D}{\frac{D}{20} - \frac{D}{36}} = \frac{D}{\frac{D(9-5)}{180}} = \frac{180}{4} = 45 \text{ minutes} \end{aligned}$$

So, they will meet first time after 45 minutes anywhere on the circular track.

Example 26: Three persons A, B and C are running on a circular track of length of 1000 m with a speed of 10 m/s, 15 m/s and 20 m/s respectively. If they start simultaneously from point P in the same direction,

after how much time will they meet for the first time?

Solution:

Time for first meeting of A and B

$$= \frac{1000}{15 - 10} = 200 \text{ seconds}$$

Time for first meeting of A and C

$$= \frac{1000}{20 - 10} = 100 \text{ seconds}$$

LCM (200, 100) = 200 seconds

So, all of them will meet after 200 seconds.

CLOCKS

Clocks follow the principle of relative speed and circular motion. Take the instance of a typical clock. There are two hands in the clock: the hour hand and the minute hand. Both are moving in the same direction. The clock is divided into 360 divisions (degrees). The minute hand travels the entire 360° in

one hour. So, the speed of minute hand is $\frac{360}{60} = 6$ degrees/minute. And the hour hand travels 360° in 12

hours. So, the speed of hour hand = $\frac{360}{12 \times 60} = \frac{1}{2}$ degrees/min.

So, the hour hand moves $(1/2)^\circ$ per minute, whereas the minute hand moves 6° per minute. The minute hand is constantly chasing the hour hand. The relative speed of the minute hand with respect to the hour

hand is $5\frac{1}{2}^\circ$ per minute.

As we know the relative speed of hour hand and minute hand, we can calculate the frequency with which both of them coincide continuously.

Difference of $5\frac{1}{2}^\circ$ comes in 1 min. So the difference

of 360° will come in $\frac{360}{5\frac{1}{2}} \text{ min.} = \frac{720}{11} \text{ min.} = 65\frac{5}{11}$

min. By using this value we can find that in a day,

hour and minute hands coincide 22 times as given below:

$$\begin{aligned} \therefore & \frac{\text{Total minutes in a day}}{\text{Time (in minutes for one coincidence)}} \\ &= \frac{24 \times 60}{720/11} = 22 \text{ times} \end{aligned}$$

The types of questions that you would encounter would be as follows:

- At a particular time what is the angle between the hands of a clock?
- When do the hands of a clock coincide or make some angle between t and $(t + 1)$ hr?
- Problems where two different clocks gain or lose certain time in a particular time period.

Example 27: At 3:45, what is the angle between the hands of a clock?

Solution:

At 3 O'clock, the minute hand of a clock would be 90° behind the hour hand. In 45 minutes, the minute hand of a clock would move $(45^\circ \times 6) = 270^\circ$ forward.

$$\left(\frac{1}{2} \times 45^\circ \right) = \left(22\frac{1}{2}^\circ \right)$$

The hour hand would move forward.

Hence, the angle between the hands would be

$$270^\circ - \left(90^\circ + 22\frac{1}{2}^\circ \right) = 157\frac{1}{2}^\circ$$

Example 28: How many minutes after 3 o'clock do the hands of the clock meet?

Solution:

At 3 o'clock, the hands are separated by a gap of 90° .

We can safely assume that the hour hand would not move and only the minute hand would move at the relative speed of $5.5^\circ/\text{min}$. Thus, the minute hand and the hour hand would travel a distance of 90° with

their relative speed in $\frac{90}{5.5} = 16\frac{4}{11}$ minutes. These many minutes after 3 o'clock will the two hands meet.

Example 29: At what time between 3:00 p.m. and 4:00 p.m., are the hands of the clock opposite to each other?

Solution:

The minute hand would need to travel (at the relative speed) a distance of 270° in order to be opposite to

the hour hand. Therefore, $\frac{270}{5.5} = 49\frac{1}{11}$ minutes after 3 O'clock will the hands be opposite to each other.

Example 30: A boy goes out to play sometime between 5:00 p.m. and 6:00 p.m. and returns from play at sometime between 6:00 p.m. and 7:00 p.m. When he comes back, he observes that the hands of the clock have exchanged their positions compared to when he moved out. For how long did he play?

Solution:

Let us say that he played for t minutes. In this time t , the minute hand moved to take the old position of hour hand and hour hand moved to take the old position of minute hand. The sum total of the distances travelled by both the hands together will be

360° . Therefore, $6t + 0.5t = 360$ or $t = \frac{360}{6.5} = 55.38$ minutes

Example 31: A clock loses 5 minutes everyday and another gains 10 minutes everyday. They both are set right at 3 : 00 p.m. on a particular day. In how many days will they show the same time?

Solution:

Everyday, the time gap between the two clocks becomes 15 minutes. When the gap between them becomes 24 hours then the two watches will show the same time.

[An argument runs in this type of questions that the gap should be of 12 hours. But note that the time mentioned is 3:00 p.m. and not 3 o'clock. When the time mentioned is 3:00 p.m., that means that the difference between 3:00 p.m. and 3:00 a.m. is important, whereas when 3 o'clock is mentioned, that difference is immaterial and hence a gap of 12 hours is taken.]

To create a gap of 15 minutes, it takes 1 day. To create a gap of 24 hours, it will take

$$\frac{1 \times 24 \times 60}{15} = 96 \text{ days}$$

Practice exercise Level 1

- The speeds of A and B are in the ratio 3 : 4. A takes 20 minutes more than B to reach the destination starting from the same point. In what time does A reach the destination?
 (A) $1\frac{1}{3}$ hours (B) 2 hours
 (C) $1\frac{2}{3}$ hours (D) $2\frac{2}{3}$ hours
- A plane left 30 minutes later than the scheduled time and in order to reach the destination 1500 km away on time, it had to increase its speed by 250 km/hr from the usual speed. What is its usual speed?
 (A) 720 km/hr (B) 730 km/hr
 (C) 740 km/hr (D) 750 km/hr
- A car traveling with $\frac{5}{7}$ th of its usual speed covers 42 km in 1 hr. 40 min. 48 sec. What is the usual speed of the car?
 (A) $17\frac{6}{7}$ km/hr (B) 25 km/hr
 (C) 30 km/hr (D) 35 km/hr
- Walking at $\frac{3}{4}$ th of his usual speed, a man covers a certain distance in 2 hours more than the time he takes to cover the distance at his

usual speed. What is the time taken by him to cover the distance at usual speed?

- (A) $4\frac{1}{2}$ hours (B) $5\frac{1}{2}$ hours
(C) 5 hours (D) 6 hours

5. A certain distance is covered at a certain speed. If half of this distance is covered in double the time, then what is the ratio of the two speeds?

- (A) 4 : 1 (B) 1 : 4
(C) 2 : 1 (D) 1 : 2

6. The speeds of three cars are in the ratio 2 : 3 : 4. What is the ratio between the times taken by these cars to travel the same distance?

- (A) 4 : 3 : 2 (B) 2 : 3 : 4

- (C) 4 : 3 : 6 (D) 6 : 4 : 3

7. An Auto travels 10 km/hr faster than a Scooty for a journey of 1000 km. The Scooty takes 5 hours more than the Auto. What is the speed of the Scooty?

- (A) 40 km/hr (B) 70 km/hr
(C) 50 km/hr (D) None of these

8. A train travels a distance of 300 km at a constant speed. If the speed of the train is increased by 5 km per hour, the journey would have taken 2 hours less. What is the original speed of the train?

- (A) 25 km/hr (B) 20 km/hr
(C) 28 km/hr (D) 30 km/hr

9. A train is scheduled to cover the distance between two stations 46 km apart in one hour. If it travels 25 km at a speed of 40 km/hr, then what should be the speed for the remaining journey to be completed in the scheduled time?

- (A) 66 km/hr (B) 56 km/hr
(C) 46 km/hr (D) 36 km/hr

10. A car takes 15 minutes less to cover a distance of 75 km, if it increases its speed by 10 km/hr

from its usual speed. How much time would it take to cover a distance of 300 km at this speed?

- (A) 5 h (B) $5\frac{1}{2}$ h
(C) 6 h (D) $6\frac{1}{2}$ h

11. If Sapna walks at 4 km/hr, she misses the bus by 10 minutes. If she walks at 5 km/hr, she reaches 5 minutes before the arrival of the bus. How far does she walk to reach the bus stand?

- (A) 5.5 km (B) 4 km
(C) 5 km (D) 4.5 km

12. The time taken by a train in traversing a certain distance between two places when it moves at 25 kmph is 36 minutes more than that when it moves at 30 kmph. Find distance between the two places.

- (A) 100 km (B) 90 km
(C) 120 km (D) 140 km

13. A pilot flies an aircraft at a certain speed for a distance of 800 km. He could have saved 40 minutes by increasing the average speed of the plane by 40 km/hr. What is the average speed of the aircraft?

- (A) 200 km/hr (B) 300 km/hr
(C) 240 km/hr (D) None of these

14. A man completes 30 km of a journey at 6 km/hr and the remaining 40 km of journey in 5 hours. His average speed for the whole journey is:

- (A) $6\frac{4}{11}$ km/hr (B) 7 km/hr
(C) $7\frac{1}{2}$ km/hr (D) 8 km/hr

15. A person travels from X to Y at a speed of 40 km/hr and returns by increasing his speed by 50%. What is his average speed for both the trips?

- (A) 36 km/hr (B) 45 km/hr
(C) 48 km/hr (D) 50 km/hr

16. During a long race, a formula one car travels 30 minutes at a speed of 90 km/hr, another 40 minutes at a speed of 120 km/hr and finally 2 hours at a speed of 140 km/hr. What is the average speed of the car during the whole race? (approx.)
 (A) 106 km/hr (B) 134 km/hr
 (C) 128 km/hr (D) 144 km/hr
17. Harman travels 500 km at a speed of 45 km/hr and he increases his speed to 60 km/hr to travel another 300 km. What is his average speed during the whole journey?
 (A) $51\frac{11}{31}$ km/hr (B) $49\frac{19}{29}$ km/hr
 (C) $53\frac{19}{31}$ km/hr (D) $52\frac{2}{3}$ km/hr
18. A car travels a distance of 45 km at the speed of 15 km/hr. It covers the next 50 km of its journey at the speed of 25 km/hr and the last 25 km of its journey at the speed of $25/3$ km/hr. Find the average speed of the car.
 (A) 40 km/hr (B) 24 km/hr
 (C) 15 km/hr (D) 18 km/hr
19. A car covers four successive 3 km stretches at 10 km/hr, 20 km/hr, 30 km/hr and 60 km/hr respectively. What is the average speed over this distance?
 (A) 10 km/hr (B) 20 km/hr
 (C) 25 km/hr (D) 30 km/hr
20. Two trains move from station A and station B towards each other at the speed of 50 km/hr and 60 km/hr respectively. At the meeting point, the driver of the second train felt that the train has covered 120 km more than the first train. What is the distance between A and B?
 (A) 1320 km (B) 1100 km
 (C) 1200 km (D) 960 km
21. Two trains, Chittaranjan Mail and Delhi Mail, start at the same time from stations Chittaranjan and Delhi respectively towards each other. After passing each other, they take 12 hours and 3 hours to reach Delhi and Chittaranjan respectively. If the Chittaranjan Mail is moving at the speed of 48 km/hr, what is the speed of the Delhi Mail?
 (A) 90 km/hr (B) 96 km/hr
 (C) 86 km/hr (D) 84 km/hr
22. The distance between two points A and B is 162 km. A train starts from A towards B and at the same time another train starts from B towards A. They meet each other after 6 hours. If the train from A is 8 km per hour faster than the other one, then what are the speed of two trains?
 (A) 13.5, 10.5 km/hr (B) 17.5, 9.5 km/hr
 (C) 13.5, 9.5 km/hr (D) 17.5, 10.5 km/hr
23. Two trains approach each other at 30 km/hr and 27 km/hr from two places 342 km apart. After how many hours will they meet?
 (A) 5 hours (B) 6 hours
 (C) 7 hours (D) 12 hours
24. R and S start walking towards each other at 10 a.m. at speeds 3 km/hr and 4 km/hr respectively. They were initially 17.5 km apart. At what time do they meet?
 (A) 11:30 a.m. (B) 12:30 p.m.
 (C) 1:30 p.m. (D) 2:30 p.m.
25. Peter walks at a uniform rate of 4 km an hour; and 4 hours after his start, Mike runs after him at the uniform rate of 10 km an hour. How far from the starting point will Mike catch Peter?
 (A) 16.7 km (B) 18.6 km
 (C) 21.5 km (D) 26.7 km
26. Two trains approach each other at 30 km/hr and 27 km/hr from two places 684 km apart. After how many hours will they meet?
 (A) 10 hours (B) 12 hours
 (C) 14 hours (D) 18 hours

27. A thief steals a car at 1:30 p.m. and drives it at 45 km/hr. The theft is discovered at 2 p.m. and the owner sets off in another car at 50 km/hr. At what time will he overtake the thief?
(A) 3:45 p.m. (B) 4:30 p.m.
(C) 5:30 p.m. (D) 6:30 p.m.
28. Two cars A and B are travelling on the same road towards each other. If car A is travelling at a speed of 120 km/hr and car B is travelling 15% slower than A, how much time will it take the cars to meet, if the initial distance between the two is 668.4 km and car A started to drive one and a half hour before car B started?
(A) 2 h and 12 min (B) 2h
(C) 1 h and 30 min. (D) 3 h and 15 min.
29. The distance between two stations A and B is 450 km. A train starts from A and moves towards B at an average speed of 15 km/hr. Another train starts from B, 20 minutes earlier than the train at A and moves towards A at an average speed of 20 km/hr. How far from A will the two trains meet?
(A) 190 km (B) 320 km
(C) 180 km (D) 260 km
30. Parikshit and Shayan start running simultaneously. Parikshit runs from point A to point B and Shayan from point B to point A. Parikshit's speed is $\frac{6}{5}$ of Shayan's speed. After crossing Shayan, if Parikshit takes $\frac{5}{2}$ hour to reach B, how much time does Shayan take to reach A after crossing Parikshit?
(A) 3 hr. 6 min. (B) 3 hr. 16 min.
(C) 13 hr. 26 min. (D) 3 hr. 36 min.
31. A train passes two persons walking in the same direction in which the train is going. These persons are walking at the rate of 3 km/hr. and 5 km/hr respectively and the train passes them completely in 10 seconds and 11 seconds respectively. What is the speed of the train?
(A) 24 km/hr (B) 25 km/hr
(C) 27 km/hr (D) 28 km/hr
32. Two guns were fired from the same place at an interval of 20 minutes but a person in a train approaching the place heard the second shot 15 minutes after the first. If speed of sound is 330 m/s, then the speed of train is:
(A) 120 m/s (B) 110 m/s
(C) 150 m/s (D) None
33. Raman and Ashok are two friends. Raman lives at a place Hubli and Ashok lives at another place Sadar. Everyday Raman goes to Sadar to meet Ashok at 120 km/hr. Thus, he takes 3 hours. On a particular day Ashok started to meet Raman so he moved towards Hubli. On that day Raman took only 2 hours to meet Ashok on the way instead at Sadar. What is the ratio of speeds of Raman is to Ashok?
(A) 2 : 1 (B) 3 : 2
(C) 1 : 2 (D) 2 : 3
34. A lives at Borivali and B lives at Lokhandwala. A usually goes to meet B at Lokhandwala. He covers the distance in 3 hours at 150 km/hr. On a particular day B started moving away from A. While A was moving towards Lokhandwala thus A took total 5 hours to meet B? What is the speed of B?
(A) 80 km/hr (B) 75 km/hr
(C) 60 km/hr (D) 70 km/hr
- Directions (35-36)** Read the following information given below and answer the questions that follow.
- The Patna Express started from Patna to Tata at 7 p.m. at a speed of 60 km/hr. Another train, Rajdhani Express, started from Tata to Patna at 4 a.m. next morning at a speed of 90 km/hr. the distance between Patna to Tata is 800 km.
35. At what time will the two trains meet?
(A) 5:32 a.m. (B) 5:28 a.m.
(C) 5:36 a.m. (D) 5:44 a.m.

36. How far from Tata will the two trains meet?

(A) 164 km (B) 156 km
(C) 132 km (D) 128 km

Directions (37-38) Read the following information given below and answer the questions that follow.

A thief, after committing the burglary, started fleeing at 12 noon, at a speed of 60 km/hr. He was then chased by a policeman Chulbul Pandey. Chulbul Pandey started the chase, 15 minute after the thief had started, at a speed of 65 km/hr.

37. At what time did Chulbul Pandey catch the thief?

(A) 3:30 p.m. (B) 3 p.m.
(C) 3:15 p.m. (D) None of these

38. If another policeman had started the same chase along with Chulbul Pandey, but at a speed of 60 km/hr, then how far behind was he when Chulbul Pandey caught the thief?

(A) 18.76 km (B) 15 km
(C) 21 km (D) 37.5 km

39. A truck crosses a man walking at 6 km/hr. The man can see the things upto 450 m only in one direction due to fog. He sees the truck which was going in the same direction for 4.5 minutes. What is the speed of the truck?

(A) 9 km/hr (B) 12 km/hr
(C) 12.5 km/hr (D) 15 km/hr

40. X express which goes from Hyderabad to Chennai, leaves Hyderabad at 5:30 a.m. and travels at a constant speed of 50 km/hr towards Nalgonda which is 100 km away. At 6:00 a.m., Y express leaves from Nalgonda for Hyderabad at a constant speed of 40 km/hr. At 6:30 a.m. Mr. Shah, the Control Officer realizes that both the trains are on the same track. How much time does Mr. Shah have to avert the accident?

(A) 20 minutes (B) 30 minutes
(C) 25 minutes (D) 15 minutes

41. Vinod and Suraj start from one end of a 1000 m track while Deepak starts from the other end. Vinod is 50% faster than Suraj and Deepak is 150% faster than Suraj. If Vinod meets Deepak in 25 seconds, how long does Suraj take to meet Deepak?

(A) 30 s (B) 35 s
(C) $28\frac{4}{7}$ s (D) $26\frac{3}{7}$ s

42. A dog finds a cat at 25 leaps away. The cat sees the dog coming towards it and starts running with the dog in hot pursuit. In every minute, the dog makes 5 leaps and the cat makes 6 leaps and one leap of the dog is equal to 2 leaps of the cat. What is the time in which the dog will catch the cat?

(A) 12.5 minutes (B) 13 minutes
(C) 11.5 minutes (D) 10.5 minutes

43. Seema leaves a point P at 6 a.m. and reaches the point Q at 10 a.m. Sapna leaves the point Q at 8 a.m. and reaches the point P at 12 noon. At what time do they meet?

(A) 8 a.m. (B) 9 a.m.
(C) 10 a.m. (D) None of these

44. The speed of a 150 m long train is 50 kmph. How much time will it take to pass 600 m long platform?

(A) 50 seconds (B) 54 seconds
(C) 60 seconds (D) 64 seconds

45. A train crosses a pole in 15 seconds, while it crosses a 100 m long platform in 25 seconds. What is the length (in metres) of the train?

(A) 125 (B) 135
(C) 150 (D) 175

46. Two trains A and B start running together from the same point in the same direction at 60 kmph and 72 kmph respectively. If the length

of each train is 240 m, how long will they take to cross each other?

- (A) 1 min. 12 sec. (B) 1 min. 24 sec.
(C) 2 min. 24 sec. (D) None

47. A train travelling at constant speed crosses a 96 m long platform in 12 seconds and another 141 m long platform in 15 seconds. What is the length and the speed of the train?

- (A) 64 m, 44 km/hr (B) 64 m, 54 km/hr
(C) 84 m, 54 km/hr (D) 84 m, 60 km/hr

48. A train passes a platform in 35 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, then what is the length of the platform?

- (A) 300 m (B) 240 m
(C) 225 m (D) 260 m

49. A train crosses a man coming from the opposite direction in 15 seconds. If the speed of man is 20 m/s and speed of train is 40 m/s, then what is the length (in metres) of the train?

- (A) 775 (B) 850
(C) 900 (D) 925

50. Ramu sees a train passing over a 1 km long bridge. The length of the train is half that of the bridge. If the train clears the bridge in 2 minutes, then what is the speed of the train?

- (A) 50 km/hr (B) 43 km/hr
(C) 45 km/hr (D) None of these

51. Two trains coming from the opposite sides cross each other in 30 seconds, if the lengths of the first train and second train are 1200 and 1800 m respectively, also the speed of first train is 180 km/hr. What is the speed of second train?

- (A) 60 km/hr (B) 108 km/hr
(C) 144 km/hr (D) 180 km/hr

52. A superfast train crosses another passenger train in 20 seconds. The speed of faster train is 72 km/hr and speed of slower train is 27 km/hr. Also, the length of superfast train is 150

m, then what is the length of the slower train if they are moving in the same direction?

- (A) 125 m (B) 100 m
(C) 150 m (D) 200 m

53. Two trains 121 metres and 99 metres in length respectively are running in opposite directions, one at the rate of 40 km/hr. and the other at the rate of 32 km/hr. What is the time they will take to clear each other completely from the moment they meet?

- (A) 9 seconds (B) 10 seconds
(C) 11 seconds (D) 12 seconds

54. A train 160 m long is running from west to east at a speed of 25 km/hr. Another train 460 m long is running on a parallel track but in opposite direction at 35 km/hr. For how much time smaller train will be completely overlapped by longer train?

- (A) 35 seconds (B) 18 seconds
(C) 36 seconds (D) 37.2 seconds

55. A moving train crosses a man standing on a platform and a bridge 300 metres long in 10 seconds and 25 seconds respectively. What will be the time taken by the train to cross a platform 200 metres long?

- (A) 15 seconds (B) 18 seconds
(C) 20 seconds (D) 22 seconds

56. A streamer goes downstream and covers the distance between two ports in 4 hours while it covers the same distance up stream in 5 hours. If the speed of the stream is 2 km/hr, then what is the speed of the streamer in still water?

- (A) 16 km/hr (B) 18 km/hr
(C) 24 km/hr (D) 36 km/hr

57. A man can row $7\frac{1}{2}$ kms an hour in still water and he finds that it takes him twice as long to row down the river than rowing against the stream. What is the rate of the stream?

- (A) 2.4 km/hr (B) 2.5 km/hr
(C) 3.4 km/hr (D) 3.5 km/hr
58. A boatman goes 2 km against the current of stream in 1 hour and goes 1 km along the current in 10 minutes. How long will he take to go 5 km in stationary water?
(A) 1 hr. 30 min. (B) 1 hr.
(C) 1 hr. 15 min. (D) 40 min.
59. A man can row 5 km/hr in still water. If the speed of current be 3 km/hr, then what is the time taken by him to row to a place 16 km upstream and back?
(A) 6.66 hours (B) 7.5 hours
(C) 8 hours (D) 10 hours
60. A boat goes 6 km in an hour in still water. It takes thrice as much time in covering the same distance against the current, then what is the Speed of the current?
(A) 2 km/hr (B) 3 km/hr
(C) 4 km/hr (D) 5 km/hr
61. A boat which sails at 20 km/hr in still water starts chasing, from 20 km behind, another one which sails at 8 km/hr in the upstream direction. After how long (in hours) will it catch up if the stream is flowing at 4 km/hr.
(A) 4 (B) 2.5
(C) 2 (D) 3.5
62. A boat takes 5 hours more while going back in upstream than downstream. If the distance between two places is 24 km and the speed of boat in still water be 5.5 km/hr. What must be the speed of boat in still water so that it can row downstream, 24 km, in 4 hours?
(A) 2.5 km/hr (B) 3 km/hr
(C) 4.5 km/hr (D) 3.5 km/hr
63. A motor cyclist goes from Mumbai to Pune, a distance of 192 km, at an average speed of 32 km/hr. Another man starts from Mumbai by car, $5/2$ hours after the first and reaches Pune half an hour earlier. What is the ratio of the speeds of the motor cycle and the car?
(A) 10 : 27 (B) 1 : 3
(C) 1 : 2 (D) 5 : 4
64. An escalator is moving at 3 steps per second. Harish walks in the same direction as a moving escalator at 2 steps per second and he takes 3 seconds less to get out the escalator than when he was moving on the stationary escalator. How many steps are there in the stationary escalator?
(A) 16 steps (B) 10 steps
(C) 25 steps (D) 20 steps
65. Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
(A) 9 minutes (B) 10 minutes
(C) 12 minutes (D) 20 minutes
66. A train travels 40% faster than a car. Both start from point A at the same time and reach point B, 140 km away at the same time. On the way the train takes 25 minutes for stopping at the stations. What is the speed (in km/hr) of the train?
(A) 67 (B) 134.4
(C) 145.9 (D) 160
67. Excluding stoppages, the speed of a bus is 72 kmph and including stoppages, it is 60 kmph. For how many minutes does the bus stop per hour?
(A) 12 (B) 8
(C) 15 (D) 10
68. A motorcyclist left $6\frac{6}{9}$ minutes later than the scheduled time but in order to reach its destination 21 km away in time, he had to increase his speed by 12 km/hr from the usual speed. What is usual speed (in km/hr) of the motorcyclist?

- (A) 28 (B) 35
(C) 42 (D) 64
69. A bus starts running with the initial speed of 33 km/hr and its speed increases every hour by certain amount. If it takes 7 hours to cover a distance of 315 km, then what will be hourly increment in the speed of the bus?
(A) 1 km/hr (B) 2 km/hr
(C) 3 km/hr (D) 4 km/hr
70. A bus starts running with some initial speed and its speed increases every hour by 9 km/hr. If it takes 11 hours to cover a distance of 572 km, then what was the initial speed of the bus?
(A) 3.5 km/hr (B) 7 km/hr
(C) 10.5 km/hr (D) 14 km/hr
71. A cyclist moving on a circular track of radius 100 m completes one lap in 2 minutes. What is the approximate speed of the cyclist?
(A) 200 m/min. (B) 314 m/min.
(C) 300 m/min. (D) 900 m/min.
72. A and B are running on a circular track of length 600 m (i.e., circumference of the track). Speed of A is 30 m/s and that of B is 20 m/s. They start from the same point at the same time in the same direction. When will they meet again for the first time?
(A) 10 s (B) 20 s
(C) 60 s (D) 30 s
73. In the previous question if they move in opposite direction, then what is the time taken by them to meet again for the first time?
(A) 20 s (B) 12 s
(C) 10 s (D) 30 s
74. Three persons A, B and C are running on a circular track of the length of 1000 m with a speed of 10 m/s, 15 m/s and 20 m/s respectively. If they start simultaneously from point P in the same direction, after how much time will they meet for the first time at the starting point?
(A) 100 seconds (B) 40 seconds
(C) 66.66 seconds (D) 200 seconds
75. 'A' walks around a circular field at the rate of one round per hour while 'B' runs around it at the rate of six rounds per hour. They start in the same direction from the same point at 7.30 a.m. At what time, they shall first cross each other?
(A) 8.30 a.m. (B) 8.10 a.m.
(C) 7.48 a.m. (D) 7.42 a.m.
76. Two cyclists start on a circular track from a given point but in opposite directions with speeds of 7 m/s and 8 m/s respectively. If the circumference of the circle is 300 m, after what time will they meet for the first time at the starting point?
(A) 20 seconds (B) 100 seconds
(C) 300 seconds (D) 200 seconds
77. A, B and C run on the circular path at the speed of 25 m/s, 30 m/s and 40 m/s respectively in the same direction. The circumference of the track (or path) is 600 m. When will they be together again for the first time at the starting point?
(A) 75 s (B) 90 s
(C) 120 s (D) 60 s
78. How many times in a day the hands of the clock are in a straight line but not together?
(A) 24 (B) 22
(C) 12 (D) 11
79. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
(A) 144° (B) 150°
(C) 168° (D) 180°
80. At what angle the hands of a clock are inclined at quarter to 5?
(A) $112\frac{1}{2}^\circ$ (B) $142\frac{1}{2}^\circ$

- (C) $97\frac{1}{2}^\circ$ (D) $127\frac{1}{2}^\circ$
81. What is the angle (smaller) between the hands of a clock when the time is 20 minutes past 9?
 (A) 150° (B) 120°
 (C) 160° (D) None of these
82. What is the value of the reflex angle between the hands of a clock at 10:25?
 (A) 150° (B) $192\frac{1}{2}^\circ$
 (C) $162\frac{1}{2}^\circ$ (D) $197\frac{1}{2}^\circ$
83. In a day, how many times the minute-hand and hour-hand make right angle between them?
 (A) 12 (B) 20
 (C) 22 (D) 44
4. A student walks from his house at a speed of $5\frac{1}{2}$ km/hr and reaches his school late by 6 minutes. Next day, he increases his speed by 1 km/hr and reaches 6 minutes before school time. How far is the school from his house?
 (A) $\frac{5}{4}$ km (B) $\frac{7}{4}$ km
 (C) $\frac{9}{4}$ km (D) $\frac{11}{4}$ km
5. A train covered a certain distance at a uniform speed. If the train had been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, the train would have taken 6 hours more than the scheduled time. What is the length of journey?
 (A) 700 km (B) 720 km
 (C) 740 km (D) 760 km
6. Salman starts in a car from Mumbai towards Pune. After sometime he realizes that he will cover only 75% of the distance in the scheduled time and therefore he doubles his speed immediately and thus manage to reach Pune exactly on time. What is the time after which Salman changed his speed, given that he could have been late by 3 hours if he had not changed his speed?
 (A) 3 h (B) 4 h
 (C) 5 h (D) 6 h
7. Ramesh travelled one third of a Journey at 10 km/hr, the next one third at 9 km/hr and the rest at 8 km/hr. If Ramesh had travelled half the journey at 10 km/hr and the other half at 8 km/hr, Ramesh would have taken half a minute more. What is the distance travelled by Ramesh?
 (A) 8 km (B) 10 km
 (C) 6 km (D) 12 km
8. A person covers a distance of 100 km in 10 hours partly by walking at 7 km/hr and the rest

Practice exercise Level 2

1. Two friends Manan and Nitin walk from P to Q at a distance of 39 km, at 3 km an hour and 3.5 km an hour respectively. Nitin reaches Q, returns immediately and meet Manan at R. What is the distance from P to R?
 (A) 20 km (B) 24 km
 (C) 36 km (D) 40 km
2. A and B walk from P to Q, a distance of 21 km at 3 km/hr and 4 km/hr. B reaches Q and immediately returns and meets A at R. What is the distance between P and R?
 (A) 3 km (B) 18 km
 (C) 15 km (D) None of these
3. Ram and Shyam start at the same time from P1 to P2 60 km away. Ram travels 4 km/hr slower than Shyam. Shyam reaches P2 and turns back and meets Ram at 12 km from P2. What is the speed of Ram?
 (A) 7 km/hr (B) 8 km/hr
 (C) 9 km/hr (D) 10 km/hr

- by running at 12 km/hr. What is the distance covered while he was running?
- (A) 64 km (B) 84 km
(C) 72 km (D) None of these
9. Suraj walked at 5 km/hr for certain part of the journey and then he took an auto for the remaining part of the journey travelling at 25 km/hr. If he took 10 hours for the entire journey. What part of journey did he travelled by auto if the average speed of the entire journey be 17 km/hr?
- (A) 170 km (B) 100 km
(C) 150 km (D) 200 km
10. An express train travels 299 km between two cities. During the first 111 km of the trip, the train travelled through mountainous terrain. The train travelled 10 km/hr slower through mountainous terrain than through level terrain. If the total time to travel between two cities was 7 h, what is the speed of the train on level terrain?
- (A) 6 km/hr (B) 55 km/hr
(C) 47 km/hr (D) 88 km/hr
11. A boy can cycle from his house to a metro station and back in a certain time at 12 km/hr. If he rides out at 8 km/hr, and returns by cycle at 15 km/hr, he takes 15 minutes longer on the double journey. Find the distance between his house and the station.
- (A) 10 km (B) 15 km
(C) 12 km (D) 5 km
12. Two horses start trotting towards each other, one from P to Q and another from Q to P. They cross each other after one hour and the first horse reaches Q $\frac{5}{6}$ hour before the second horse reaches P. If the distance between P and Q is 50 km. What is the speed of the slower horse?
- (A) 30 km/hr (B) 15 km/hr
(C) 25 km/hr (D) 20 km/hr
13. Bombay Express left Delhi for Bombay at 14:30 hours, traveling at a speed of 60 kmph and Rajdhani Express left Delhi for Bombay on the same day at 16.30 hours, traveling at a speed of 80 km/hr, how far away from Delhi will two trains meet?
- (A) 120 km (B) 360 km
(C) 480 km (D) 500 km
- Directions (14-15)** Solve the questions according to the conditions given.
- Smruti, a resident of California, started to meet Nilesh, who lives in New York, at 5 P.M. at a speed of 20 kmph. Nilesh also started after half an hour to meet Smruti at the speed of 10 kmph in the opposite direction.
14. If they meet at 8:30 P.M., then what is the distance between two cities?
- (A) 105 km (B) 100 km
(C) 120 km (D) 90 km
15. If her father also started to meet Nilesh at 6 P.M. from California, at what speed should he go so as to meet them at 8:30 pm?
- (A) 35 kmph (B) 50 kmph
(C) 45 kmph (D) 28 kmph
16. Two trains, 100 km apart, travel towards each other on the same track. One train travels at a speed of 40 kmph; the other travels at a speed of 60 kmph. A bird starts flying at a speed of 90 kmph, at the same location of the faster train. When it reaches the slower train, it turns around and flies in the opposite direction at the same speed. When it reaches the faster train, again it turns around and so on, till the two trains collide. What is the total distance traveled by the bird?
- (A) 90 km (B) 45 km
(C) 180 km (D) 135 km
17. Aryan runs at a speed of 40 metre/minute. Rahul follows him after an interval of 5 minutes

and runs at a speed of 50 metre/minute. Rahul's dog runs at a speed of 60 metre/minute and start along with Rahul. The dog reaches Aryan and then comes back to Rahul, and continues to do so till Rahul reaches Aryan. What is the total distance covered by the dog?

- (A) 600 metres (B) 750 metres
(C) 980 metres (D) 1200 metres

18. A train starts traveling from A at 7 a.m. and reaches B at 12 noon. Another train starts traveling from B at 9 a.m. and reaches A at 4 PM. At what time do the two trains cross each other?

- (A) 10:45 a.m. (B) 11 a.m.
(C) 9:45 a.m. (D) None of these

19. Points 'A' and 'B' are 70 km apart on a highway. A car starts from 'A' and another from 'B' at the same time. If they travel in the same direction, they meet in 7 hours, but if they travel towards each-other, they meet in one hour. What are the speeds of the two cars (in km/hr)?

- (A) 20, 30 (B) 40, 30
(C) 30, 50 (D) 20, 40

20. A bee and a fly start simultaneously from two trees that are located 180 km apart towards each other and meet in 5 hours time. If they resume their journey after meeting and proceed to the other tree, it is found that the time taken by the fly is 10 hours lesser than the time taken by the bee after they had met. How quick is the fly as compared to the bee?

- (A) 4 times as quick (B) 5 times as quick
(C) thrice as quick (D) 2.5 time as quick

21. A bird sitting on a wall, spots an approaching train, which is 720 km away, and moving at 15 m/s. The bird starts flying towards the train, and as soon as it reaches the train, it flies back towards the wall. It continues to do this to and

fro motion till the time the train crashes to the wall. If the bird can fly at a speed of 120 kmph, then what is the total distance covered by the bird?

- (A) 1600 km (B) 1500 km
(C) 1400 km (D) 1100 km

22. A thief escaped from police custody. Since he was a sprinter, he could run at a speed of 40 km/hr. The police realized it after 3 hr and started chasing him in the same direction at 50 km/hr. The police had a dog which could run at 60 km/hr. The dog would run to the thief and then return to the police and then would turn back towards the thief. It kept on doing so till the police caught the thief. What is the total distance travelled by the dog in the direction of the thief?

- (A) 720 km (B) 600 km
(C) 640 km (D) 660 km

23. A car is climbing up a hill at a speed of 90 kmph. A rabbit, sitting at the top of hill, spots the car when it is 1170 km away, and start running down the hill towards the car at a speed of 140 kmph. As soon as it meets the car, the rabbit turns back towards the top of the hill at a speed of 120 kmph. The rabbit continues this to and fro motion from the top of the hill towards the car and again back at the top of the hill till the car reaches the top of the hill. What is the total distance covered by the rabbit?

- (A) 1840 km (B) 1340 km
(C) 1640 km (D) 1680 km

24. Two cyclists start from the same place to ride in the same direction. A start's at noon at the rate of 8 km/hr. While B starts at 2 pm at the rate of 10 km per hour. How far will A have travelled before he is overtaken by B? Also find, at what times A and B will be 5 km apart before A is overtaken by B?

- (A) 64 km, 6 p.m. (B) 80 km, 7 p.m.
(C) 80 km, 7:30 p.m. (D) 16 km, 7:30 p.m.

25. Andy, Mandy and Sandy travel from the same place at the rates of 5, 6 and 8 km an hour respectively, if Mandy starts 2 hours after Andy, how long after Mandy must Sandy start in order that they may overtake Andy at the same instant?

- (A) $\frac{3}{2}$ hours (B) $\frac{7}{2}$ hours
(C) $\frac{5}{2}$ hours (D) $\frac{11}{2}$ hours

Directions (26-27) A and B start to run from the same point. A starts 4 days after B. B travels 1 km the first day, 2 km the second, 3 km the third, and so on. A travels 15 km a day.

26. On which day will they be together?
(A) 5th (B) 6th
(C) 7th (D) 8th
27. On which day will they be together again?
(A) 24th (B) 25th
(C) 26th (D) 27th
28. Two trains each 80 metres long, pass each other on parallel lines. If they are moving in the same direction, the faster one takes one minute to pass the slower one completely. If they are moving in the opposite direction they completely pass each other in 3 seconds. What is the speed of the faster train?
(A) 22 m/s (B) 24 m/s
(C) 26 m/s (D) 28 m/s
29. An express train passes completely the platform of length 1000 metres traveling with a speed of 90 kmph in 90 seconds. If the express train completely crosses the other train moving in parallel (in same direction) in 135 seconds, given that the length of the other train is the same as that of the platform, what is the speed of the other train?

- (A) 60 kmph (B) 30 kmph
(C) 45 kmph (D) 90 kmph

30. One local and another Express trains were proceeding in the same direction on parallel tracks at 29 km/hour and 65 km/hour respectively. The driver of the former notices that it took exactly 16 seconds for the faster train to pass by him. What is the length of the faster train?

- (A) 60 m (B) 120 m
(C) 160 m (D) 240 m

31. There are two trains running on two parallel tracks. Length of each train is 120 m. When they are running in opposite directions, they cross each other in 4 seconds and when they are running in the same direction they cross in 12 seconds. What is the speed of the faster train?

- (A) 40 km/hr (B) 72 km/hr
(C) 120 km/hr (D) 144 km/hr

32. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/hr. The other one walks at 5.4 km/hr. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?

- (A) 66 km/hr (B) 72 km/hr
(C) 78 km/hr (D) 81 km/hr

33. A train travelling at 48 km/hr completely crosses another train having half its length and travelling in opposite direction at 42 km/hr in 12 seconds. It also passes a railway platform in 45 seconds. What is the length of the platform?

- (A) 400 m (B) 450 m
(C) 560 m (D) 600 m

34. Two trains have their respective lengths as 230 m and 190 m. They cross each other completely in 21 s, if they are travelling in the opposite direction and in 42 s, if they are

travelling in the same direction. What is the ratio of the speeds of the two trains?

- (A) 3 : 1 (B) 4 : 1
(C) 3 : 2 (D) None of these

35. An express train 32 yards long takes 3 seconds to pass a slow train 56 yards long which runs in the opposite direction on a parallel line. If the trains had been going in the same direction they would have taken 9 seconds to pass each other. What are the rates at which each train travels (in yards/seconds)?

- (A) $\frac{88}{9}, \frac{176}{9}$ (B) 88, 176
(C) $\frac{44}{9}, \frac{44}{9}$ (D) $\frac{44}{3}, \frac{44}{3}$

36. A man can row 30 km upstream and 44 km downstream in 10 hours. Also, he can row 40 km upstream and 55 km downstream in 13 hours. What is the rate of the current?

- (A) 3 km/hr (B) 3.5 km/hr
(C) 4 km/hr (D) 4.5 km/hr

Directions (37-39) Two swimmers in a river started off from two different points at same time along the course of the river separated by 100 m and moving towards each other crossed each other to reach opposite ends and then reversed their directions to come back to their respective start positions. Speed of one swimmer is 6 m/s and the second is 4 m/s. The ratio of the distances covered by them is 4 : 1 respectively during the first meeting.

37. What was their first meeting time?
(A) 10 seconds (B) 12 seconds
(C) 15 seconds (D) None
38. What is the river flow speed?
(A) 2 m/s (B) 3 m/s
(C) 4 m/s (D) None
39. After what time they will meet for the second time from the starting?

- (A) 12.5 seconds (B) 15 seconds
(C) 20 seconds (D) 25 seconds

40. A motorboat going downstream overcame a raft at a point A, 1 hour later; it turned back and after some time passed meets the raft at a distance of 6 km from the point A. What is the speed with which the river is flowing?

- (A) 6 km/hr (B) 5 km/hr
(C) 3 km/hr (D) None of these

41. The difference between downstream speed and upstream speed is 3 km/hr and the total time taken during upstream and downstream is 3 hours. What is the downstream speed, if the downstream and upstream distance are 3 km each?

- (A) 3.5 km/hr (B) 4.33 km/hr
(C) 4 km/hr (D) 3.3 km/hr

42. A man can cross a downstream river by steamer in 40 minutes and same by a boat in 1 hour. If the time of crossing the river in upstream direction by steamer is 50% more than downstream time by the steamer and the time required by boat to cross the same river by boat in upstream is 50% more than the time required in downstream by boat. What is the time taken for the man to cross the river downstream by steamer and then return to same place by boat half the way and by steamer the rest of the way?

- (A) 85 minutes (B) 115 minutes
(C) 120 minutes (D) 125 minutes

43. A boy who can swim 48 metre/minute in still water swims 200 m against the current and 200 m with the current. If the difference between these two times is 10 minutes, then what is the speed of the current?

- (A) 30 metres/minute (B) 32 metres/minute
(C) 33 metres/minute (D) 36 metres/minute

44. A boat goes 15 km upstream and 22 km downstream in 5 hours. It goes 20 km

upstream and $\frac{55}{2}$ km, downstream in $\frac{13}{2}$

hours. What is the speed (in km/hr) of stream?

- (A) 3 (B) 5
(C) 8 (D) 11

45. Himkund Express travels for 30 minutes, then stops for 45 minutes due to an accident, then travels at two-third of the original speed. It reaches 1 hr 30 min. late. Had the accident occurred 60 km later, it would have been only 1 hr late. What is the speed of train and the distance of journey?

- (A) 48 km/hr, 140 km (B) 60 km/hr, 120 km
(C) 60 km/hr, 124 km (D) 72 km/hr, 132 km

46. Only a single rail track exists between station A and B on a railway line. One hour after the north bound super fast train N leaves station A for station B, a south bound passenger train S reaches station A from station B. The speed of the super fast train is twice that of a normal express train E, while the speed of a passenger train is half that of E. On a particular day N leaves for station B from Station A, 20 minutes behind the normal schedule. In order to maintain the schedule both N and S increased their speed. If the super fast train doubles its speed, what should be the ratio (approximately) of the speed of passenger train to that of the super fast train so that passenger train S reaches exactly at the scheduled time at station A on that day.

- (A) 1 : 3 (B) 1 : 4
(C) 1 : 5 (D) 1 : 6

47. A train X departs from station A at 11:00 am for station B, which is 180 km away. Another train Y departs from station B at 11:00 am for station A. Train X travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for

15 min at station C, which is 60 km away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest (in km), from station A to the point where the trains cross each other?

- (A) 112 (B) 118
(C) 120 (D) None of these

48. A candle of 6 cm long burns at the rate of 5 cm in 5 hours and another candle of 8 cm long burns at the rate of 6 cm in 4 hours. What is the time when both the candles are of equal lengths?

- (A) 1 h (B) 1.5 h
(C) 2 h (D) None of these

49. The Shri Shakti Express from Delhi to Katra was delayed by fog for 16 minutes and made up for the delay on a section of 80 km travelling with a speed 10 km per hour higher than its normal speed. Find the original speed of the Shri Shakti Express (according to the schedule)

- (A) 40 km/hr (B) 50 km/hr
(C) 60 km/hr (D) 66.66 km/hr

50. Nikhil goes to school everyday at 4 p.m. to pickup his children. On a Saturday, the school got over at 3 p.m. and the children started walking towards home. Nikhil met them on their way and returned home 20 minutes earlier. How long did the children walk?

- (A) 40 minutes (B) 45 minutes
(C) 50 minutes (D) 55 minutes

51. A ship is moving at a speed of 30 km/hr. To know the depth of the ocean beneath it, it sends a radio wave which travel at a speed of 200m/s. The ship receives the signal after it has moved 500m. What is the depth of the ocean?

- (A) $\frac{\sqrt{143}}{2}$ km (B) 12 km

- (C) 9 km (D) 8 km

52. On a 20 km tunnel, connecting two cities A and B, there are three gutters (1, 2 and 3). The

distance between gutters 1 and 2 is half the distance between gutters 2 and 3. The distance from city A to its nearest gutter, gutter 1, is equal to the distance of city B from gutter 3. On a particular day, the hospital in city A receives information that an accident has happened at gutter 3. The victim can be saved only if an operation is started within 40 min. An ambulance started from city A at 30 km/hr and crossed gutter 1 after 5 min. If the driver had doubled the speed after that, what is the maximum amount of time would the doctor get to attend the patient at the hospital. Assume 1 min is elapsed for taking the patient into and out of the ambulance?

- (A) 4 minutes
(B) 2.5 minutes
(C) 1.5 minutes
(D) The patient died before reaching the hospital.

Directions (53-54) Read the following information given below and answer the questions that follows.

Kishan left Delhi by his Hayabusa motorcycle. After travelling for 150 km which was 62.5% of the distance, he got stuck in a jam for 8 minutes. Once the jam got cleared, he increased his speed by $\frac{1}{3}^{\text{rd}}$ and hence reached his destination 7 minutes before the anticipated time.

53. What was the initial speed of the motor cycle?
(A) 72 km/hr (B) 90 km/hr
(C) 120 km/hr (D) 105 km/hr
54. What is the average speed of the motorcycle over the entire journey?
(A) 98.23 km/hr (B) 108.94 km/hr
(C) 94.11 km/hr (D) None of these
55. Mukesh, Suresh and Dinesh travel from Delhi to Mathura to attend Janmashmi Utsav. They

have a bike which can carry only two riders at a time as per traffic rules. Bike can be driven only by Mukesh. Mathura is 300 km from Delhi. All of them can walk at 15 km/hr. All of them start their journey from Delhi simultaneously and are required to reach Mathura at the same time. If the speed of bike is 60 km/hr then what is the shortest possible time in which all three can reach Mathura at the same time.

- (A) $8\frac{2}{7}$ hours (B) $9\frac{2}{7}$ hours
(C) 10 hours (D) None of these

56. Two cyclists met at 10 a.m. at Connaught place. After their meeting, one of them proceeded in the East direction while the other proceeded in the North direction. Exactly at noon, they were 30 km apart. What is the speed of the slower cyclist, if the difference of their speeds is 6 km/hr?

- (A) 6 km/hr (B) 12 km/hr
(C) 18 km/hr (D) 24 km/hr

57. It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the car is:

- (A) 3 : 5 (B) 3 : 4
(C) 4 : 3 (D) 4 : 5

Directions (58-61) Bhim and Arjun were exercising during their Vanvaas. They start running on a circular track simultaneously from the same starting point and in the same direction. If Bhim takes 4 minutes to complete one full round, and Arjun takes 7 minutes to complete one full round.

58. After how much time will they meet for the first time at the starting point?
(A) 4 minutes (B) 7 minutes
(C) 28 minutes (D) 56 minutes

59. After how much time will they meet for the first time?
 (A) $\frac{28}{3}$ minutes (B) 14 minutes
 (C) 28 minutes (D) 56 minutes
60. After how much time would they meet for the first time at a point diametrically opposite to the starting point on the track?
 (A) 4 minutes (B) 7 minutes
 (C) 28 minutes (D) Not Possible
61. If Bhim gives Arjun a lead of 4 min., when would they meet for the first time?
 (A) $\frac{16}{3}$ minutes (B) $\frac{22}{3}$ minutes
 (C) 28 minutes (D) 56 minutes
62. Raman and Rohit run with the speeds of 30 m/s and 20 m/s around a circular track of 600 m. They participate in a 5000 m race. What is the distance covered by Raman when he passes Rohit for the 2nd time?
 (A) 2200 m (B) 2250 m
 (C) 2850 m (D) None of these
63. A and B start together from the same point on a walking match round a circular course. After half an hour, A has walked three complete circuits and B, four and a half. Assuming that each walks with uniform speed, when would B overtakes A?
 (A) 10 minutes (B) 24 minutes
 (C) 18 minutes (D) 20 minutes
64. At what time, in minutes, between 3 o' clock and 4 o' clock, both the needles will coincide each other?
 (A) $5\frac{1}{11}$ past 3 (B) $12\frac{4}{11}$ past 3
 (C) $13\frac{4}{11}$ past 3 (D) $16\frac{4}{11}$ past 3
65. At what time between 7 and 8 o' clock will the hands of a clock be in the same straight line but, not together?
 (A) 5 min. past 7 (B) $5\frac{2}{11}$ min. past 7
 (C) $5\frac{3}{11}$ min. past 7 (D) $5\frac{5}{11}$ min. past 7
66. A watch which gains uniformly is 2 minutes slow at noon on Monday and is 4 min. 48 sec. fast at 2 p.m. on the following Monday. When was it correct?
 (A) 2 p.m. on Wednesday
 (B) 2 p.m. on Tuesday
 (C) 3 p.m. on Thursday
 (D) 1 p.m. on Friday
67. A clock loses 5 minutes everyday and another gains 10 min. everyday. They both are set right at 3:00 p.m. on a particular day. In how many days will they show the same time?
 (A) 24 (B) 36
 (C) 72 (D) 96
68. At a railway station, a 24 hour watch loses 3 minutes in 4 hours. If it is set correctly on Sunday noon when will the watch show the correct time again?
 (A) 6 p.m. after 40 days
 (B) 12 noon after 75 days
 (C) 12 p.m. after 100 days
 (D) 12 noon after 80 days
69. A Greek watch is being shown in a museum which has a very peculiar property. It gains as much in the day as it loses during night between 8 p.m. to 8 a.m. In a week how many times will the clock show the correct time?
 (A) 6 times (B) 14 times
 (C) 7 times (D) 8 times

Solution

Practice Exercise Level 1

1.(A) As the distance is constant

$$\text{So, } \frac{S_1}{S_2} = \frac{t_2}{t_1}$$

$$\frac{3}{4} = \frac{x}{x+20}$$

$$3x + 60 = 4x$$

$$x = 60$$

$$\text{Time of A} = 60 + 20 = 80 \text{ minutes or } 1\frac{1}{3} \text{ hours}$$

Ratio of speeds = Inverse ratio of time taken. So, the ratio of time will be 4 : 3. Now difference is 20 minutes, so we can say that A takes 4x minutes and B takes 3x minutes.

$$\therefore 4x - 3x = 20 ; x = 20$$

$$4x = 80 \text{ minutes}$$

2.(D) Let the usual speed be x km/hr.

$$\frac{1500}{x} - \frac{1500}{(x+250)} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{(x+250)} = \frac{1}{3000}$$

$$\Rightarrow \frac{(x+250) - x}{x(x+250)} = \frac{1}{3000}$$

$$\Rightarrow x(x+250) = 750000$$

$$\Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow x(x+1000) - 750(x+1000) = 0$$

$$\Rightarrow (x+1000)(x-750) = 0$$

$$\Rightarrow x = 750$$

$$\therefore \text{Usual speed} = 750 \text{ km/hr}$$

3.(D) As distance is constant so let the usual time be x minutes

$$1 \text{ hr } 40 \text{ min } 48 \text{ sec} = 100\frac{4}{5} \text{ minutes}$$

$$\frac{S_1}{S_2} = \frac{t_2}{t_1}$$

$$\frac{7}{5} = \frac{504}{5x}$$

$$7x = 504$$

$$x = 72 \text{ minutes}$$

$$\text{Now, usual speed} = \frac{42}{72} \times 60$$

$$= 35 \text{ km/hr}$$

As the speed became 5/7 of its former speed, so time will become 7/5 of usual time.

Let usual time be x.

$$\text{So, } \frac{7}{5}x = \frac{504}{5} \text{ minutes}$$

$$x = 72 \text{ minutes}$$

$$\text{Usual speed} = \frac{42 \times 60}{72} = 35 \text{ km/hr}$$

4.(D) As the distance is constant.

$$\text{So, } \frac{S_1}{S_2} = \frac{t_2}{t_1}$$

Let usual time be x hours.

$$\frac{4}{3} = \frac{x+2}{x}$$

$$x = 6 \text{ hours}$$

As the speed became 3/4 of the usual speed.

So, the time became 4/3 of the usual time.

Let the usual time be t.

$$\text{So, new time} = \frac{4}{3}t.$$

$$\text{i.e. } \left(t + \frac{1}{3}t\right) = \frac{4}{3}t$$

$$\text{So, } \frac{1}{3}t = 2 \text{ hours}$$

$$t = 6 \text{ hours}$$

5.(A) Let x kms be covered in y hr.

$$\text{Then, first speed} = \frac{x}{y} \text{ km/hr}$$

$$\text{Again, } \frac{x}{2} \text{ km is covered in } 2y \text{ hrs.}$$

$$\therefore \text{New speed} = \left(\frac{x}{2} \times \frac{1}{2y} \right) \text{ km/hr}$$

$$= \left(\frac{x}{4y} \right) \text{ km/hr}$$

Ratio of speeds

$$= \frac{x}{y} : \frac{x}{4y} = 1 : \frac{1}{4} = 4 : 1$$

6.(D) Ratio of speeds = 2 : 3 : 4

$$\text{Ratio of times} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$

$$= 6 : 4 : 3$$

7.(A) Let the speed of Scooty be x km/hr.

Time taken by Scooty,

$$t = \frac{1000}{x} \quad \dots (1)$$

Time taken by Auto,

$$(t-5) = \frac{1000}{x+10} \quad \dots (2)$$

From equation (1) and (2),

$$\frac{10000}{x(x+10)} = 5$$

$$x(x+10) = 2000$$

$$\text{So, } x = 40 \text{ km/hr}$$

8.(A) Let the normal speed of train = x km/hr

Let the normal time of train = T h

$$\text{Then, } \frac{300}{x} = T \quad \dots (1)$$

$$\text{and } \frac{300}{x+5} = T-2 \quad \dots (2)$$

Solving equation (1) and (2),

$$x = 25 \text{ or } (-30)$$

Discarding the negative value,

Speed of train = 25 km/hr

9.(B) Time taken to cover 25 km

$$= \frac{25}{40} \text{ h} = \frac{5}{8} \text{ h}$$

Required speed

$$= \frac{\text{Remaining distance}}{\text{Remaining time}}$$

$$\frac{46-25}{1-\frac{5}{8}} = \frac{21}{\frac{3}{8}} = 56 \text{ km/h}$$

10.(A) Let the usual speed of car be x km/hr.

$$\text{Then, } \frac{75}{x} = \frac{75}{x+10} + \frac{15}{60}$$

$$\Rightarrow 75 \left[\frac{10}{x(x+10)} \right] = \frac{1}{4}$$

So, $x(x+10) = 3000 = 50 \times 60$

$$x = 50 \text{ km/hr}$$

$$\text{Required time} = \frac{300}{50+10} = 5 \text{ h}$$

11.(C) $S_1 : S_2 = 4 : 5$

As the distance is constant,

$$t_1 : t_2 = 5 : 4$$

Now, the difference between t_1 and t_2 is 15 minutes i.e.

$$5x - 4x = 15$$

$$x = 15 \text{ minutes}$$

Required distance

$$\text{Required distance} = 5 \times \frac{4 \times 15}{60}$$

$$= 5 \text{ km}$$

12.(B) Let the distance between two places = D

Time difference = 36 minutes

$$\text{So, } \frac{D}{25} - \frac{D}{30} = \frac{36}{60}$$

$$D \left(\frac{30-25}{30 \times 25} \right) = \frac{3}{5}$$

$$d = 90 \text{ km}$$

13.(A) Let the average speed be x km/hr.

Time taken by aircraft,

$$t = \frac{800}{x} \quad \dots (1)$$

$$\text{also, } t - \frac{40}{60} = \frac{800}{x+40} \quad \dots (2)$$

From equation (1) and (2),

$$\frac{800}{x} = \frac{800}{x+40} + \frac{2}{3}$$

$$\frac{32000}{x(x+40)} = \frac{2}{3}$$

$$x(x+40) = 48000$$

$$\text{So, } x = 200 \text{ km/hr}$$

14.(B) Total distance = $(30 + 40) \text{ km} = 70 \text{ km}$

Total time taken

$$= \left(\frac{30}{6} + 5 \right) \text{ hrs} = 10 \text{ hrs}$$

Average speed

$$\frac{70}{10} \text{ km/hr} = 7 \text{ km/hr}$$

15.(C) Speed from X to Y

$$= 40 \text{ km/hr}$$

Speed for return journey

$$= 60 \text{ km/hr}$$

Let the distance be D.

Average Speed

$$= \frac{\text{Total Distance travelled}}{\text{Total time taken}}$$

$$= \frac{D+D}{\frac{D}{40} + \frac{D}{60}} = \frac{2}{\frac{1}{40} + \frac{1}{60}} = 48 \text{ km/hr}$$

16.(C) Using the formula,

$$S_a = \frac{S_1 t_1 + S_2 t_2 + S_3 t_3}{t_1 + t_2 + t_3}$$

We get,

$$S_a = \frac{\left(\frac{30}{60} \times 90 \right) + \left(\frac{40}{60} \times 120 \right) + (2 \times 140)}{\frac{30}{60} + \frac{40}{60} + 2}$$

$$= 127.89 \text{ km/hr}$$

Hence, the average speed of the car during the whole race is approximately 128 km/hr.

17.(B)

$$\begin{aligned} \frac{500+300}{\frac{500}{45} + \frac{300}{60}} &= \frac{800}{\frac{100}{9} + 5} \\ &= \frac{800 \times 9}{145} = \frac{1440}{29} = 49 \frac{19}{29} \text{ kmph} \end{aligned}$$

Hence, the required average speed is $49 \frac{19}{29}$

kmph.

18.(C) Average speed of car

$$\begin{aligned} &= \frac{45+50+25}{\frac{45}{15} + \frac{50}{25} + \frac{25 \times 3}{25}} \\ &= \frac{120}{3+2+3} = \frac{120}{8} = 15 \text{ km/hr} \end{aligned}$$

19.(B) Total distance = $(3 \times 4) \text{ km}$

$$= 12 \text{ km}$$

Total time taken

$$\begin{aligned} &= \left(\frac{3}{10} + \frac{3}{20} + \frac{3}{30} + \frac{3}{60} \right) \text{ hr} \\ &= \frac{(36+18+12+6)}{120} \text{ hr} = \frac{72}{120} \text{ hr} \end{aligned}$$

$$\text{Average speed} = \frac{12}{\left(\frac{3}{5} \right)} \text{ km/hr}$$

$$= \frac{(12 \times 5)}{3} \text{ km/hr} = 20 \text{ km/hr}$$

20.(A) Let the distance covered by first train be x km.

Then, the distance covered by second train

$$= (x + 120) \text{ km}$$

As both trains have travelled for same time.

$$\text{So, } \frac{x}{50} = \frac{x+120}{60}$$

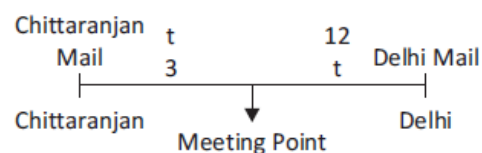
$$60x = 50x + 6000$$

$$x = 600$$

$$\text{Total distance} = x + (x + 120)$$

$$= 1320 \text{ km}$$

21.(B) If we assume that the initial stretch of track is covered by the two trains in time t each, the following figure will give a clear picture.



From the above figure, it can be deduced that,

$$\frac{t}{3} = \frac{12}{t}$$

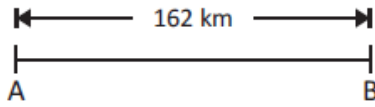
$$t^2 = 36$$

$$t = 6$$

Hence, the distance between Chittaranjan to the starting point is covered by Chittaranjan Mail in 6 hours, while the same distance is covered by the Delhi Mail in 3 hours.

Hence, the ratio of their speeds would be 1 : 2. Hence, the Delhi mail would travel at 96 km/hr.

22.(B)



Let the speed of train A = x km/hr
and train B = y km/hr

So, total time taken to cover the distance

$$\Rightarrow \frac{162}{x+y} = 6$$

$$x + y = 27 \quad \dots (1)$$

$$\text{Given that, } x - y = 8 \quad \dots (2)$$

So, now solving eq. (1) and (2),

And get $x = 17.5$ km/hr

and $y = 9.5$ km/hr

23.(B) Relative speed = $30 + 27 = 57$ km/hr

$$\text{Required time} = \frac{342}{57} = 6 \text{ hours}$$

24.(B) $\text{Time} = \frac{\text{Distance}}{\text{Relative speed}}$

$$\text{Time} = \frac{17.5}{3+4}$$

$$= \frac{17.5}{7} = 2.5 \text{ hours}$$

$$\text{Required time} = 10 \text{ AM} + 2.5 \text{ Hours}$$

$$= 12:30 \text{ PM}$$

25.(D) Distance covered by Peter in 4 hours = $4 \times 4 = 16$ km

Relative speed of Mike with respect to Peter
= $10 - 4 = 6$ km/hr

So, time taken to catch Peter

$$= \frac{16}{6} = \frac{8}{3} \text{ hours}$$

So, required distance

$$= \frac{8}{3} \times 10 = \frac{80}{3} = 26.67 \text{ km}$$

26.(B) Required time

$$= \frac{684}{30+27} = 12 \text{ hr}$$

27.(D) Here, there is a gap of half an hour, as the theft is discovered at 2 p.m.

Distance covered by the thief in $\frac{1}{2}$ hour

$$= 45 \times \frac{1}{2} = 22.5 \text{ km}$$

This distance has to be covered by the owner but since the thief is also running away we have to find relative speed, which is $50 - 45 = 5$ km/hr, since relative speed in same direction is the difference in speed.

Hence, time taken

$$= \frac{\text{Distance}}{\text{Relative speed}} = \frac{22.5}{5} = 4.5$$

Hence, the owner will overtake the thief after 4.5 hours,

$$2 \text{ p.m.} + 4.5 \text{ hours} = 6.30 \text{ p.m.}$$

Speed of car A = 120 km/hr

Speed of car B = 120×0.85

$$= 102 \text{ km/hr}$$

Distance travelled by car A in $1\frac{1}{2}$ h

$$= 120 \times 1\frac{1}{2} = 180 \text{ km}$$

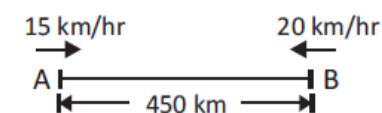
As the cars are travelling towards each other.

So, required time

$$= \frac{668.4 - 180}{120 + 102} = 2.2 \text{ h}$$

$$= 2 \text{ h } 12 \text{ min.}$$

Let the time of meet = t h



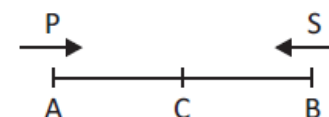
$$15\left(t - \frac{20}{60}\right) + 20t = 450$$

$$t = 13 \text{ h}$$

Distance from A

$$= 15\left(13 - \frac{1}{3}\right) = 190 \text{ km}$$

30.(D)



Let speed of Shayan = x km/hr

Speed of Parikshit = $\frac{6}{5}x$

Let Parikshit and Shayan meet at point C.
Also, given that Parikshit takes $5/2$ hour to reach B.

$$\therefore BC = \frac{6}{5}x \times \frac{5}{2} = 3x$$

So, time taken by Shayan to reach C
= 3 hours

Hence, distance AC

$$= \frac{6}{5}x \times 3 = \frac{18}{5}x$$

$\therefore$ Time taken by Shayan to reach A from C

$$= \frac{18x}{5x} \text{ hr} = \frac{18}{5} \text{ hr} = 3 \text{ hr } 36 \text{ min.}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{t_2}{t_1}}$$

$$\frac{6}{5} = \sqrt{\frac{t_2}{5/2}}$$

$$t_2 = 3.6 \text{ hr} = 3 \text{ hr } 36 \text{ min.}$$

- 31.(B)** Let the length of the train be x km and its speed be y km/hr. Speed of the train relative to first man = $(y - 3)$ km/hr
Speed of the train relative to second man = $(y - 5)$ km/hr.

$$\frac{x}{(y-3)} = \frac{10}{(60 \times 60)} \text{ and}$$

$$\frac{x}{(y-5)} = \frac{11}{60 \times 60}$$

$$\Rightarrow y - 3 = 360x \text{ and } 11y - 55 = 3600x$$

$$= 3600x$$

$$\Rightarrow 10y - 30 = 3600x \text{ and } 11y - 55 = 3600x$$

$$\Rightarrow 10y - 30 = 11y - 55 \Rightarrow y = 25$$

$\therefore$ Speed of the train = 25 km/hr

- 32.(B)** Sound would take 5 more min to reach initial position of man but this distance was travelled by train in 15 min. So, now that particular distance is constant.

$$\text{So, } \frac{T_s}{T_r} = \frac{S_r}{S_s}$$

$$\frac{5}{15} = \frac{S_r}{S_s}$$

So, speed of train is $1/3$ of speed of sound
i.e. 110 m/s.

- 33.(A)** Distance between Hubli and Sadar = 120×3
= 360 km

Let the speed of Ashok be S_{Ashok} , then

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$2 = \frac{360}{(120 + S_{\text{Ashok}})}$$

$$\therefore S_{\text{Ashok}} = 60 \text{ km/hr}$$

Here, Raman and Ashok are moving towards each other. So, the relative speed will be the sum of the speeds of Raman and Ashok both.

Therefore, ratio of speeds of Raman : Ashok = 2 : 1

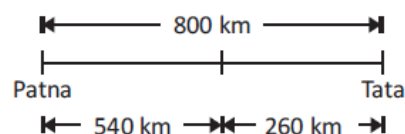
- 34.(C)** Distance = 3×150
= 450 km

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}}$$

$$5 = \frac{450}{(150 - S_B)}$$

$$S_B = 60 \text{ km/hr}$$

- 35.(D)** Till 4 a.m. Patna Express would have covered 540 km (60×9). So, remaining distance is covered in = $\frac{260}{90 + 60} = 1.73$ hours
i.e. 104 minutes (i.e. 5:44 a.m.)



- 36.(B)** After 1.73 hours, the two trains meet at 644 km ($540 + 1.73 \times 60$) from Patna. So, the trains meet at 156 km from Tata.

- 37.(C)** Distance travelled by the thief in 15 minutes.

$$= 60 \times \frac{15}{60} = 15 \text{ km}$$

Hence, distance between police and thief when police started to chase = 15 km.

$$\text{Relative speed} = (65 - 60)$$

$$= 5 \text{ km/hr}$$

Hence, time taken by police to catch the thief = $15 \div 5 = 3 \text{ hr}$

Hence, required time

$$= (12 \text{ h} + 15 \text{ min.} + 3 \text{ h}) = 3:15 \text{ p.m.}$$

- 38.(B)** Since, the speed of the another policeman is same as that of thief. Hence, distance between thief and him will be 15 km. And this is the required distance.

39.(B) $\text{Time} = \frac{4.5}{60} = \frac{450/1000}{x}$

where x is the relative speed.

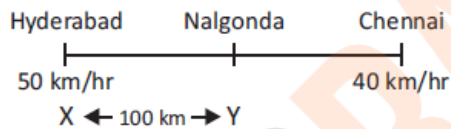
$$\Rightarrow x = 6 \text{ km/hr}$$

Relative speed = Speed of truck - Speed of man

$$6 = x - 6$$

$$x = 12 \text{ km/hr}$$

- 40.(A)** Distance travelled by X in 1 hour = 50 km
Distance travelled by Y in half an hour = 20 km



At 6:30, distance between 2 trains = 30 km

Time taken to travel this 30 km

$$= \frac{30}{50 + 40} = \frac{1}{3} \text{ h}$$

$$= \frac{1}{3} \times 60 = 20 \text{ minutes}$$

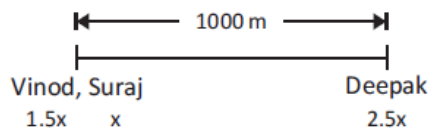
- 41.(C)** Let the speed of Suraj be $x \text{ m/s}$.

Then speed of Vinod will be

$$= 1.5x \text{ m/s}$$

Speed of Deepak will be

$$= 2.5x \text{ m/s}$$



$$\text{So, } 25 = \frac{1000}{1.5x + 2.5x}$$

$$x = 10 \text{ m/s}$$

So, Suraj will meet Deepak in

$$= \frac{1000}{10 + 25} = \frac{1000}{35}$$

$$= \frac{200}{7} = 28 \frac{4}{7} \text{ seconds}$$

- 42.(A)** Initial distance = 25 dog leaps

Per minute dog makes 5 dog leaps.

Per minute cat makes 6 cat leaps = 3 dog leaps

Relative speed = 2 dog leaps per minutes

Thus, an initial distance of 25 dog leaps would be covered in 12.5 minutes.

- 43.(B)** Let the distance PQ = $y \text{ km}$

And they meet x hours after Seema starts.

Average speed of Seema

$$= \frac{y}{10 - 6} = \frac{y}{4} \text{ km/hr}$$

Average speed of Sapna

$$= \frac{y}{12 - 8} = \frac{y}{4} \text{ km/hr}$$

Distance travelled by Seema

$$= \frac{yx}{4} \text{ km}$$

They meet x hours after Seema starts.

Sapna, as she starts 2 hours late, meets after $(x - 2)$ hours from her start. Therefore, the distance travelled by Sapna

$$= \frac{y(x - 2)}{4} \text{ km}$$

$$\text{Now, } \frac{yx}{4} + \frac{y(x - 2)}{4} = y$$

$$x + x - 2 = 4$$

$$\therefore x = \frac{6}{2} = 3 \text{ hours}$$

Thus, they meet at 6 a.m. + 3 hours = 9 a.m.

Seema takes time to complete the journey = 4 hours

Sapna takes time to complete the journey = 4 hours

Ratio of time = 4 : 4

Ratio of speed = 1 : 1

Let distance = 4 km

Then Seema cover 2 km with the speed of 1 km/hr.

Now, Seema covered the distance in 2 hours = 2 km

Remaining distance = 2 km

$$\text{Required time} = \frac{2}{(1+1)} = 1 \text{ hour}$$

So, they meet at (8:00 a.m. + 1 hour) 9 a.m.

44.(B) $\text{Required time} = \frac{2}{(1+1)} = 1 \text{ hour}$

$$= \frac{125}{9} \text{ m/s}$$

$$\text{Required time} = \frac{(600+150)}{(125/9)} \text{ s}$$

$$= \left(750 \times \frac{9}{125} \right) \text{ s} = 54 \text{ s}$$

45.(C) Let the length of the train be x metres.

Then,

Case of pole:

$$15 = \frac{x}{S_T}; S_T = \frac{x}{15}$$

Case of platform:

$$25 = \frac{x+100}{S_T}; S_T = \frac{x+100}{25}$$

Equating the value of S_T

$$\frac{x}{15} = \frac{(100+x)}{25}$$

$$\Rightarrow 25x = 1500 + 15x$$

$$\Rightarrow 10x = 1500 \Rightarrow x = 150$$

Hence, the length of the train is 150 m.

46.(C) Relative speed = $(72 - 60)$

$$= \left(12 \times \frac{5}{18} \right) \text{ m/s}$$

$$= \frac{10}{3} \text{ m/s}$$

Sum of lengths = $(240 + 240) \text{ m} = 480 \text{ m}$

$$\text{Required time} = \left(480 \times \frac{3}{10} \right) \text{ s}$$

$$= 144 \text{ s}$$

$$= 2 \text{ min. } 24 \text{ s}$$

47.(C) Let the length of the train be ' x ' metre.

The speed of the train is same in both the cases.

$$\text{So, } \frac{x+96}{12} = \frac{x+141}{15}$$

By solving, we get

$$x = 84 \text{ m}$$

Now, we put x in the above equation to get the speed.

$$\text{Speed} = \frac{(84+96)}{12} \times \frac{18}{5} = 54 \text{ km/h}$$

48.(C) Speed of the train S_t

$$= 54 \text{ km/hr}$$

$$= 54 \times \frac{5}{18} = 15 \text{ m/s}$$

Let L_t be the length of the train and L' be the length of the platform in metres.

A train passes a platform in 35 seconds and a man in 20 seconds.

$$L_t = S_t \times 20$$

$$= 15 \times 20 = 300 \text{ m}$$

$$\text{Now, } 35 = \frac{300+L}{15}$$

$$525 = 300 + L$$

$$L = 225 \text{ m}$$

49.(C) Length of train = Time $\times$ Relative speed

$$= 15 \times (20 + 40) = 15 \times 60 = 900 \text{ m}$$

50.(C) Length of bridge = 1000 m

$$\text{Length of train} = 500 \text{ m}$$

$$\text{Total length} = 1000 + 500$$

$$= 1500 \text{ m}$$

Speed of train

$$= \frac{1500}{1000} \times \frac{60}{2} = 45 \text{ km/h}$$

51.(D) Speed of first train = 180 km/hr = 50 m/s

Now,

$$\text{Time} = \frac{\text{Sum of length of two trains}}{\text{Sum of their speeds}}$$

$$30 = \frac{1200+1800}{(50+x)}$$

$$x = 50 \text{ m/s} = 180 \text{ km/hr}$$

52.(B) Relative speed = $(72 - 27)$

$$= 45 \text{ km/hr}$$

$$= 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/s}$$

$$\text{Time} = \frac{\left(\begin{array}{c} \text{Sum of length of} \\ \text{the two trains} \end{array} \right)}{\text{Relative speed}}$$

$$20 = \frac{(150 + x)}{25/2}$$

$$x = 100 \text{ m}$$

- 53.(C)** As the two trains are moving in opposite directions their relative speed = $40 + 32 = 72$ km/hr, i.e., they are approaching each other at 72 km/hr or 20 m/s.

Therefore, the required time

$$\therefore \text{Required time} = \frac{\text{Total length}}{\text{Relative speed}}$$

$$= \frac{121 + 99}{20} = \frac{220}{20} = 11 \text{ seconds}$$

Hence, trains shall take 11 seconds to clear each other completely.

- 54.(B)** Required Time

$$= \frac{460 - 160}{(25 + 35) \times \frac{5}{18}} = 18 \text{ sec.}$$

- 55.(C)** Let the length of the train = x metre

When a train crosses a platform the distance covered

= Length of (train + Platform)

According to the question,

$$\text{Speed of train} \rightarrow \frac{x}{10} = \frac{x + 300}{25}$$

$$25x = 10x + 3000$$

$$15x = 3000$$

$$x = \frac{3000}{15} = 200 \text{ m}$$

Length of train = 200 metres

Speed of train

$$= \frac{x}{10} = \frac{200}{10} = 20 \text{ m/s}$$

So, time taken in crossing a 200 m long platform

$$= \frac{200 + 200}{20} = 20 \text{ sec.}$$

- 56.(B)** Let the speed of the steamer in still water be x km/hr.

Downstream speed = $(x + 2)$ km/hr,

Upstream speed = $(x - 2)$ km/hr

Distance is constant.

$$\text{Then, } 4(x + 2) = 5(x - 2)$$

$$\Rightarrow x = 18$$

Hence, the speed of the steamer in still water is 18 km/hr.

- 57.(B)** Let the rate of stream be x km/hr. Then,

$$\text{Speed downstream} = \left(\frac{15}{2} + x \right) \text{ km/hr.}$$

$$\text{Speed upstream} = \left(\frac{15}{2} - x \right) \text{ km/hr}$$

Distance is constant

$$= \left(\frac{15}{2} + x \right) = 2 \left(\frac{15}{2} - x \right) \Rightarrow 3x = \frac{15}{2}$$

$$\Rightarrow x = \frac{15}{2 \times 3} = \frac{5}{2} = 2.5 \text{ km/hr}$$

- 58.(C)** Speed upstream = 2 km/hr,

Speed downstream = 6 km/hr

Speed in stationary water

$$= \frac{1}{2}(6 + 2) \text{ km/hr} = 4 \text{ km/hr}$$

Time taken to cover 5 km in stationary water

$$= \left(\frac{1}{4} \times 5 \right) = 1 \text{ hr } 15 \text{ min.}$$

- 59.(D)** Time required

$$= \frac{16}{8} + \frac{16}{2} = 10 \text{ hr}$$

- 60.(C)** Speed of boat in still water = 6 km/hr

Ratio of time taken in upstream and still water = 3 : 1.

The ratio of speed = 1 : 3.

So, $3x = 6$ km/hr ; $x = 2$ km/hr

Upstream Speed = 2 km/hr

Speed of stream = $6 - 2 = 4$ km/hr

- 61.(B)** Upstream speed of first boat = 16 km/hr

Upstream speed of second boat

$$= 8 \text{ km/hr}$$

So, Relative speed = 8 km/hr

$$\text{Required time} = \frac{20}{8} = 2.5 \text{ hr}$$

62.(D) Let R be the speed of stream.

$$\text{Then, } \frac{24}{(5.5-R)} - \frac{24}{(5.5+R)} = 5$$

$$\Rightarrow R = 2.5 \text{ km/h ;}$$

$$\text{Again } (B+R) = \frac{24}{4} = 6$$

where B is the required boat speed.

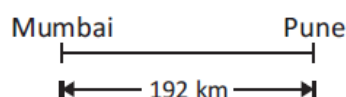
= downstream speed

$$\Rightarrow (B + 2.5) = 6$$

$$B = 3.5 \text{ km/hr}$$

63.(C) Time taken by motor cyclist

$$= \frac{192}{32} = 6 \text{ h}$$



$$\text{Time taken by car} = 6 - (2.5 + 0.5) = 3 \text{ h}$$

Ratio of the speeds of motor cycle : car

$$= 3 : 6 = 1 : 2$$

64.(B) Due to movement of the escalator, the speed has become $\frac{5}{2}$ of original speed, hence time should become $\frac{2}{5}$ of the original time, thereby saving $\frac{3}{5}$ of original time. Hence, $\frac{3}{5}$ of original time = 3 seconds

Original time = 5 seconds

So, in 5 seconds, Harish can cover $5 \times 2 = 10$ steps

Therefore, there are 10 steps in the escalator.

65.(B) Due to stoppages, it covers 9 km less per hour.

Time taken to cover 9 km

$$= \left(\frac{9}{54} \times 60 \right) \text{ min.} = 10 \text{ min.}$$

Required time:

$$= \frac{\left(\text{Speed without stoppage} - \text{Speed with stoppage} \right)}{\text{Speed without stoppage}}$$

$$\text{Required time} = \frac{54 - 45}{54}$$

$$= \frac{1}{6} \text{ hours or } \frac{1}{6} \times 60 = 10 \text{ min.}$$

66.(B) Let the speed of train and car be $7x$ and $5x$ respectively.

$$\frac{140}{5x} - \frac{140}{7x} = \frac{25}{60}$$

$$x = 19.2$$

Hence, speed of train = 7×19.2

$$= 134.4 \text{ km/hr.}$$

67.(D) Stoppage time

$$= \frac{72 - 60}{72} \times 60 = 10 \text{ minutes}$$

68.(C) Let the usual speed of the motorcyclist be S km/hr. Then,

$$\frac{21}{S} - \frac{21}{S+12} = \frac{1}{9}$$

$$S = 42 \text{ km/hr}$$

69.(D) Let the increment in the speed per hour be d km/hr.

Then, speed of bus each hour is 33 km/hr, $(33 + d)$ km/hr,, $(33 + 6d)$ km/hr

$$\text{Average speed} = \frac{315}{7} = 45 \text{ km/hr}$$

Then,

$$\frac{33 + (33 + d) + \dots + (33 + 6d)}{7} = 45$$

$$\Rightarrow \frac{7 \times (33 + 33 + 6d)}{2 \times 7} = 45$$

$$\Rightarrow d = 4 \text{ km/hr}$$

70.(B) Let the initial speed be S km/hr.

$$\text{Average speed} = \frac{572}{11} = 52 \text{ km/hr}$$

$$52 \times 11 = \frac{11}{2} (2S + 10 \times 9)$$

$$\Rightarrow S = 7 \text{ km/hr}$$

71.(B) Distance travelled

$$= 2 \times \frac{22}{7} \times 100$$

$$\therefore \text{Speed} = \frac{2 \times \frac{22}{7} \times 100}{2}$$

$$= 314.29 \approx 314 \text{ m/min.}$$

72.(C) Required time

$$= \frac{600}{(30-20)} = \frac{600}{10} = 60 \text{ s}$$

73.(B) Required time

$$= \frac{600}{(30+20)} = \frac{600}{50} = 12 \text{ s}$$

74.(D) Time taken by A to complete one full round

$$= \frac{1000}{10} = 100 \text{ sec.}$$

Time take by B to complete one full round

$$= \frac{1000}{15} = \frac{200}{3} \text{ sec.}$$

Time taken by C to complete one full round

$$= \frac{1000}{20} = 50 \text{ sec.}$$

$$\text{The LCM of } \left(100, \frac{200}{3}, 50\right) = 200$$

So, all three of them will come at the starting point after 200 seconds.

75.(D) Let the length of the circular track be x km.

$$\text{Speed of A} = \frac{x}{1} = x \text{ km/h}$$

$$\text{Speed of B} = \frac{6x}{1} = 6x \text{ km/h}$$

Time after which they will cross each other

$$= \frac{x}{6x-x} = \frac{x}{5x} = \frac{1}{5} \text{ hr}$$

$$\text{i.e. } \frac{1}{5} \times 60 = 12 \text{ min.}$$

So, they will cross each other for the first time at 7:42 a.m.

76.(C) Time taken by first cyclist to complete a track $= \frac{300}{7} \text{ s}$

Time taken by second cyclist to complete a track $= \frac{300}{8} \text{ s}$

$$\therefore \text{Required time}$$

$$= \text{LCM of } \frac{300}{7} \text{ and } \frac{300}{8} = 300 \text{ s}$$

77.(C) A takes $\frac{600}{25} = 24 \text{ s}$ to complete one round

B takes $\frac{600}{30} = 20 \text{ s}$ to complete one round

C takes $\frac{600}{40} = 15 \text{ s}$ to complete one round

LCM of 24, 20 and 15 is 120 seconds

Hence, they would meet for the first time at the starting point after 120 seconds.

78.(B) In 12 hours they are 11 times in a straight line. So, in 24 hours they will be 22 times.

79.(D) Total time hour hand has to travel = 6 hours. Hence, Angle traced by hour hand in

$$6 \text{ hours} = \left(\frac{360}{12} \times 6\right)^\circ = 180^\circ$$

80.(D) At 5'o clock the angle between hour and minute hand will be 150° .

The minute hand have to cover $15 \times 6^\circ = 90^\circ$

$$\text{In this } 15 \text{ min hour hand will also move} \\ = 15 \times \frac{1}{2} = 7.5^\circ$$

So, required angle

$$360 - (150^\circ + 90^\circ - 7.5^\circ) = 127\frac{1}{2}^\circ$$

81.(C) Required angle

$$= 270^\circ - (6^\circ \times 20) + \left(\frac{1}{2}^\circ \times 20\right) = 160^\circ$$

82.(D) At 10'o clock the angle between hour hand and minute hand will be 60° .

To be at 10:25, the minute hand will cover = $25 \times 6^\circ = 150^\circ$.

Hour hand will travel in this

$$25 \text{ min} = 25 \times \frac{1}{2}^\circ = 12.5^\circ$$

So, angle between them at 10 : 25.

$$= 150^\circ + 12.5^\circ = 162.5^\circ$$

So, reflex angle = $360^\circ - 162.5^\circ$

$$= 197.5^\circ$$

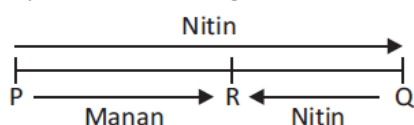
83.(D) Since between 2 – 4 O' clock and 8 – 10 O' clock two hands of a clock make 90° angle only 2 times while in rest of the hours two

hands make 90° angle 2 times every one hour.

Hence, they are at right angle 22 times in 12 hours and 44 times in 24 hours.

Practice Exercise Level 2

- 1.(C) When Nitin meets Manan at R, Nitin has walked the distance PQ + QR and Manan has walked the distance PR. Let us represent it in a diagram.



So, both Manan and Nitin have walked together a distance

$$= 2 \times PQ = 2 \times 39 = 78 \text{ km}$$

The ratio of the speeds of Manan and Nitin

$$\text{is } 3 : 3.5 \text{ i.e. } \frac{6}{7}.$$

Hence, the distance travelled by Manan = PR

$$\frac{6}{6+7} \times 78 = 36 \text{ km}$$

Hence, the required distance from P to R is 36 km.

2.(B)



Both A and B will reach the point R after the same time.

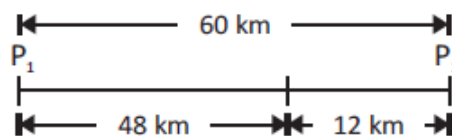
Now, when time is same,

$$\frac{S_1}{S_2} = \frac{D_1}{D_2}$$

$$\Rightarrow \frac{3}{4} = \frac{x}{21+21-x}$$

By solving we get, $x = 18$

3.(B)



Given that,

Difference between their speeds = 4 km/hr

Now, in this case time is constant.

$$\text{So, } \frac{S_1}{S_2} = \frac{D_1}{D_2}$$

D_1 = Distance travel by Ram

D_2 = Distance travel by Shyam

$$D_1 = 48, D_2 = 72$$

$$\frac{S_1}{S_2} = \frac{48}{72} = \frac{2}{3}$$

Now, $S_1 = 2x, S_2 = 3x$

$$S_2 - S_1 = x = 4$$

So, speeds $S_1 = 2 \times 4 = 8 \text{ km/hr}$

Speed of Ram = 8 km/hr

4.(B)

The distance is constant and let usual time be x .

$$\text{So, } \frac{S_1}{S_2} = \frac{T_2}{T_1}$$

$$\frac{5}{7} = \frac{x-6}{x+6}$$

$$5x + 30 = 7x - 42$$

$$x = 36 \text{ minutes}$$

Time he takes while walking at speed of $2\frac{1}{2} \text{ km/hr}$ is 42 minutes.

So, Required distance = Speed $\times$ Time

$$= \frac{5}{2} \times \frac{42}{60} = \frac{7}{4} \text{ km}$$

Let the required distance be $x \text{ km}$.

$$\text{Then, } \frac{x}{(5/2)} - \frac{x}{(7/2)}$$

$$= \frac{12}{60} \left[\begin{array}{l} \text{Difference in two} \\ \text{times is 12 min.} \end{array} \right]$$

$$\Rightarrow \frac{2x}{5} - \frac{2x}{7} = \frac{1}{5}$$

$$\Rightarrow 14x - 10x = 7$$

$$\Rightarrow 4x = 7 \Rightarrow x = \frac{7}{4}$$

$$\text{Required distance} = \frac{7}{4} \text{ km}$$

- 5.(B) Let the uniform speed be x km/hr and the scheduled time be t hours.
as the distance is constant.

$$\text{Also, } \frac{S_1}{S_2} = \frac{t_2}{t_1}$$

$$\frac{x}{x+6} = \frac{t-4}{t}$$

By applying componendo and Dividendo,

$$2x - 3t = -12 \quad \dots (1)$$

$$\text{Also, } \frac{S_1}{S_2} = \frac{t_2}{t_1}$$

$$\frac{x}{x-6} = \frac{t+6}{t}$$

By applying Componendo and Dividendo,

$$x - t = 6 \quad \dots (2)$$

Solving (1) and (2),

$$t = 24 \text{ hours}$$

$$x = 30 \text{ km/hr}$$

Now, required distance

$$= 30 \times 24 = 720 \text{ km}$$

- 6.(D) Let the original speed be s km/hr and scheduled time = t hours.

And total distance = D km

$$\text{then, } s \times t = \frac{3}{4}D \quad \dots (1)$$

$$\text{and } s \times (t + 3) = D \quad \dots (2)$$

From equation (1) and (2), we get,

$$\frac{t}{t+3} = \frac{3}{4} \Rightarrow t = 9 \text{ h}$$

and let $s = 1$ km/hr, then

$$D = 12 \text{ km}$$

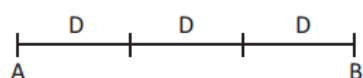
Again, since he doubles his speed after k hours then,

$$s_1 t_1 + s_2 t_2 = D$$

$$1 \times k + 2 \times (9 - k) = 12$$

$$k = 6 \text{ h}$$

- 7.(C) Let the distance between both places = $3D$



Case I:

Distance D is travelled at 10 kmph so time =

$$\frac{D}{10}$$

Same as $\frac{D}{9}$ and $\frac{D}{8}$

Case II:

Distance $\frac{3D}{2}$ is travelled at 10 kmph so time

$$\frac{3D}{20} \text{ same as } \frac{3D}{16}$$

Now, given that time difference in both cases.

$$\Rightarrow \frac{3D}{20} + \frac{3D}{16} - \left(\frac{D}{10} + \frac{D}{9} + \frac{D}{8} \right) = \frac{1}{2} \times \frac{1}{60}$$

$$\Rightarrow D \left(\frac{108 + 135 - 72 - 80 - 90}{720} \right) = \frac{1}{120}$$

$$D = 6 \text{ km}$$

- 8.(C) Let the time taken when walking = x

So, distance = $7x$

Then, time taken when running

$$= 10 - x$$

So, distance = $12(10 - x)$

$$\text{Now, } 7x + 12(10 - x) = 100$$

$$x = 4 \text{ hours}$$

Time taken when running

$$= 6 \text{ hours}$$

$$\text{Distance} = 12 \times 6 = 72 \text{ km}$$

- 9.(C) Let he walked for x hours,

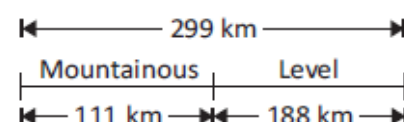
$$\text{then, } 5x + 25(10 - x) = 17 \times 10$$

$$x = 4$$

$$10 - x = 6 \text{ hours travelled by auto.}$$

$$\text{Hence, distance travelled by auto} = 25 \times 6 = 150 \text{ km}$$

- 10.(C) Let the speed of train on level terrain = x km/hr



Then, the speed of train through mountainous = $(x - 10)$ km/hr

According to the question,

$$\frac{188}{x} + \frac{111}{x-10} = 7$$

$$\frac{188x - 1880 + 111x}{x(x-10)} = 7$$

$$7x^2 - 369x + 1880 = 0$$

$$x = 47 \text{ km/hr}$$

- 11.(A)** Let the distance between house and metro station = D

And time difference between both cases.

$$\frac{D}{8} + \frac{D}{15} - \frac{2D}{12} = \frac{15}{60}$$

$$D \left(\frac{15+8-20}{120} \right) = \frac{1}{4}$$

$$D \times \frac{3}{120} = \frac{1}{4}$$

$$D = 10 \text{ km}$$

- 12.(D)** Let the speed of faster horse be f_s and that of slower horse be s_s , then

$$= f_s + s_s = \frac{50}{1} = 50$$

$$\text{and } \frac{50}{s_s} - \frac{50}{f_s} = \frac{5}{6}$$

Now, you can go through the options or solve the two equations.

The speed of slower horse is 20 km/hr.

$$\text{Since, } 20 + 30 = 50$$

$$\text{and } \frac{50}{20} - \frac{50}{30} = \frac{5}{6}$$

- 13.(C)** In 2 hours Bombay Express will travel 120 km. Now, the time required once the Rajdhani starts = $\frac{120}{80-60} = 6$ hours to catch up with the other train. Distance travelled by Rajdhani Express in 6 hours is $80 \times 6 = 480 \text{ km}$

- 14.(B)** Smruti started 5 p.m. and in half an hr she will cover distance $20 \times \frac{1}{2} = 10 \text{ km}$.

Now, after half an hour Nilesh also started, so the relative speed of them will be $20 + 10 = 30 \text{ kmph}$. So, in 3 hrs distance covered will

be $30 \times 3 = 90$. So, total distance will be $10 + 90 = 100 \text{ km}$.

- 15.(D)** Distance to be covered by father
 $= 20 \times \frac{7}{2} = 70 \text{ km}$

Time he is left with = 8:30 p.m. – 6:00 p.m.

$$= 2\frac{1}{2} \text{ hour}$$

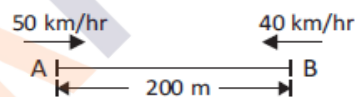
$$\text{Required speed} = \frac{70 \times 2}{5} = 28 \text{ km/hr}$$

- 16.(A)** Train will collide after

$$= \frac{100}{60+40} = 1 \text{ hour}$$

The bird must be flying at 90 kmph during this one hour, so the bird will travel $90 \times 1 = 90 \text{ km}$

- 17.(D)** In 5 minutes Aryan will go 40×5 i.e. 200 metres.



Now, Rahul will catch Aryan after $\frac{200}{50-40} =$

20 minutes. We can also say that the dog will run only for 20 minutes. So, total distance travelled by the dog = $60 \times 20 = 1200 \text{ metres}$

- 18.(A)**



Train starting from A takes 5 hours to reach the point B whereas the train starting from B takes 7 hours to reach the point A.

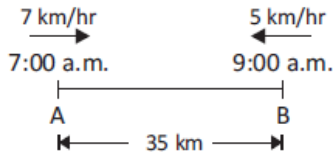
Now, to find the speed of the two trains, the distance AB has to be divided by 5 and 7 for the two trains. So, let us assume the distance to be 35 (LCM of 5 and 7) so that the speed of the two trains should be an integer value.

Speed of train starting from

$$A = \frac{35}{5} = 7 \text{ km/h}$$

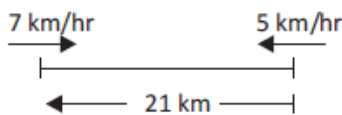
Speed of train starting from

$$B = \frac{35}{7} = 5 \text{ km/h}$$



Trains starting from A will travel for 2 hours and then both the trains will proceed towards each other. In the first 2 hours the train from A will move 7×2 i.e. 14 km.

Then, the remaining distance between the two trains will be $35 - 14 = 21$ km

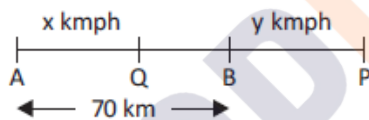


Now, they will meet each other after

$$\frac{21}{7+5} = \frac{21}{12} = 1\frac{3}{4} \text{ or 1 hour 45 minutes.}$$

So, the two trains will cross each other 1 hour 45 minutes after 9 a.m. i.e. 10:45 a.m.

19.(B)



Speed of car starting from A
= x kmph

Speed of car starting from B
= y kmph

Case I:

When cars meet at P,

$$7x = AP = AB + BP = 70 + 7y$$

$$7x - 7y = 70$$

$$x - y = 10 \quad \dots (1)$$

Case II:

When cars meet at Q,

$$x + y = 70 \quad \dots (2)$$

On adding these equations,

$$x = 40 \text{ kmph}$$

Putting the value of x in equation (1),

$$y = 40 - 10 = 30 \text{ kmph}$$

20.(D) They meet 5 hours after start i.e. they travel 180 km simultaneously after starting.

$$\text{So, } f + b = \frac{180}{5}$$

$$= 36 \text{ kmph} \quad \dots (1)$$

where, f = speed of fly and

b = speed of bee.

In 5 hours before meeting,

$$\text{Fly travelled} = 5f$$

$$\text{Bee travelled} = 180 - 5f$$

So,

$$\Rightarrow \frac{5f}{b} - \frac{180 - 5f}{f} = 10$$

$$\Rightarrow 5\left(\frac{f}{b}\right) - \frac{180}{f} + 5 = 10$$

$$\Rightarrow 5\left(\frac{f}{b}\right) - \frac{180}{f} = 5 \quad \dots (2)$$

Solving (1) and (2),

$$\frac{5f}{(36-f)} - \frac{180}{f} = 5$$

$$\frac{5f^2 - 6480 + 180f}{36f - f^2} = 5$$

$$\Rightarrow 5f^2 - 6480 + 180f = 180f - 5f^2$$

$$\Rightarrow 10f^2 = 6480$$

$$\Rightarrow f^2 = 648$$

$$\Rightarrow f = 25.45$$

$$\therefore b = 36 - 25.45 = 10.54$$

$$= 2.5 \text{ times approx.}$$

21.(A) 15 m/s = 54 kmph

The train will crash at the wall after

$$= \frac{720}{54} = \frac{40}{3} \text{ hours}$$

The bird will cover a distance of $\frac{40}{3} \times 120 = 1600$ km during this time.

22.(D) Total time taken by the police in catching the thief

$$= \frac{40 \times 3}{50 - 40} = 12 \text{ hours}$$

Total distance travelled by the dog = 12×60
= 720 km

Total distance travelled by the dog in the direction of the thief

$$= 720 \times \frac{60 + 50}{(60 + 50) + (60 - 50)}$$

$$= 720 \times \frac{11}{12} = 660 \text{ km}$$

23.(D) The car will reach to the top of the hill after

$$= \frac{1170}{90}$$

= 13 hours

Let total time during which the rabbit climbed downhill and uphill are t_1 and t_2 hours respectively. Obviously, during every to and from motion, the distance covered downhill is equal to distance covered uphill.

Therefore, $140 \times t_1 = 120 \times t_2$

$$\text{or } \frac{t_1}{t_2} = \frac{6}{7}$$

Also, $t_1 + t_2 = 13$

Solving we get,

$t_1 = 6$ and $t_2 = 7$

Total distance covered by the rabbit = $140 \times t_1 + 120 \times t_2$
= $840 + 840 = 1680$ km

24.(C) Distance travelled by A in 2 hours = 8×2

= 16 km

Then, time taken by B to overtake A

$$= \frac{16}{(10 - 8)} = 8 \text{ hours}$$

Hence, total distance travelled by A

= $16 + 8 \times 8 = 80$ km

When A and B are 5 km apart.

Then, distance B has to travel

= $16 - 5 = 11$ km

Required time = $\frac{11}{2} = 5.5$ hours

i.e. at 2:00 p.m. + 5.5 hours

= 7:30 p.m.

25.(C) As Mandy starts 2 hours after Andy, distance travelled by Andy in 2 hours = 5×2
= 10 km

Time taken by Mandy to overtake Andy

$$= \frac{10}{6 - 5} = 10 \text{ hours}$$

Total distance covered by Andy

= $10 + 5 \times 10 = 60$ km

This distance is covered by Sandy

$$= \frac{60}{8} = 7.5 \text{ hours}$$

Therefore, Sandy should start 2.5 hours ($10 - 7.5$) before Mandy.

26.(A) B will travel $1 + 2 + 3 + 4 = 10$ km in first 4 days, 5th day he will travel 5 km, so total distance in 5 days will be 15 km. While A will travel 15 km in 1st day. So, they will meet after 5th day.

27.(A) Sum of distance travelled by B is $\frac{n(n+1)}{2}$ and the distance travelled by A is $(n - 4) \times 15$.

According to the question,

$$= \frac{n(n+1)}{2} = (n - 4) \times 15$$

$$\Rightarrow n^2 - 29n + 120 = 0$$

$$\Rightarrow (n - 24)(n - 5) = 0, n = 24 \text{ or } 5$$

28.(D) Let the speed of faster train be x m/s and that of slower train be y m/s.

When they are moving in the same direction, relative speed

= $(x - y)$ m/s

$$\therefore x - y = \frac{80 + 80}{60}$$

$$x - y = \frac{8}{3} \quad \dots (1)$$

When they are moving in the opposite directions,

relative speed = $(x + y)$ m/s

$$x + y = \frac{80 + 80}{3} = \frac{160}{3} \quad \dots (2)$$

Adding (1) and (2), we get for

$$= 2x = \frac{8}{3} + \frac{160}{3}$$

$$x = \frac{56}{2} = 28 \text{ m/s}$$

From (1), we get

$$y = x - \frac{8}{3} = 25\frac{1}{3} \text{ m/s}$$

Hence, the speed of the faster train is 28 m/s.

29.(B) Let L be the length of train.

$$\text{Then, } \frac{(1000+L) \times 18}{90 \times 5} = 90$$

$$\text{So, } \frac{1000+L}{90} = \frac{5}{18} \times 90$$

$$L = 1250 \text{ m}$$

Length of other train = length of platform = 1000 m

$$\frac{(1000+1250) \times 18}{(90-x) \times 5} = 135$$

where x is the speed of other train.

$$\therefore \frac{1000+1250}{135} \times \frac{18}{5} = 90 - x$$

$$60 = 90 - x$$

$$x = 30 \text{ km/hr}$$

30.(C) Let the length of the faster train be 'x' m.

The faster train has passed the driver of the slower train in 16 seconds, then

$$(65-29) \times \frac{5}{18} = \frac{x}{16}$$

$$x = 160 \text{ m}$$

31.(D) Let the speed of faster train be S_1 m/s and that of slower train by S_2 m/s.

$$\text{then, } S_1 + S_2 = \frac{240}{4} = 60$$

$$\text{and } S_1 - S_2 = \frac{240}{12} = 20$$

$$\text{So, } S_1 = 40 \text{ m/s and } S_2 = 20 \text{ m/s}$$

$$S_1 = 40 \times \frac{18}{5} = 144 \text{ km/hr}$$

32.(D) Let the speed of train be x km/hr.

As both the person are walking in the same direction of train.

$$\text{So, } (x - 4.5) \times 8.4 = (x - 5.4) \times 8.5$$

$$0.1x = 8.1$$

$$x = 81 \text{ km/hr}$$

33.(A) Let the length of platform be x m, length of first train be y m and length of second train be $y/2$. As both trains are travelling in opposite direction.

$$\text{So, } y + \frac{y}{2} = (48+42) \times \frac{5}{18} \times 12$$

$$\Rightarrow \frac{3}{2}y = 300$$

$$y = 200 \text{ m}$$

$$\text{Now, } y + x = 48 \times \frac{5}{18} \times 45 = 600$$

$$x = 600 - 200 = 400 \text{ m}$$

34.(A) Let the speeds of two trains be x and y km/hr respectively. So, if trains are travelling in opposite direction

$$21 = \frac{230+190}{x+y}$$

$$\Rightarrow x + y = 20 \quad \dots (1)$$

If they are travelling in same direction

$$42 = \frac{230+190}{x-y}$$

$$\Rightarrow x - y = 10 \quad \dots (2)$$

Solving these two equations, we get

$$x = 15 \text{ km/hr and}$$

$$y = 5 \text{ km/hr}$$

$$\text{So, } \frac{x}{y} = 3:1$$

35.(A) Let S_1 and S_2 be the speed of two trains where $S_1 > S_2$.

$$\text{Then, } \frac{32+56}{3} = S_1 + S_2$$

$$\frac{32+56}{9} = S_1 - S_2$$

$$S_1 + S_2 = \frac{88}{3}$$

$$S_1 - S_2 = \frac{88}{9}$$

$$2S_1 = \frac{88}{3} + \frac{88}{9}$$

$$2S_1 = 88 \left(\frac{9+3}{9 \times 3} \right)$$

$$2S_1 = \frac{88 \times 12}{9 \times 3}$$

$$S_1 = \frac{176}{9} \text{ yards/sec.}$$

$$\text{Same as } S_2 = \frac{88}{9} \text{ yards/sec.}$$

- 36.(A)** Let the speed in upstream be S_U and the speed in downstream be S_D .

$$\text{Then, } \frac{30}{S_U} + \frac{44}{S_D} = 10 \quad \dots (1)$$

$$\text{and } \frac{40}{S_U} + \frac{55}{S_D} = 13 \quad \dots (2)$$

$$\text{Let } x = \frac{1}{S_U} \text{ and } y = \frac{1}{S_D}$$

$$\text{So, } 30x + 44y = 10, 40x + 55y = 13$$

Now, by solving these equations we get

$$S_U = 5 \text{ and } S_D = 11$$

$$\text{Speed of current} = \frac{S_D - S_U}{2}$$

$$= \frac{11 - 5}{2} = 3 \text{ km/hr}$$

Ans.(37-39) The ratio of the speeds of two swimmer is 3 : 2, but the ratio of the distance covered by them during the first meeting is 4 : 1. It means the person with greater speed is in the downstream motion.

Let the speed of the stream is x m/s.



Time is constant

$$\frac{S_1}{S_2} = \frac{D_1}{D_2}$$

$$\frac{6+x}{4-x} = \frac{4}{1} \text{ or } 6+x = 16-4x$$

$$\text{or } 5x = 10 \Rightarrow x = 2$$

Now, the speed of the two swimmers be $6 + 2 = 8$ m/s and $4 - 2 = 2$ m/s

$$\text{Time for first meeting} = \frac{100}{8+2} = 10 \text{ s}$$

Now, for the second meeting, the swimmer started from A will reach B after 12.5 sec.

$\left(\frac{100}{8}\right)$ and reverse the direction and will travel at the speed of 4 m/s ($6 - 2$) (upstream motion). The other swimmer in 12.5 sec. will travel 25 m (12.5×2). The distance between them now is 25 m. Now,

the second meeting will be after $\frac{25}{4-2}$ i.e. 12.5 s. So, the second meeting time after them start is $(12.5 + 12.5)$ i.e. 25 s.

37.(A)

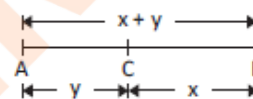
38.(A)

39.(D)

40.(C) Let the speed of motorboat in still water = x

Speed of river (speed of raft) = y

So, speed of motorboat in downstream = $x + y$



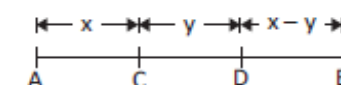
In 1 hr the motorboat will travel $x + y$ and the raft will travel y .

Now, the motorboat reverse the direction

So, the speed became $x - y$.

Now, the motorboat and the raft will meet

once again at point D after $\frac{x}{y+x-y} = 1$ hr



$$\text{Now, } 2y = 6$$

$$\text{So, } y = 3 \text{ km/h}$$

41.(D) Let x be upstream speed, then the downstream speed will be $(x + 3)$.

$$\frac{3}{x} + \frac{3}{x+3} = 3$$

$$\text{So, } x^2 + 3x - 1 = 0$$

$$\frac{3}{x} + \frac{3}{x+3} = 3$$

$$= \frac{-3+3.6}{2} = 0.3 \text{ km/hr}$$

$$(x+3) = 3.3 \text{ km/hr}$$

- 42.(B)** Time taken by stream (downstream) = 40 minutes

Time taken by stream (upstream) = 60 minutes

Time taken by boat (downstream) = 60 minutes

Time taken by boat (upstream) = 90 minutes

Total time = Time by stream (downstream) + $\frac{1}{2}$ time by stream (upstream) + $\frac{1}{2}$ time by boat (upstream)

$$= 40 + \frac{60}{2} + \frac{90}{2} = 115 \text{ min.}$$

- 43.(B)** Let the speed of current = x metres/minute

$$\text{Now, } \frac{200}{48-x} - \frac{200}{48+x} = 10$$

Then, put the options and now get $x = 32$ metres/minute

- 44.(A)** Let the upstream speed be u and downstream speed be d .

$$\text{Then, } \frac{15}{u} + \frac{22}{d} = 5$$

$$\text{Then, } \frac{15}{u} + \frac{22}{d} = 5$$

On solving above equations,

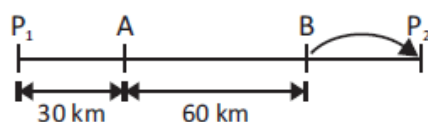
$$u = 5 \text{ km/h}$$

$$d = 11 \text{ km/h}$$

$$\text{Speed of stream} = \frac{d-u}{2}$$

$$= 3 \text{ km/hr}$$

- 45.(B)**



Let original speed = $3x$ kmph

After accident = $2x$ kmph

If accident occurred at point A then time taken by train = $t + \frac{3}{4}$

If accident occurred at point B then time taken by train = $t + \frac{1}{4}$

Given that time difference from A to B.

$$\frac{60}{2x} - \frac{60}{3x} = \frac{3}{4} - \frac{1}{4}$$

$$60 \left(\frac{x}{6x^2} \right) = \frac{1}{2}$$

$$x = 20$$

So, original speed = 60 kmph

Take accident happened at point B.

Then, speed ratio = 2 : 3

Time ratio = 3 : 2

Given that difference of time = 30 minutes

Real time = 30 minutes

So, distance from B to P_2 covered in 30 minutes. So, distance = 30 km

Total distance = 120 km

- 46.(D)** Let the speed of the superfast train be 4 unit, hence speed of passenger train would be 1 unit.

$$\text{Average speed} = \frac{2 \times 4 \times 1}{4+1}$$

$$= 1.6 \text{ units}$$

Since, train N is already late by 20 min, hence available time would be $(60 - 20) = 40$ min or $\frac{2}{3}$ h. If train has to reach the station at schedule time. Now, average speed would be $\frac{3}{2} \times 1.6 = 2.4$ units.

Now, given that new speed of superfast train = 8 unit

Let new speed of passenger train be y , then

$$= \frac{2 \times 8 \times y}{8+y} = 2.4$$

$$= y = 1.4$$

Hence, required ratio = 1.4 : 8

= 1 : 6 (approx.)

- 47.(A) Time taken by Y to cover

$$60 \text{ km} = \frac{60}{50} = \frac{6}{5} \text{ h}$$

Time taken by Y at station C

$$= \frac{15}{60} = \frac{1}{4} \text{ h}$$

Now, distance travelled by train X in

$$\left(\frac{6}{5} + \frac{1}{4}\right) = \frac{29}{20} \text{ h}$$

$$= 70 \times \frac{29}{20} = 101.5 \text{ km}$$

Distance between X and Y when Y starts from station C

$$= 180 - (101.5 + 60) = 18.5 \text{ km}$$

Hence, time taken by them in crossing one another

$$= \frac{18.5}{120} = 0.15 \text{ h}$$

Now, distance travelled by X in 0.15 h

$$= 70 \times 0.15 = 10.5 \text{ km}$$

Therefore, distance of C from station A, where they meet

$$= 112 \text{ km}$$

- 48.(D) Candle 1 burns at rate 1 cm/hr.
Candle 2 burns at rate 1.5 cm/hr.
Let us assume they are of same length after 't' hour.

So,

$$6 \text{ cm} - (1 \times t) = 8 \text{ cm} - (1.5 \times t)$$

$$1.5t - t = 8 - 6$$

$$0.5t = 2$$

$$t = 4 \text{ hours}$$

- 49.(B) The train saves 16 minutes by travelling faster over a section of 80 km.

$$\text{Thus, } \frac{80}{S} - \frac{80}{S+10} = \frac{16}{60}$$

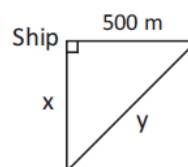
$$= \frac{80(S+10) - 80S}{S(S+10)} = \frac{4}{15}$$

$$S = 50 \text{ km/hr}$$

- 50.(C) Nikhil saved 20 minutes in total, so he must have saved 10 minutes on either side of journey. Now, his children got picked 10

minutes early means at 4 p.m. – 10 minutes = 3:50 p.m. Now the school got over at 3:00 p.m., so they were walking for 50 minutes.

- 51.(A)



Ship is moving at 8.33 m/s.

Time taken to go 500 m

$$= \frac{500}{8.33} = 60 \text{ sec.}$$

In 60 seconds, Radio wave travels = $60 \times 200 = 12000 \text{ m}$

$$\text{So, } x + y = 12000 \text{ m} \quad \dots (1)$$

$$\text{and } y^2 = x^2 + (500)^2 \quad \dots (2)$$

Solving equation (1) and (2),

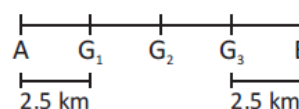
$$x = \frac{\sqrt{143}}{2} \text{ km}$$

- 52.(C)

$$\text{Distance } AG_1 = BG_3$$

$$= 30 \times \frac{5}{60} = 2.5 \text{ km}$$

$$\text{Distance } G_1G_3 = (20 - 2.5 - 2.5) = 15 \text{ km}$$



$$\text{Given, } G_1G_2 : G_2G_3 = 1 : 2$$

$$G_1G_2 = 5 \text{ m and } G_2G_3 = 10 \text{ km}$$

Now, time taken from reaching A to G_3 and back to A

From, A to $G_1 = 5 \text{ min}$ (given).

From G_1 to G_3

$$= \frac{15}{60} \times 60 = 15 \text{ min.}$$

From G_3 to A

$$= \frac{17.5}{60} \times 60 = 17.5 \text{ min.}$$

and time elapsed for taking the patient into and out of the ambulance = 1 min.

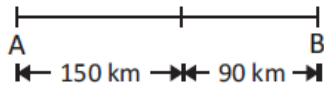
Total time taken

$$= (5 + 15 + 17.5 + 1) = 38.5$$

$$\text{Remaining time} = (40 - 38.5)$$

$$= 1.5 \text{ min.}$$

53.(B)



$$\text{Total distance} = AB$$

$$= 150 \times \frac{100}{62.5} \text{ km} = 240 \text{ km}$$

Let t be the required time and s be the speed.

$$s = \frac{240}{t} \quad \dots (1)$$

Total time required to reach B,

$$= t - \frac{7}{60} = \frac{150}{s} + \frac{8}{60} + \frac{90}{4/3s}$$

[Using equation (1)]

$$\frac{240}{s} - \frac{150}{s} - \frac{270}{4s} = \frac{15}{60}$$

$$\frac{90}{4s} = \frac{1}{4}$$

$$s = 90$$

Initial speed of motorcycle

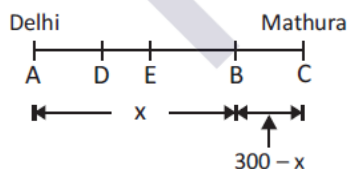
$$s = 90 \text{ km/hr}$$

54.(C) Average Speed

$$= \frac{150 + 90}{\frac{150}{90} + \frac{90}{120} + \frac{8}{60}}$$

$$= \frac{240 \text{ km}}{\frac{153}{60} \text{ hr}} = 94.11 \text{ km/hr}$$

55.(B)



Let Mukesh take Suresh on his bike till B and leave him there to walk till C (Mathura). In the meanwhile, Dinesh keeps walking to reach D, Mukesh comes back picks Dinesh and then both ride to Mathura.

When Mukesh comes back, let us say he meets Dinesh at E.

Let $AB = x$, then $BC = 300 - x$

Since Dinesh walks at 15 kmph and bike's speed is 60 kmph, we have

$$AD = \frac{x}{4}$$

$$\therefore BD = \frac{3x}{4}$$

$$\therefore BE = \left(\frac{60}{75} \right) \left(\frac{3x}{4} \right) = \frac{3x}{5}$$

Hence,

$$\frac{300 - x}{15} = \frac{\frac{3x}{5} + \frac{3x}{5} + 300 - x}{60}$$

On solving,

$$x = \frac{1500}{7} \text{ km}$$

Hence, minimum time

$$= \frac{x}{60} + \frac{300 - x}{15}$$

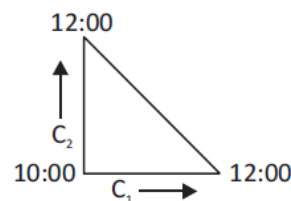
$$= \frac{1500}{60 \times 7} + 20 - \frac{1500}{15 \times 7}$$

$$= \frac{25}{7} + 20 - \frac{100}{7}$$

$$= 20 - \frac{75}{7}$$

$$= \frac{65}{7} = 9\frac{2}{7} \text{ hours}$$

56.(C)



The distance between the cyclists can be represented by the hypotenuse. Using the 3, 4, 5 Pythagoras triplet and the condition that the two speeds are 6 km/hr different from each other, we will get the triplet as: 18, 24, 30. Hence, the slower cyclist travelled at 18 km/hr.

- 57.(B)** Let speed of train be x km/hr and speed of car be y km/hr.

According to the question,

$$\frac{120}{x} + \frac{480}{y} = 480 \quad \dots (1)$$

$$\frac{200}{x} + \frac{400}{y} = 500 \quad \dots (2)$$

Dividing (1) by (2),

$$\frac{120 \left(\frac{1}{x} + \frac{4}{y} \right)}{100 \left(\frac{2}{x} + \frac{4}{y} \right)} = \frac{480}{500}$$

$$\frac{y + 4x}{2y + 4x} = \frac{4}{5}$$

$$5y + 20x = 8y + 16x$$

$$4x = 3y$$

$$\frac{x}{y} = \frac{3}{4}$$

$$\frac{\text{Speed of train}}{\text{Speed of car}} = \frac{3}{4}$$

- 58.(C)** Time for first at starting point.

LCM of (4 and 7) = 28 minutes

- 59.(A)** Let the circular track be D .

Speed of Bhim and Arjun be $\frac{D}{4}$ and $\frac{D}{7}$.

They will meet after

$$\frac{\frac{D}{4} - \frac{D}{7}}{\frac{D}{4} - \frac{D}{7}} = \frac{\frac{D}{7 - 4}}{\frac{D}{28}} = \frac{28}{3} \text{ min.}$$

i.e. $9\frac{1}{3}$ min. or 9 min. 20 sec.

- 60.(D)** They will never meet at a point diametrically opposite to the starting point. Because they will always meet at $\frac{1}{3}$ rd part of the track.

- 61.(A)** Speed of Bhim = 7 m/min., Speed of Arjun = 4 m/min.

$$\text{Lead} = 4 \text{ m/min} \times 4 \text{ min.} = 16 \text{ m}$$

$$\text{Time to meet} = \frac{\text{lead}}{\text{relative speed}}$$

$$= \frac{16}{7-4} = \frac{16}{3} \text{ min.}$$

- 62.(D)** Time taken by them to meet for the first time

$$= \frac{600}{30-20} = 60 \text{ sec.}$$

Time taken to meet 2nd time

$$= 2 \times 60 = 120 \text{ sec.}$$

Distance travelled by Raman in 120 seconds

$$= 120 \times 30 = 3600 \text{ m}$$

- 63.(A)** Distance travelled by A in half an hour = 3 circuits

So, 1 hour = 6 circuits/hr

By B = 9 circuits/hr

Relative speed = 3 circuits/hr

Difference between A and B

$$= 0.5 \text{ circuit}$$

$$\text{Now, time} = \frac{0.5}{3} = 10 \text{ minutes}$$

- 64.(D)** To coincide, At 3'o clock minute hand to travel for 15 min to cover 90° .

Degrees covered by minute hand in one

$$\text{minute} = \frac{360}{60} = 6^\circ$$

$$\text{Relative speed} = 6 - \frac{1}{2} = \left(5\frac{1}{2} \right)^\circ$$

By keeping hour hand on hold,

$$\text{Required time} = \frac{90 \times 2}{11}$$

$$= 16\frac{4}{11} \text{ minutes.}$$

- 65.(D)** To be in a straight line, the minute hand has to travel just 30° .

$$\text{Relative speed} = 5\frac{1}{2}^\circ$$

By keeping hour hand on hold,

$$\text{Required time} = \frac{30 \times 2}{11}$$

$$= 5\frac{5}{11} \text{ minutes}$$

- 66.(A)** Time from 12 p.m. on Monday to 2 p.m. on the following Monday = 7 days 2 hours = 170 hours

∴ The watch gains $\left(2 + 4\frac{4}{5}\right)$ min.

or $\frac{34}{5}$ min. in 170 hr.

Now, $\frac{34}{5}$ min. are gained in 170 hrs

∴ 2 min. are gained in

$$\left(170 \times \frac{5}{34} \times 2\right) \text{ hrs} = 50 \text{ hrs}$$

So, the watch is correct 2 days 2 hrs after 12 p.m. on Monday i.e. it will be correct at 2 p.m. on Wednesday.

- 67.(D)** Every day, the time gap between the two clocks becomes 15 minutes. When the gap between them becomes 24 hrs then the two watches will show same time.
[An argument runs in this type of questions that the gap should be of 12 hours. But note that the time mentioned is 3:00 p.m. and not 3 o'clock. When the time mentioned is 3:00 p.m., that means that the difference

between 3:00 p.m. and 3:00 a.m. is important, whereas when 3 o'clock is mentioned, that difference is immaterial and hence a gap of 12 hours is taken.]

To create a gap of 15 minutes, it takes 1 day. To create a gap of 24 hours, it will take

$$= \frac{1 \times 24 \times 60}{15} = 96 \text{ days}$$

- 68.(D)** To show the correct time again, watch must create 24 hours difference.

So, the required time

$$= \frac{4}{3} \times \frac{60 \times 24}{24} = 80 \text{ days}$$

- 69.(D)** The clock starts by showing correct time and after every 24 hours and hence $(n + 1)$ times in n days.

So, it will show correct time 8 times in 7 days.

Algebra

10

Algebra is the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations. In this chapter, you will go through the concepts of quadratic equations, polynomials, functions, maxima & minima and series & progression.

ALGEBRAIC EXPRESSION

The various combinations of variables and constants form an algebraic expression. For example let 'x' be any variable and '5' be a constant.

Then $5x$, $x/5$, x^5 , $x - 5$, $5 - x$ etc. are all algebraic expressions.

POLYNOMIAL

An expression of the form

$$f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$$

where $a_0, a_1, a_2, \dots, a_n$ are constants ($a_0 \neq 0$), and n is a positive integer, is called a polynomial in x of degree n .

The polynomial $f(x) = 0$ is called an **algebraic equation of degree n** .

The value of x which satisfies $f(x) = 0$ is called its root. Geometrically, a root of a equation is that value of x where the graph of $y = f(x)$ crosses the x -axis. The process of finding the roots of an equation is known as solution of that equation.

General properties of Polynomials

- If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ and vice versa.

For instance, 3 is a root of the equation $x^4 - 6x^2 - 8x - 3 = 0$, because $x = 3$ satisfies this equation.

$\therefore x - 3$ divides $x^4 - 6x^2 - 8x - 3$ completely, i.e. $x - 3$ is its factor.

- Every equation of the n^{th} degree has n roots (real or imaginary).

Conversely, if $\alpha_1, \alpha_2, \dots, \alpha_n$ are the roots of the n^{th} degree equation $f(x) = 0$, then

$f(x) = A(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n)$ where A is a constant.

Example 1: Find the remainder when $x^{100} + x + 2$ is divided by $x - 1$.

Solution:

$$\text{Let } f(x) = x^{100} + x + 2$$

$$f(1) = 1^{100} + 1 + 2 = 4. \text{ So, the remainder is 4.}$$

Example 2: For what value of p , is the expression $px^4 - 3x^3 + 4x^2 - 5x + 6 = 0$, divisible by $x - 1$?

Solution:

$$\text{Let } f(x) = px^4 - 3x^3 + 4x^2 - 5x + 6$$

$$f(1) = p(1)^4 - 3(1)^3 + 4(1)^2 - 5(1) + 6$$

$$= p + 2$$

For this expression to be divisible by $x - 1$,

$$f(1) = 0$$

$$\Rightarrow p + 2 = 0$$

$$\Rightarrow p = -2$$

EQUATION

Equation is defined as a mathematical statement of equality. If the equality is true for a certain value of the variable involved, the equation is often called a conditional equation and equality sign '=' is used.

For example,

$$\frac{3x+2}{7} = 5 \text{ holds true only for } x = 11.$$

So, it is a conditional equation.

Determination of value of the variable which satisfies an equation is called solution of the equation or finding root of the equation.

Degree of an equation

It is the highest or maximum sum of all the powers of a variable in an algebraic equation.

For example,

$$x^3 + 5x^2 + 6x + 1 = 0$$

In this equation, the maximum power of the variable i.e. x is '3'. Hence its degree is 3.

$$\Rightarrow x^3 y^2 z^3 + x^4 y^2 z + x^3 y^3 z^1 + 3xyz + 9 = 0$$

The degree of the above expression is power of x + power of y + power of z of $x^3 y^2 z^3$ i.e. $3 + 2 + 3 = 8$ as this is the maximum sum of all the power of variable in this expression.

LINEAR EQUATION

An equation in which the highest power of the variable is one is called a linear (or a simple) equation. This is also called as the equation of degree one.

Linear Equation in one variable

A linear equation in one variable x is of the form $ax + b = 0$, where a, b are known constants and $a \neq 0$. It has only one root.

For example, $x - 5 = 10$, $3x + 2 = 7$, $5x - 5 = 25$, etc.

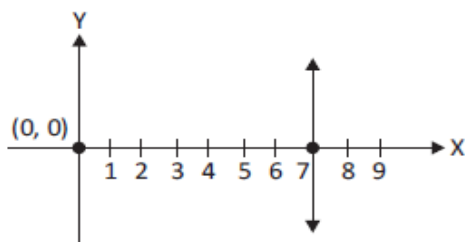
▪ **Graph of linear equation in one variable**

Consider the equation $2x - 9 = 5$

$$2x = 14 \Rightarrow x = 7$$

So, there is only one value i.e. $x = 7$ which satisfies the given equation. Hence, the graph of $x = 7$ will be nothing but a straight line parallel to y -axis and 7 units from the origin.

So, the solution of the linear equation when plotted on the $X - Y$ plane will be represented by the given line parallel to y -axis as given below.



Example 3: Find a number such that the difference between 8 times the number and 3 times the number is 55.

Solution:

Let the number be x .

$$8x - 3x = 55$$

$$x = 11$$

So, the number is 11.

Example 4: A father is now 3 times as old as his son. 5 years ago, he was 4 times as old as his son. Find their present ages.

Solution:

If ' x ' be the son's present age, then $3x$ is the father's present age.

$$5 \text{ years ago, son's age} = x - 5.$$

$$5 \text{ years ago, father's age} = 3x - 5$$

$$\therefore 3x - 5 = 4(x - 5)$$

$$\therefore 3x - 5 = 4x - 20$$

$$\therefore x = 15$$

$$\therefore 3x = 45$$

Hence, son's present age is 15 years and father's is 45 years.

Example 5: A man rode a distance of 61 miles in 7 hours. For a part of the time, he rode at 8 mph and the rest he rode at 10 mph. For how long did he ride at 8 mph?

Solution:

Let ' x ' be the required time in hours during which he rode at the rate of 8 mph.

Then, he rode for $(7 - x)$ hours at 10 mph.

$$8x + (7 - x)10 = 61$$

$$\Rightarrow 8x - 10x = 61 - 70$$

$$\therefore x = 4\frac{1}{2} \text{ hours}$$

So, he rode for $4\frac{1}{2}$ hours at 8 mph.

Linear Equation in two variables

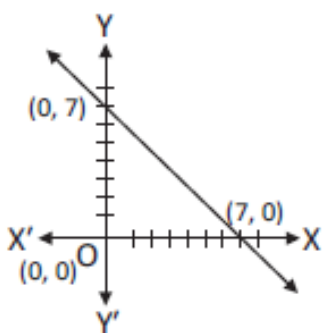
The general form of a linear equation in two variables x and y is $ax + by + c = 0$ where a and b are non-zero coefficients and c is a constant.

For example, $x + y = 7$ is a linear equation in two variables.

Each linear equation in two variables depicts a straight line and can be plotted on a graph. For $x + y = 7$, we can take different values of x and y that satisfies this equation and plot a graph.

x	0	1	2	-1	-2
y	7	6	5	8	9

The above values will give the following graph.



Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of **simultaneous equations** in x and y . A value for each unknown which satisfies both the equations simultaneously is known as the solution of the equations.

▪ **How to solve simultaneous linear equation?**

Let's take two linear equations and solve them.

Solving means that we need to find the value of ' x ' and ' y ' for which they satisfy the equations.

$$x + y = 5 \quad \dots (1)$$

$$3x + 2y = 6 \quad \dots (2)$$

In equation (1), coefficient of ' x ' is 1 and in equation (2) coefficient of x is 3

We multiply (1) by 3 and equation (2) by 1, to make coefficients of x same in both the equation.

We get,

$$3x + 3y = 15 \quad \dots (3)$$

$$3x + 2y = 6 \quad \dots (4)$$

Now, subtracting (4) from (3), we get

$$3x + 3y - 3x - 2y = 15 - 6$$

$$\therefore y = 9$$

Putting $y = 9$ in (1), we get

$$x + 9 = 5$$

$$\therefore x = 5 - 9 = -4$$

Hence, $x = -4$ and $y = 9$ is the solution.

Note:

Root and the solution of the equation are one and the same thing.

Example 6: Solve the equation.

$$2x + 7y - 11 = 0$$

$$5x - 3y - 7 = 0$$

Solution:

$$2x + 7y = 11 \quad \dots (1)$$

$$5x - 3y = 7 \quad \dots (2)$$

Equation (1) is multiplied by 5 and equation (2) is multiplied by 2 and subtracting equation (2) from equation (1).

$$41y = 41$$

$$\therefore y = 1$$

Now, putting $y = 1$ in equation (1), we get

$$\therefore x = 2$$

So, $x = 2$ and $y = 1$ is the solution.

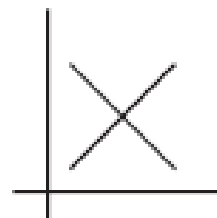
▪ **Graphical representation of two simultaneous equations**

For two simultaneous equations

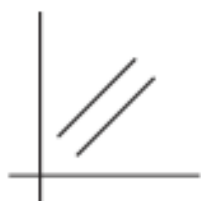
$$a_1x + b_1y + c_1 = 0 \text{ and}$$

$$a_2x + b_2y + c_2 = 0$$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the equations are **consistent** and have a unique solution for x and y . The lines shown by the two equations are intersecting.



2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the equations are **inconsistent**. The lines are parallel to each other and there is no solution.



3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the equations are **dependent**. The lines are coincident and there will be infinite solutions as both the equations are basically one and the same or they intersect each other at infinite points.



Example 7: For what value of K , the equations $2x - 7y = 4$ and $3x + Ky = 3$ will have a unique solution?

Solution:

$$2x - 7y = 4$$

$$3x + Ky = 3$$

For a unique solution,

$$\frac{2}{3} \neq \frac{-7}{K}$$

$$\text{So, } K \neq \frac{-21}{2}$$

Example 8: Solve for x and y :

$$5x + 9y = 17, 15x + 27y = 51$$

Solution:

First we check the ratio of coefficients of x and y .

And we get,

$$\frac{5}{15} = \frac{9}{27} = \frac{17}{51}$$

$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

$\Rightarrow$ The two equations are dependent, the lines are coincident and there will be infinitely many solutions.

Note:

We can also make out this from the equations, where we get the second equation and simply by multiplying the first by '3'. Both the equations are basically the same.

Example 9: Solve for m and n :

$$41m - 47n = 35; 47m - 41n = 53$$

Solution:

$$41m - 47n = 35 \quad \dots (1)$$

$$47m - 41n = 53 \quad \dots (2)$$

Adding (1) and (2),

$$88m - 88n = 88 \quad \dots (3)$$

Dividing (3) by 88,

$$m - n = 1 \quad \dots (4)$$

Subtracting (2) from (1),

$$-6m - 6n = -18 \quad \dots (5)$$

Dividing (5) by 6,

$$-m - n = -3 \quad \dots (6)$$

Adding (4) and (6),

$$-2n = -2$$

$$\therefore n = 1$$

Substituting the value of 'n' in equation (4),

$$m - 1 = 1$$

$$\therefore m = 2$$

So, $m = 2$ and $n = 1$ is the solution.

Example 10: Solve: $\frac{x}{3} + \frac{y}{4} = -\frac{1}{2}$ and $\frac{x}{3} - \frac{y}{4} = -\frac{3}{2}$

Solution:

$$\frac{x}{3} + \frac{y}{4} = -\frac{1}{2} \quad \dots (1)$$

$$\frac{x}{3} - \frac{y}{4} = -\frac{3}{2} \quad \dots (2)$$

Adding (1) and (2),

$$\frac{2x}{3} = -\frac{4}{2}$$

$$\therefore x = -3$$

Substituting the value of 'x' in equation (1),

$$-1 + \frac{y}{4} = -\frac{1}{2}$$

$$\frac{y}{4} = \frac{1}{2}$$

$$\therefore y = 2$$

So, $x = -3$ and $y = 2$ is the solution.

Example 11: Solve: $\frac{x-5}{3} = \frac{y+2}{7}$; $2x = 1 + 3y$

Solution:

$$\frac{x-5}{3} = \frac{y+2}{7}$$

$$7x - 35 = 3y + 6$$

$$7x - 3y = 41 \quad \dots (1)$$

$$2x = 1 + 3y$$

$$2x - 3y = 1 \quad \dots (2)$$

Subtracting (2) from (1),

$$5x = 40$$

$$\therefore x = 8$$

Substituting the value of 'x' in equation (2),

$$16 - 3y = 1$$

$$3y = 15$$

$$\therefore y = 5$$

So, $x = 8$ and $y = 5$ is the solution.

Method of solving linear equations in three variables

We will understand the method of solving three variable linear equations with the help of an example.

Example 12: Solve for x , y and z :

$$2x - y + z = 3, x + 3y + 2z = 7, 3x - 2y + 3z = 2$$

Solution:

$$2x - y + z = 3 \quad \dots (1)$$

$$x + 3y + 2z = 7 \quad \dots (2)$$

$$3x - 2y + 3z = 2 \quad \dots (3)$$

From equation (1) $\times 2$, we get

$$4x - 2y + 2z = 6 \quad \dots (4)$$

Now, equation (2) $-$ (4), we get

$$-3x + 5y = 1 \quad \dots (5)$$

From equation (1) $\times 3$, we get

$$6x - 3y + 3z = 9 \quad \dots (6)$$

Now, equation (6) $-$ (3), we get

$$3x - y = 7 \quad \dots (7)$$

Now, equation (5) and (7), we get

$$4y = 8$$

$$\therefore y = 2$$

Putting $y = 2$ in equation (5), we get

$$\therefore x = 3$$

Now, by putting the values of x and y in any of the given equations we get $z = -1$

So, $x = 3$, $y = 2$ and $z = -1$ is the solution.

Example 13: Solve for a , b and c :

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 5, \frac{2}{a} - \frac{3}{b} - \frac{4}{c} = -11, \frac{3}{a} + \frac{2}{b} - \frac{1}{c} = -6$$

Solution:

We put $u = \frac{1}{a}$, $v = \frac{1}{b}$, $w = \frac{1}{c}$ and get

$$u + v + w = 5 \quad \dots (1)$$

$$2u - 3v - 4w = -11 \quad \dots (2)$$

$$3u + 2v - w = -6 \quad \dots (3)$$

From equation (1) $+$ (3),

$$4u + 3v = -1 \quad \dots (4)$$

From equation (3) $\times 4$,

$$12u + 8v - 4w = -24 \quad \dots (5)$$

From equation (2) $-$ (5),

$$-10u - 11v = 13$$

$$\text{or } 10u + 11v = -13 \quad \dots (6)$$

From equation (4) $\times 11$,

$$44u + 33v = -11 \quad \dots (7)$$

From equation (6) $\times 3$,

$$30u + 33v = -39 \quad \dots (8)$$

From equation (7) $-$ (8),

$$14u = 28$$

$$\therefore u = 2$$

Putting $u = 2$ in equation (4),

$$4 \times 2 + 3v = -1$$

$$8 + 3v = -1$$

$$3v = -9$$

$$\therefore v = -3$$

Putting $u = 2$, $v = -3$ in equation (1),

$$2 - 3 + w = 5$$

$$-1 + w = 5$$

$$w = 5 + 1$$

$$\therefore w = 6$$

So, $a = \frac{1}{u} = \frac{1}{2}$, $b = \frac{1}{v} = \frac{1}{-3}$ and $c = \frac{1}{w} = \frac{1}{6}$ is the solution.

QUADRATIC EQUATION

An equation of the form $ax^2 + bx + c = 0$, where a, b, c are constants and $a \neq 0$ is called a quadratic equation. The numbers a, b, c are called the coefficients of this equation.

For example, $x^2 + 3x + 2 = 0$, $x^2 + 4 = 0$, $x^2 - 5x - 6 = 0$ are quadratic equations.

Roots and their nature

A root of the given quadratic equation is a complex number α such that $a\alpha^2 + b\alpha + c = 0$.

The discriminant 'D' of the quadratic equation $= b^2 - 4ac$

The roots of the given equation are given by the formula

$$x = \frac{-b \pm \sqrt{D}}{2a} \text{ or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 14: Find the roots of the equation $7p^2 - 5p - 2 = 0$.

Solution:

$$7p^2 - 5p - 2 = 0$$

$$a = 7, b = -5, c = -2$$

$$\therefore p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 7 \times (-2)}}{2 \times 7}$$

$$= \frac{5 \pm \sqrt{25 + 56}}{14} = \frac{5 \pm \sqrt{81}}{14} = \frac{5 \pm 9}{14}$$

$$p = \frac{5+9}{14} \text{ or } p = \frac{5-9}{14}$$

$$p = 1 \text{ or } p = -\frac{2}{7}$$

$\therefore$ The roots of $7p^2 - 5p - 2 = 0$ are $p = 1$ and $-\frac{2}{7}$.

Example 15: Find the roots of $x^2 + 7x + 12 = 0$.

Solution:

Now, sum of roots = -7

Product of the roots = $12 \times 1 = 12$

Which is only possible when the values are -3 and -4 .

$$\text{Hence, } x^2 + 3x + 4x + 12 = 0$$

$$x(x + 3) + 4(x + 3) = 0$$

$$(x + 4)(x + 3) = 0$$

Hence, $x = -4$ or $x = -3$ is the solution or the roots of the equation.

Example 16: Find the roots of $x^2 - 2x - 120 = 0$.

Solution:

Sum of the roots = 2

Product of the roots = -120

Which is only possible when the values are -10 and 12 .

$$\text{Hence, } x^2 + 10x - 12x - 120 = 0$$

$$x(x + 10) - 12(x + 10) = 0$$

$$(x - 12)(x + 10) = 0$$

Hence, $x = 12$ or $x = -10$ is the solution or the roots of the equation.

Example 17: Solve: $\frac{1}{4}(2x - 1)^2 = 4$

Solution:

$$(2x - 1)^2 = 16$$

$$2x - 1 = \pm 4$$

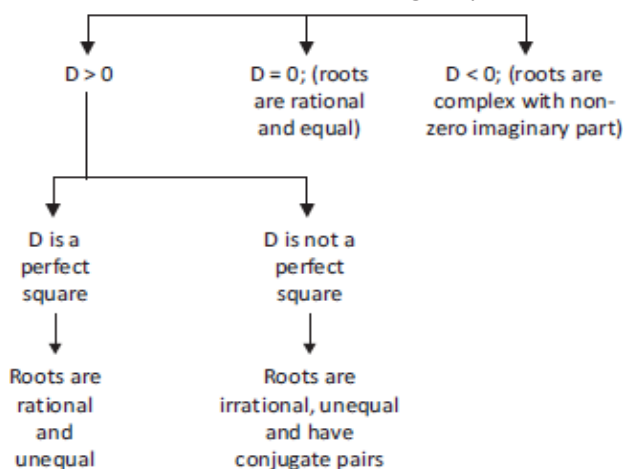
$$2x - 1 = 4 \text{ or } 2x - 1 = -4$$

$$x = \frac{5}{2} \text{ or } x = -\frac{3}{2}$$

For a quadratic equation $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$ and $D = b^2 - 4ac$.

I. If $D \geq 0$, then the roots are real.

II. If $D < 0$, then the roots are imaginary.



Example 18: Examine the nature of the roots of the following equations:

- I. $x^2 - 2x - 7 = 0$
- II. $x^2 - 8x + 7 = 0$
- III. $2x^2 + 5x + 21 = 0$

Solution:

I. Discriminant $(D) = (-2)^2 - 4(1)(-7)$
 $= 4 + 28 = 32$

As $D > 0$ and is not a perfect square; so the roots are real, distinct and irrational.

II. Discriminant $(D) = (-8)^2 - 4(1)(7) = 36$
 As $D > 0$ and is a perfect square.

So, the roots are real, distinct and rational.

III. Discriminant $(D) = (5)^2 - 4(2)(21)$
 $= 25 - 168 = -143$

As $D < 0$, so the roots are imaginary.

Sum and Product of the roots of a Quadratic Equation

Let α, β be the roots of a quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$ then

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = -\left(\frac{\text{coefficient of } x}{\text{coefficient of } x^2}\right) \text{ and}$$

$$\text{Product of roots} = \alpha \times \beta = \frac{c}{a} = \left(\frac{\text{constant term}}{\text{coefficient of } x^2}\right)$$

Sum of the roots	Product of the roots	Sign of the roots
+ve	+ve	Both the roots are positive
-ve	+ve	Both the roots are negative
+ve	-ve	The numerically larger root is positive and the other root is negative
-ve	-ve	The numerically larger root is negative and the other root is positive

Formation of Quadratic Equation from its roots

Suppose we have to form an equation whose roots are α and β . Since α and β are the roots of the equation, so $(x - \alpha) = 0$ and $(x - \beta) = 0$

$$\therefore (x - \alpha)(x - \beta) = 0$$

$$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{i.e. } x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0.$$

Example 19: Form a quadratic equation whose roots are 9 and 5.

Solution:

$$\text{Sum of the roots} = 9 + 5 = 14$$

$$\text{Product of the roots} = 9 \times 5 = 45$$

We know that the general form of a quadratic equation is:

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

$$\Rightarrow x^2 - 14x + 45 = 0$$

This is the required equation.

Example 20: The equation $x^2 - 3x + 1 = 0$ has α, β as its roots. Find the quadratic equation whose roots are:

I. α^4, β^4

II. α^3, β^3

III. $\alpha^2 + \beta, \beta^2 + \alpha$

IV. $\frac{4}{\alpha}, \frac{4}{\beta}$

Solution:

I. Now, $x^2 - 3x + 1 = 0$

$$\alpha + \beta = 3$$

$$\alpha\beta = 1$$

$$\Rightarrow (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 3^2$$

$$\Rightarrow \alpha^2 + \beta^2 = 9 - 2 = 7$$

$$\Rightarrow (\alpha^2 + \beta^2)^2 = 7^2$$

$$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 49$$

$$\Rightarrow \alpha^4 + \beta^4 = 47 \text{ and } \alpha^4 \times \beta^4 = 1$$

Since, the quadratic equation can be written as:

$$x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$$

$\therefore$ The equation will be

$$x^2 - 47x + 1 = 0$$

II. Sum of the roots

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$$

$$= 3(7 - 1)$$

$$= 18 \text{ and product of the roots } \alpha^3 \times \beta^3 = 1$$

$\therefore$ The equation will be

$$x^2 - 18x + 1 = 0$$

III. Sum of the roots $= \alpha^2 + \beta + \beta^2 + \alpha$

$$= 7 + 3 = 10$$

$$\text{Product of the roots} = (\alpha^2 + \beta)(\beta^2 + \alpha)$$

$$= \alpha^2 \beta^2 + \alpha^3 + \beta^3 + \alpha\beta$$

$$= 1 + 18 + 1$$

$$= 20$$

$$\therefore \text{The equation will be } x^2 - 10x + 20 = 0$$

IV. Product of the roots $= \frac{4}{\alpha} \times \frac{4}{\beta} = \frac{16}{\alpha\beta} = 16$

$$\text{Sum of the roots} = \frac{4}{\alpha} + \frac{4}{\beta} = \frac{4\alpha + 4\beta}{\alpha\beta}$$

$$= \frac{4(\alpha + \beta)}{\alpha\beta} = 12$$

$$\therefore \text{The equation will be } x^2 - 12x + 16 = 0$$

Example 21: If α and β are the roots of $x^2 - 7x + 12 = 0$, then find the quadratic equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Solution:

$$x^2 - 7x + 12 = 0$$

$$\alpha + \beta = 7 \text{ and } \alpha\beta = 12$$

$$\text{Sum of the roots} = (\alpha + \beta)^2 + (\alpha - \beta)^2$$

$$= 2(\alpha^2 + \beta^2)$$

$$= 2((\alpha + \beta)^2 - 2\alpha\beta)$$

$$= 2((7)^2 - 2(12))$$

$$= 2(49 - 24) = 50$$

$$(\text{As } 50 = (\alpha + \beta)^2 + (\alpha - \beta)^2)$$

$$\text{So, } (\alpha - \beta)^2 = 50 - 49 = 1$$

$$\text{Product of the roots} = (\alpha + \beta)^2 \times (\alpha - \beta)^2$$

$$= (7)^2 \times (1)$$

$$= 49$$

$$\text{So, the equation is } x^2 - 50x + 49 = 0$$

Example 22: One root of $x^2 + kx - 8 = 0$ is the square of the other. Then, the value of k is:

Solution:

If the roots are a and a^2 , then product of the roots $= a^3 = -8$.

$$\therefore a = -2. \text{ Hence, sum of the roots} = k = -(a + a^2)$$

$$= -(-2 + 4) = -2$$

Example 23: Ujagar and Keshab attempted to solve a quadratic equation. Ujagar made a mistake in writing down the constant term. He ended up with the roots $(4, 3)$. Keshab made a mistake in writing down the coefficient of x . He got the roots $(3, 2)$. What will be the exact roots of the original quadratic equation?

Solution:

First roots are $(4, 3)$

$$\text{Sum of the roots} = \frac{-b}{a} = -7 \Rightarrow \frac{b}{a} = 7$$

$$\text{Product of the roots} = \frac{c}{a} = 12 \Rightarrow \frac{c}{a} = 12$$

$$\therefore \text{Equation formed } x^2 - 7x + 12 = 0 \quad \dots (1)$$

The other boy gets the wrong roots $(3, 2)$.

$$\text{Sum of the roots} = \frac{-b}{a} = -5 \Rightarrow \frac{b}{a} = 5$$

$$\text{Product of the roots} = \frac{c}{a} = 6 \Rightarrow \frac{c}{a} = 6$$

$$\therefore \text{Equation formed } x^2 - 5x + 6 = 0 \quad \dots (2)$$

$$\therefore \frac{c}{a} = 6$$

$$\frac{b}{a} = 7$$

and $\frac{b}{a}$ are the right coefficients.

Hence, the correct equation is $x^2 - 7x + 6 = 0$.

$$\Rightarrow x^2 - 6x - x + 6 = 0$$

$$\Rightarrow x(x - 6) - 1(x - 6) = 0$$

$$\Rightarrow (x - 6)(x - 1) = 0$$

$$\therefore x = 6, 1$$

Hence, the actual roots are $(6, 1)$.

Alternate Method:

Since, constant $= 6[3 \times 2]$ and

Coefficient of $x = [-4 - 3] = -7$

And quadratic equation is given by

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

$$\text{So, the equation formed will be: } x^2 - 7x + 6 = 0$$

Solving the equation, we get

$$(x - 6)(x - 1) = 0 \text{ or } x = (6, 1)$$

Constructing a new Quadratic Equation by changing the roots of a given Quadratic Equation

If we are given a quadratic equation, we can build a new quadratic equation by changing the roots of this equation in the manner specified to us.

For example, let us take a quadratic equation $ax^2 + bx + c = 0$ and let its roots be α and β respectively. Then we can build new quadratic equation as per the following patterns:

- I. A quadratic equation whose roots are m more than the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $(\alpha + m)$ and $(\beta + m)$

These can be obtained by substituting $(x - m)$ in place of x in the given equation.

- II. A quadratic equation whose roots are m less than the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $(\alpha - m)$ and $(\beta - m)$

These can be obtained by substituting $(x + m)$ in place of x in the given equation.

- III. A quadratic equation whose roots are the reciprocals of the roots of the given equation $ax^2 + bx + c = 0$, i.e., the roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

These can be obtained by substituting $\frac{1}{x}$ in place of x in the given equation giving us $cx^2 + bx + a = 0$, i.e., we get the required equation by interchanging the coefficients of x^2 and the constant term.

- IV. A quadratic equation whose roots are m times the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $m\alpha$ and $m\beta$.

These can be obtained by substituting $\frac{x}{m}$ in place of x in the given equation.

- V. A quadratic equation whose roots are $\frac{1}{m}$ times the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $\frac{\alpha}{m}$ and $\frac{\beta}{m}$.

These can be obtained by substituting ' mx ' in place of x .

Example 24: Form a quadratic equation whose roots are 6 more than the roots of $x^2 + 5x + 18 = 0$.

Solution:

We have to replace x by $(x - 6)$ to form the equation whose roots are 6 more than that of $x^2 + 5x + 18 = 0$.

$$\therefore \text{The equation will be } (x - 6)^2 + 5(x - 6) + 18 = 0$$

$$\Rightarrow x^2 + 36 - 12x + 5x - 30 + 18 = 0$$

$$\therefore x^2 - 7x + 24 = 0$$

Example 25: Form a quadratic equation whose roots are reciprocals of the roots of the equations $2x^2 + 7x + 12 = 0$.

Solution:

We have to replace x by $1/x$ to get the new quadratic equation.

$\therefore$ The equation will be

$$2\left(\frac{1}{x}\right)^2 + 7\left(\frac{1}{x}\right) + 12 = 0$$

$$\Rightarrow \frac{2}{x^2} + \frac{7}{x} + 12 = 0$$

$$\Rightarrow 2 + 7x + 12x^2 = 0$$

$$\therefore 12x^2 + 7x + 2 = 0$$

Example 26: Form a quadratic equation whose roots are one third of the roots of the equation $3x^2 + 8x + 7 = 0$.

Solution:

We have to replace ' x ' by ' $3x$ ' to get the new equation.

$\therefore$ The equation will be

$$3(3x)^2 + 8(3x) + 7 = 0$$

$$\therefore 27x^2 + 24x + 7 = 0$$

HIGHER DEGREE EQUATIONS

Cubic Equation

$$f(x) = ax^3 + bx^2 + cx + d = 0$$

Assume that the roots are α , β and γ .

$$\alpha + \beta + \gamma = -\left(\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}\right) = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}\right) = \frac{c}{a}$$

$$\alpha\beta\gamma = -\left(\frac{\text{Constant term}}{\text{Coefficient of } x^3}\right) = \frac{-d}{a}$$

Bi-Quadratic Equation

$$f(x) = ax^4 + bx^3 + cx^2 + dx + e = 0$$

Assume that the roots are α , β , γ and δ .

$$\alpha + \beta + \gamma + \delta = -\left(\frac{\text{Coefficient of } x^3}{\text{Coefficient of } x^4}\right) = \frac{-b}{a}$$

$$\alpha\beta + \gamma\delta + \alpha\delta + \beta\gamma + \alpha\gamma + \delta\beta = \left(\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^4}\right) = \frac{c}{a}$$

$$\alpha\beta\gamma + \delta\alpha\beta + \gamma\delta\alpha + \beta\gamma\delta = -\left(\frac{\text{Coefficient of } x}{\text{Coefficient of } x^4}\right) = \frac{-d}{a}$$

$$\alpha\beta\gamma\delta = \left(\frac{\text{Constant term}}{\text{Coefficient of } x^4}\right) = \frac{e}{a}$$

Example 27: Solve the equation: $x^3 - 9x^2 + 11x + 21 = 0$

Solution:

We will try to find the first root of the equation by using the constant term. We find factors of 21 that are $\pm 1, \pm 3, \pm 7, \pm 21$.

By hit and trial we find that $x = -1$ satisfies the equation as $(-1)^3 - 9(-1)^2 + 11(-1) + 21 = -1 - 9 - 11 + 21 = 0$ so $x + 1$ is a factor of $x^3 - 9x^2 + 11x + 21$

Now, dividing $x^3 - 9x^2 + 11x + 21$ by $(x + 1)$, we get $x^2 - 10x + 21$

$$\text{So } x^3 - 9x^2 + 11x + 21 = 0$$

$$\text{or } (x + 1)(x^2 - 10x + 21) = 0$$

$$(x + 1)(x^2 - 3x - 7x + 21) = 0$$

$$\Rightarrow (x + 1)(x(x - 3) - 7(x - 3)) = 0$$

$$\Rightarrow (x + 1)(x - 3)(x - 7) = 0$$

So, the roots are $x = -1, 3$ or 7

Example 28: If α , β and γ are the roots of the equation $x^3 - 3x^2 + x + 1 = 0$ find the value of

I. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

II. $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$

III. $(\alpha + 1)(\beta + 1)(\gamma + 1)$

Solution:

$$\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 1 \text{ and } \alpha\beta\gamma = -1$$

I. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma} = \frac{1}{-1} = -1$

II. $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{(\alpha + \beta + \gamma)}{\alpha\beta\gamma} = \frac{3}{(-1)} = -3$

III. If α , β and γ are the roots of the equation, then:

$$f(x) = x^3 - 3x^2 + x + 1 = (x - \alpha)(x - \beta)(x - \gamma) = 0$$

$$f(-1) = -1 - 3 - 1 + 1 = (-1 - \alpha)(-1 - \beta)(-1 - \gamma)$$

$$\Rightarrow -4 = -(\alpha + 1)(\beta + 1)(\gamma + 1)$$

$$\text{So, } (\alpha + 1)(\beta + 1)(\gamma + 1) = 4$$

Example 29: If α , β and γ are the roots of the equation, $x^3 + 7x - 11 = 0$, then find the value of $\alpha^3 + \beta^3 + \gamma^3$

Solution:

As in the given equation the coefficient of x^2 is not present so $\alpha + \beta + \gamma = 0$

$$\text{Now, if } \alpha + \beta + \gamma = 0,$$

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

$$\text{Here, } \alpha\beta\gamma = 11$$

$$\text{So, } \alpha^3 + \beta^3 + \gamma^3 = 3(11) = 33$$

Example 30: Find all the roots of the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ if one root is $2 + \sqrt{3}$.

Solution:

All coefficients are real, irrational roots and so they will occur in conjugate pairs.

Hence, another root is $2 - \sqrt{3}$.

$$\therefore \text{Product of the roots} = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

$$= (x - 2)^2 - 3$$

$$= x^2 - 4x + 1$$

Dividing $x^4 + 2x^3 - 16x^2 - 22x + 7$ by $x^2 - 4x + 1$, then the other quadratic factor is $x^2 + 6x + 7$.

Then the given equation reduces to $(x^2 - 4x + 1)$

$$(x^2 + 6x + 7) = 0$$

$$\therefore x^2 + 6x + 7 = 0$$

$$\text{Then, } x = \frac{-6 \pm \sqrt{36 - 28}}{2} = -3 \pm \sqrt{2}$$

Hence, the roots are $2 \pm \sqrt{3}$, $-3 \pm \sqrt{2}$.

Example 31: $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ has roots 1, 2, 3 and 4; $P(0) = 48$, then what is $P(5)$?

Solution:

As, 1, 2, 3 and 4 are roots, we can write $P(x)$ as

$$P(x) = a(x-1)(x-2)(x-3)(x-4)$$

$$\text{Now, } P(0) = 48$$

$$\Rightarrow 48 = a(0-1)(0-2)(0-3)(0-4)$$

$$\Rightarrow 48 = 24a$$

$$\Rightarrow a = 2$$

$$\text{Now, } P(5) = a(5-1)(5-2)(5-3)(5-4)$$

$$P(5) = 24a$$

$$\text{So, } P(5) = 24 \times 2 = 48$$

ALGEBRAIC IDENTITIES

Algebraic identities refer to various formulas and expansions of various degree equations. Let us see some basic identities:

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- $(a+b)^2 - (a-b)^2 = 4ab$
- $(a^2 - b^2) = (a-b)(a+b)$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $(a+b)^3 + (a-b)^3 = 2a^3 + 6ab^2$
- $(a+b)^3 - (a-b)^3 = 2b^3 + 6a^2b$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a+b)(b+c)(c+a) = [(a+b+c)(bc+ca+ab)] - abc$

- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- If $a+b+c=0$, then $a^3 + b^3 + c^3 - 3abc = 0$
- $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$
- If $x + \frac{1}{x} = a$, then $x^2 + \frac{1}{x^2} = a^2 - 2$
- If $x - \frac{1}{x} = a$, then $x^2 + \frac{1}{x^2} = a^2 + 2$
- If $x + \frac{1}{x} = a$, then $x^3 + \frac{1}{x^3} = a^3 - 3a$
- If $x - \frac{1}{x} = a$, then $x^3 - \frac{1}{x^3} = a^3 + 3a$
- If $x + \frac{1}{x} = a$, then $x^4 + \frac{1}{x^4} = a^4 - 4a + 2$
- If $x - \frac{1}{x} = a$, then $x^4 + \frac{1}{x^4} = a^4 + 4a + 2$

Example 32: If $a^2 - b^2 = 158$, $a^3 - b^3 = 79$ and $ab = 33$, then find the value of $(a+b)$.

Solution:

$$\frac{a^2 - b^2}{a^3 - b^3} = \frac{158}{79}$$

$$\therefore \frac{(a+b)(a-b)}{(a-b)(a^2 + ab + b^2)} = 2$$

$$\begin{aligned} \therefore (a+b) &= 2(a^2 + ab + b^2) \\ &= 2(a^2 + ab + b^2) + 2ab - 2ab \\ &= 2(a^2 + 2ab + b^2) - 2ab \\ \therefore (a+b) &= 2(a+b)^2 - 2ab \end{aligned}$$

Let $(a+b) = x$. Also, $ab = 33$.

Hence the above equation becomes a quadratic equation.

$$2x^2 - x - 66 = 0$$

$$\therefore (x-6)(2x+11) = 0$$

$$\therefore x = 6 \text{ or } x = -5.5$$

Hence, $(a+b) = 6$ or $(a+b) = -5.5$

Example 33: If $x - y = 1$, evaluate $x^3 - y^3 - 3xy$.

Solution:

$$\begin{aligned} x^3 - y^3 - 3xy \\ = x^3 - y^3 - 3xy(x-y) = (x-y)^3 \end{aligned}$$

As $(x - y) = 1$

$$\therefore (x - y)^3 = 1$$

Example 34: If $a + b + c = 10$, $ab + bc + ca = 30$, then what is the value of $a^2 + b^2 + c^2$?

Solution:

$$(a + b + c)^2 = 10^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 100$$

$$\therefore a^2 + b^2 + c^2 + 2 \times 30 = 100$$

$$\therefore a^2 + b^2 + c^2$$

$$= 100 - 60 = 40$$

Example 35: If $x^4 + \frac{1}{x^4} = 98$ and $x > 1$, then what is the

value of $x - \frac{1}{x}$?

Solution:

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\left(x - \frac{1}{x}\right)^2 + 2 = x^2 + \frac{1}{x^2}$$

$$\left[\left(x - \frac{1}{x}\right)^2 + 2\right]^2 = x^4 + \frac{1}{x^4} + 2$$

$$\left[\left(x - \frac{1}{x}\right)^2 + 2\right]^2 = 98 + 2$$

$$\left(x - \frac{1}{x}\right)^2 + 2 = 10$$

$$\left(x - \frac{1}{x}\right)^2 = 8$$

$$\left(x - \frac{1}{x}\right) = 2\sqrt{2}$$

Example 36: What is the value of

$$\frac{(a^2 + b^2)(a - b) - (a^3 - b^3)}{a^2b - ab^2}?$$

Solution:

$$\frac{(a^2 + b^2)(a - b) - (a - b)(a^2 + b^2 + ab)}{ab(a - b)}$$

$$\frac{(a - b) \left[(a^2 + b^2) - (a^2 + b^2 + ab) \right]}{ab(a - b)}$$

$$\therefore \frac{a^2 + b^2 - a^2 - b^2 - ab}{ab} = \frac{-ab}{ab} = -1$$

Example 37: If $x^2 + y^2 + z^2 = 14$ and $xy + yz + zx = 11$, then what is the value of $(x + y + z)^2$?

Solution:

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$= 14 + 2(xy + yz + zx)$$

$$= 14 + (2 \times 11)$$

$$= 36$$

Example 38: If $x^2 - 4x + 1 = 0$, then what is the value

of $\frac{x^6 + 1}{x^3}$?

Solution:

$$x^2 - 4x + 1 = 0$$

Dividing both sides by x , we get

$$\Rightarrow x - 4 + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 4$$

On cubing both sides, we get

$$\therefore \left(x + \frac{1}{x}\right)^3 = 4^3$$

$$\therefore x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 64$$

$$\therefore \frac{x^6 + 1}{x^3} + 3(4) = 64$$

$$\therefore \frac{x^6 + 1}{x^3} = 52$$

Example 39: If $x + \frac{1}{x} = 4$, then what is the value of

$$x^6 + \frac{1}{x^6}?$$

Solution:

$$x + \frac{1}{x} = 4$$

On squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 14$$

On cubing both sides, we get

$$\therefore \left(x^2 + \frac{1}{x^2}\right)^3 = 14^3$$

$$\therefore x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 2744$$

$$\therefore x^6 + \frac{1}{x^6} = 2744 - 3 \times 14$$

$$\therefore x^6 + \frac{1}{x^6} = 2702$$

Example 40: If $(x + y + z) = 12$, $xy + yz + zx = 44$ and $xyz = 48$, then what is the value of $x^3 + y^3 + z^3$?

Solution:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\text{Now, } (x + y + z)^2 = 12^2$$

$$\Rightarrow x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 144$$

$$\Rightarrow x^2 + y^2 + z^2 + 2(44) = 144$$

$$\Rightarrow x^2 + y^2 + z^2 = 144 - 88 = 56$$

$$\text{Now, } x^3 + y^3 + z^3 - 3(48) = 12(56 - 44)$$

$$\therefore x^3 + y^3 + z^3 = 144 + 144 = 288$$

Example 41: If $x + \frac{1}{x} = 2$, then what is the value of x^{13}

$$+ 3x^7 + 7x^3 - 8x + 1?$$

Solution:

$$x + \frac{1}{x} = 2, \text{ is possible only when } x = 1.$$

So, putting $x = 1$ in $x^{13} + 3x^7 + 7x^3 - 8x + 1$, we get

$$= (1)^{13} + 3(1)^7 + 7(1)^3 - 8(1) + 1 = 4$$

Example 42: If $x + \frac{1}{x} = -2$, then what is the value of

$$6x^3 + 2x^2 + 3x + 7 + \frac{8}{x} + \frac{2}{x^2}?$$

Solution:

$$x + \frac{1}{x} = -2, \text{ is possible only when } x = -1$$

So, putting $x = -1$ in $6x^3 + 2x^2 + 3x + 7 + \frac{8}{x} + \frac{2}{x^2}$, we get

$$\begin{aligned} &= 6(-1)^3 + 2(-1)^2 + 3(-1) + 7 + \frac{8}{(-1)} + \frac{2}{(-1)^2} \\ &= -6 + 2 - 3 + 7 - 8 + 2 = -6 \end{aligned}$$

SEQUENCES AND SERIES

An expression of the form $a_1 + a_2 + a_3 + \dots + a_n +$

..... which is the sum of the elements of the

sequence $\{a_n\}$ is called a Series. If the series contains

a finite number of elements, it is called a Finite

Series, otherwise it is called an Infinite Series.

If $S_n = a_1 + a_2 + a_3 + a_4 + \dots + a_n$, then S_n is called the sum of n terms (or the sum of the first n terms) of the series and the term sum is denoted by the Greek letter Σ .

Thus, $S_n = \Sigma a_n$

For example,

I. $1 + 3 + 5 + 7 + \dots$ is a series in which 1st term = 1, 2nd term = 3, and so on.

II. $2 - 4 + 8 - 16 + \dots$ is also a series in which 1st term = 2, 2nd term = -4, and so on.

Arithmetic Sequence or Arithmetic Progression (A.P.)

An Arithmetic Progression (A.P.) is a sequence in which the difference between any term and its immediate preceding term is constant throughout.

This constant is called the **common difference** of the A.P.

or

An Arithmetic Progression (A.P.) is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the **common difference** of the Arithmetic Progression.

If a sequence $\{t_1, t_2, t_3, \dots, t_n\}$ is such that $t_n - t_{n-1} =$ constant where $n \in \mathbb{N}$, then it is an Arithmetic Progression.

Note:

If a is the first term and d is the common difference then A.P. can be written as $a, a + d, a + 2d, \dots, a + (n - 1)d, \dots$

▪ **The n^{th} term of an A.P.**

Let a be the first term and d be the common difference of an A.P., then

$$1^{\text{st}} \text{ term} = a = a + (1 - 1)d$$

$$2^{\text{nd}} \text{ term} = a + d = a + (2 - 1)d$$

$$3^{\text{rd}} \text{ term} = a + 2d = a + (3 - 1)d$$

$$4^{\text{th}} \text{ term} = a + 3d = a + (4 - 1)d$$

.....

.....

.....

$$\therefore n^{\text{th}} \text{ term} = a + (n - 1)d$$

Hence, the n^{th} term of A.P., $T_n = a + (n - 1)d$

If the number of terms in an A.P. is n and the value of the last term is l , then similarly,

$$\text{Then, } l = T_n = a + (n - 1)d$$

1. n^{th} term of this A.P., $T_n = a + (n - 1)d$

$$\text{where } d = T_n - T_{n-1}$$

2. n^{th} term of this A.P. from last

$$T_n' = l - (n - 1)d, (l \text{ is the last term})$$

3. The sum of first n term of this A.P. is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l) \quad [l \text{ is the last term}]$$

$$4. T_n = S_n - S_{n-1}$$

Example 43: Find the 17^{th} term of the arithmetic progression 8, 5, 2, -1, -4,

Solution:

$$\text{Here, } a = 8, d = 5 - 8 = -3$$

$$t_n = a + (n - 1)d$$

$$\text{For } n = 17$$

$$t_{17} = 8 + (17 - 1)d$$

$$= 8 + (17 - 1)(-3)$$

$$= 8 + 16(-3)$$

$$= 8 - 48$$

$$= -40$$

Example 44: If 5^{th} and 12^{th} terms of an arithmetic progression are 14 and 35 respectively. Find the common difference and the first term.

Solution:

Let a be the first term and d be the common difference of A.P.

$$t_5 = a + 4d = 14$$

$$t_{12} = a + 11d = 35$$

On solving the above equations,

$$7d = 35 - 14$$

$$7d = 21$$

$$d = 3$$

$$\text{and } a = 14 - (4 \times 3) = 14 - 12 = 2$$

$$\text{So, } a = 2 \text{ and } d = 3$$

$$\therefore \text{A.P. is } 2, 5, 8, 11, 14, \dots$$

Example 45: Is -150 a term of the series 11, 8, 5, 2, ...?

Solution:

$$\text{Here, } a = 11, d = 8 - 11 = -3.$$

$$\text{Let } a_n = -150$$

$$\text{Therefore, } a + (n - 1)d = -150$$

$$11 + (n - 1)(-3) = -150$$

$$-3(n - 1) = -150 - 11 = -161$$

$$\text{So, } (n - 1) = \frac{-161}{-3} = 53\frac{2}{3}$$

As n cannot be a fraction, so we can say that -150 is not the term of the given series.

Geometric Sequence or Geometric Progression (G.P.)

A Geometric Progression (G.P.) is a sequence if the ratio of any term and its immediate preceding term is constant throughout. This constant factor is called the **common ratio** of the G.P. or

A Geometric Progression (G.P.) is a sequence of numbers, whose first term is non zero and each of the term is obtained by multiplying its immediate preceding term by a constant quantity. This constant quantity is called the **common ratio** of the G.P.

Thus if $T_1, T_2, T_3, \dots$ are in G.P., then the common ratio

$$r = \frac{T_n}{T_{n-1}}, n \in \mathbb{N}.$$

If a is the first term and r is the common ratio then G.P. can be written as: $a, ar, ar^2, \dots, ar^{n-1}, \dots (a \neq 0)$

1. n^{th} term of the G.P.

$$T_n = ar^{n-1}, \text{ where } r = \frac{T_n}{T_{n-1}}$$

2. n^{th} term of this G.P. from end $T_n' = \frac{l}{r^{n-1}}$ [l is the last term]
3. Sum of the first n terms of this G.P. is given by

$$S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{lr - a}{r - 1}, \text{ if } r \neq 1$$

Also, $S_n = na$ if $r = 1$

Example 46: If the fifth term of a GP is 243 and the third term is 81, find the first term of the series.

Solution:

Given that the fifth term of a GP is 243 and the third term is 81, then the first term is obtained from dividing both, we get

$$ar^2 = 81 \text{ and } ar^4 = 243$$

$$r^2 = 3$$

$$ar^2 = 81 \text{ or } a = 27$$

Example 47: Find the first term and the common ratio of the geometric progression where 4th term is 8

$$\text{and } 8^{\text{th}} \text{ term is } \frac{128}{625}.$$

Solution:

Let a be the first term and r be the common ratio.

$$\text{From the question, we have } t_4 = 8 \text{ and } t_8 = \frac{128}{625}$$

$$\text{So, } ar^3 = 8 \text{ and } ar^7 = \frac{128}{625}$$

$$\frac{ar^7}{ar^3} = \frac{128}{8 \times 625}$$

$$r^4 = \frac{16}{625}$$

$$r = \frac{2}{5}$$

$$\text{Now, } a \times \frac{8}{125} = 8$$

$$\therefore a = 125$$

$$\text{So, } a = 125 \text{ and } r = \frac{2}{5}$$

Example 48: Find the sum of the n terms of the series:

$$6 + 66 + 666 + \dots$$

Solution:

Let S denote the required sum.

i.e. $S = 6 + 66 + 666 + \dots$ upto n terms

$$= 6(1 + 11 + 111 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{6}{9} \{(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots + (10^n - 1)\}$$

$$= \frac{6}{9} \{(10 + 10^2 + 10^3 + \dots + 10^n) - n\}$$

$$= \frac{6}{9} \{10(1 + 10 + 10^2 + \dots + 10^{n-1}) - n\}$$

$$= \frac{6}{9} \left[10 \left(\frac{10^n - 1}{10 - 1} \right) - n \right]$$

$$= \frac{6}{81} (10^{n+1} - 10 - 9n)$$

$$= \frac{2}{27} (10^{n+1} - 9n - 10)$$

▪ **Sum of an infinite G.P. when $|r| < 1$ and $n \rightarrow \infty$**

$$\therefore S_{\infty} = \frac{a}{1-r} \quad (|r| < 1)$$

Example 49: What is the sum upto infinity of the series 8, 4, 2, 1, ?

Solution:

$$a = 8 \text{ and } r = \frac{1}{2}$$

$$\text{Sum upto infinity} = \frac{a}{1-r}$$

$$= \frac{8}{1 - \frac{1}{2}} = \frac{8}{\frac{1}{2}} = 16$$

Example 50: The sum of first two terms of a GP is $\frac{5}{3}$

and the sum upto infinity of the series is 3. What is the first term?

Solution:

Let the first two terms be a and ar .

$$a + ar = \frac{5}{3} \Rightarrow a(1+r) = \frac{5}{3} \quad \dots (1)$$

$$\text{and } \frac{a}{1-r} = 3 \quad \dots (2)$$

Dividing equation (1) by equation (2), we get

$$(1+r)(1-r) = \frac{5}{9}$$

$$\Rightarrow 1-r^2 = \frac{5}{9}$$

$$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \pm \frac{2}{3}$$

If $r = \frac{2}{3}$, then $a = 1$ and if $r = -\frac{2}{3}$, then $a = 5$.

Example 51: What is the sum upto infinity of the series $3, \frac{3}{7}, \frac{3}{7^2}, \dots$?

Solution:

$$a = 3 \text{ and } r = \frac{1}{7}$$

$$\text{Sum upto infinity} = \frac{a}{1-r}$$

$$\text{Sum upto infinity} = \frac{a}{1-r}$$

Example 52: A boy throws a ball to the ground with force from a height of 10 m. After hitting the ground for the first time it rises to a height of 20 m. There after it rises only upto half the prior height on hitting the ground. What is the total distance travelled by the ball till it comes to rest?

Solution:

Distance travelled by the ball ground till it rises 20 m and then comes back to the ground = 40 m

Next it rises 10 m

Distance from ground to top to ground = 20 m, this continues

So, the series = $40 + 20 + 10 + \dots$

$$= \frac{40}{1 - \frac{1}{2}} = 80 \text{ m}$$

But the ball was first thrown from a height of 10 m.

Total distance = $80 + 10 = 90 \text{ m}$

Harmonic Sequence or Harmonic Progression H.P.

A sequence is said to be in Harmonic Progression (H.P.) if the reciprocal of its terms are in Arithmetic Progression (A.P.)

If $a_1, a_2, a_3, \dots, a_n$ are in H.P. then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in A.P.

1. n^{th} term of this H.P.

$$T_n = \frac{1}{\frac{1}{a_1} + (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)}$$

$$= \frac{a_1 a_2}{a_2 + (n-1)(a_1 - a_2)}$$

2. n^{th} term of this H.P. from end

$$T_n' = \frac{1}{\frac{1}{a_n} - (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)}$$

$$= \frac{a_1 a_2 a_n}{a_1 a_2 - a_n (n-1)(a_1 - a_2)}$$

Note:

Important points about Harmonic Progression (H.P.)

- There is no general formula for finding out the sum of n terms of H.P.
- No term of H.P. can be zero.

Example 53: If the 4^{th} and the 7^{th} term of an H.P. are $\frac{1}{2}$ and $\frac{2}{7}$ respectively. Find the first term.

Solution:

$$4^{\text{th}} \text{ term in H.P.} = \frac{1}{2}$$

$$\therefore 4^{\text{th}} \text{ term in A.P.} = 2$$

$$7^{\text{th}} \text{ term in H.P.} = \frac{2}{7}$$

$$\therefore 7^{\text{th}} \text{ term in A.P.} = \frac{7}{2}$$

$$\therefore T_4 = a + 3d = 2 \quad \dots (1)$$

$$\text{and } T_7 = a + 6d = \frac{7}{2} \quad \dots (2)$$

Subtracting equation (2) from (1), we get

$$3d = \frac{3}{2} \Rightarrow d = \frac{1}{2}$$

First term of A.P. = $T_4 - 3d$

$$= 2 - \frac{3}{2} \Rightarrow \frac{1}{2}$$

Hence, the first term of H.P. is 2.

Practice Exercise level 1

- What is the remainder when $x^3 + 3x + 7 = 0$ is divided by $x + 1$?
(A) 3 (B) 4
(C) 5 (D) 7
- If $x^4 + 2x^3 - 3x^2 + x - 1$ is divided by $x - 2$, then what is the remainder?
(A) 12 (B) 14
(C) 16 (D) 21
- If $(x - 5)$ is a factor of $2x^2 + 2px - 2p = 0$, then what is the value of p ?
(A) -4 (B) -25/4
(C) 25/4 (D) 4
- If $(x + 1)$ is a factor of $2x^3 - ax^2 - (2a - 3)x + 2$, then what is the value of 'a'?
(A) 3 (B) 2
(C) $\frac{3}{2}$ (D) $\frac{1}{2}$
- If $x + 4$ is a factor of $3x^2 + kx + 8$, then what is the value of k ?
(A) 4 (B) -4
(C) -14 (D) 14
- If the roots of the equation $2x^2 + 8x - k^3 = 0$ are equal, then what is the value of k ?
(A) -3 (B) -1
(C) 1 (D) -2
- The one third part of a number exceeds the sixth part by 4. Then, the number is:
(A) 12 (B) 24
(C) 48 (D) 72
- The sum of two numbers be 18 and their difference is 4. Then, the numbers are:
(A) (12, 8) (B) (13, 9)
(C) (11, 7) (D) (8, 4)
- The solution set of the equations $4x - y = 3$ and $3x + 4y = 7$ is:
(A) (2, 1) (B) (1, 1)
(C) (1, -1) (D) (-1, -1)
- A number consists of two digits, the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number, the digits are reversed. What is the number?
(A) 21 (B) 42
(C) 84 (D) 63
- Ten years ago, the age of a father was four times that of his son. Ten years later the age of the father will be twice that of his son. What are the present ages of the father and the son?
(A) (50, 20) (B) (60, 20)
(C) (55, 25) (D) None of these
- Four years ago, the ratio of the age of a father and son was 9 : 4. If their combined age was 52. Then what is ratio of their ages after another 3 years?
(A) 2 : 1 (B) 37 : 21
(C) 43 : 23 (D) 55 : 17
- A sum of ₹ 312 was divided among 100 boys and girls in such a way that each boys gets ₹ 3.60 and each girls gets ₹ 2.40. The number of girls is:
(A) 40 (B) 60
(C) 35 (D) 65

14. If the price of three apples and four bananas is ₹ 20 and it is also equal to the price of five apples and three bananas, then how many apples can be purchased in the same price as that used to purchase 18 bananas?
 (A) 18 (B) 36
 (C) 24 (D) 40
15. The ages of Anand and Vishal differ by 20 years. If 5 years ago, Anand's age be five times as old as Vishal's, then their present ages, in years, are:
 (A) 25, 5 (B) 30, 10
 (C) 35, 15 (D) 50, 30
16. Find the value of y which satisfies the pair of equations $2x + 4y = 6$ and $3x + 15y = 25$
 (A) $\frac{4}{3}$ (B) $\frac{16}{9}$
 (C) $\frac{5}{6}$ (D) $\frac{6}{5}$
17. The total cost of 6 books and 4 pencils is ₹ 34 and that of 5 books and 5 pencils is ₹ 30. The cost of each book and each pencil respectively is:
 (A) ₹ 1 and ₹ 5 (B) ₹ 5 and ₹ 1
 (C) ₹ 6 and ₹ 1 (D) ₹ 1 and ₹ 6
18. The sum of two numbers is 80. If the larger number exceeds four times the smaller one by 45, then the smaller number is:
 (A) 5 (B) 7
 (C) 20 (D) 25
19. A man has some hens and cows. If the number of heads are 48 and number of feet equals to 140, the number of hens will be:
 (A) 26 (B) 24
 (C) 23 (D) 22
20. The sum of a rational number and its reciprocal is $\frac{13}{6}$, find the number.
 (A) $\frac{2}{3}$ or $\frac{3}{2}$ (B) $\frac{3}{4}$ or $\frac{4}{3}$
 (C) $\frac{2}{5}$ or $\frac{5}{2}$ (D) None of these
21. The system of equations $Kx - y = 2$ and $6x - 2y = 3$ has a unique solution when
 (A) $K = 0$ (B) $K \neq 0$
 (C) $K = 3$ (D) $K \neq 3$
22. The equations $3X - 5Y + 2 = 0$ and $6X + 4 = 10Y$ have:
 (A) no solution (B) a single solution
 (C) two solutions (D) infinite number of solutions
23. Find the roots of the equation $x^2 - 9x + 20 = 0$.
 (A) 3, 5 (B) 4, 5
 (C) 3, 4 (D) None of these
24. The roots of the equation $x^2 - 8x + 15 = 0$ are:
 (A) 2, 3 (B) 3, 5
 (C) 8, 15 (D) 6, 5
25. If one root of $x^2 - 6kx + 5 = 0$ is 5, then the value of k is:
 (A) $-\frac{1}{2}$ (B) -1
 (C) 1 (D) 2
26. The roots of the quadratic equation $x^2 - 7x + 11 = 0$ are
 (A) real and unequal
 (B) rational and unequal
 (C) imaginary
 (D) irrational
27. Form a quadratic equation whose roots are 9 and 5.
 (A) $x^2 - 14x + 35 = 0$ (B) $x^2 - 14x + 45 = 0$
 (C) $x^2 + 14x + 35 = 0$ (D) $x^2 + 14x + 45 = 0$
- Directions (28-29):** The equation $x^2 - 3x + 1 = 0$ has α , β as its roots. Find the quadratic equation whose roots are:
28. $\frac{4}{\alpha}, \frac{4}{\beta}$
 (A) $x^2 - 12x + 16 = 0$ (B) $x^2 - 12x - 16 = 0$
 (C) $x^2 + 12x + 16 = 0$ (D) $x^2 + 12x - 16 = 0$

29. $\alpha^2 + \beta, \beta^2 + \alpha$
 (A) $x^2 - 20x + 10 = 0$ (B) $x^2 - 10x + 20 = 0$
 (C) $x^2 + 10x + 20 = 0$ (D) $x^2 - 10x - 20 = 0$
30. If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is:
 (A) $2\sqrt{5}$ (B) $\sqrt{5}$
 (C) $3\sqrt{5}$ (D) $-2\sqrt{5}$
31. Form a quadratic equation whose roots are 6 more than the roots of $x^2 + 5x + 18 = 0$.
 (A) $x^2 - 7x + 34 = 0$ (B) $x^2 - 5x + 24 = 0$
 (C) $x^2 - 7x + 24 = 0$ (D) $x^2 - 7x - 24 = 0$
32. Form a quadratic equation whose roots are one third of the roots of the equation $3x^2 + 8x + 7 = 0$.
 (A) $27x^2 + 24x + 7 = 0$ (B) $27x^2 - 24x - 7 = 0$
 (C) $x^2 - 24x + 7 = 0$ (D) $24x^2 - 27x + 7 = 0$
33. If the root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4 then the value of m is:
 (A) $m = 10$ (B) $m = 11$
 (C) $m = 9$ (D) $m = 12$
34. The values of k for which the equation $2x^2 + kx + x + 8 = 0$ will have real and equal roots are:
 (A) 7 and -9 (B) -7 and 9
 (C) 7 and 9 (D) -7 and -9
35. If a and b are the roots of the equation $x^2 - 6x + 6 = 0$, then the value of $a^2 + b^2$ is:
 (A) 36 (B) 24
 (C) 12 (D) 6
36. If one root of $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is:
 (A) $\frac{49}{4}$ (B) $\frac{4}{49}$
 (C) 4 (D) $\frac{1}{4}$
37. Of the following quadratic equations, which is the one whose roots are 2 and -15?
 (A) $x^2 - 2x + 15 = 0$ (B) $x^2 + 15x - 2 = 0$
 (C) $x^2 + 13x - 30 = 0$ (D) $x^2 - 30 = 0$
38. If α and β are the two roots of the equation $2x^2 - 7x - 3 = 0$, then find the value of $(\alpha + 2)(\beta + 2)$.
 (A) 9 (B) -9.5
 (C) 9.5 (D) 6
39. If 4 is a solution of the equation $x^2 + 3x + k = 0$, where k is a constant, what is the other solution?
 (A) -18 (B) -7
 (C) -28 (D) None of these
40. The equation $x^2 - (k + 4)x + 2k + 5 = 0$ has equal roots then the value of k is:
 (A) ± 1 (B) 2
 (C) ± 2 (D) -2
41. Find the equation each of whose root is 2 less than the roots of the equation $x^2 - 2x - 195 = 0$.
 (A) $x^2 - 2x - \frac{195}{2} = 0$ (B) $x^2 - 4x - 195 = 0$
 (C) $x^2 + 2x - 195 = 0$ (D) None of these
42. Find the equation whose roots are 1 less than the roots of the equation $x^2 - 5x + 8 = 0$.
 (A) $x^2 - 5x + 14 = 0$ (B) $x^2 - 7x + 14 = 0$
 (C) $x^2 - 7x - 12 = 0$ (D) None of these
43. If one of the roots of the equation $x^2 - 7x + S = 0$ is double the other, find the value of S .
 (A) $\frac{98}{9}$ (B) $\frac{98}{3}$
 (C) $\frac{14}{3}$ (D) 7
44. If α, β be the roots of the equation $ax^2 + bx + c = 0$, then the value of $\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)$ is:
 (A) $\frac{(ab - b^2c)}{2b^2c}$ (B) $\frac{(3ac - b^2)}{a^3c}$
 (C) $\frac{(3abc - b^3)}{a^2c}$ (D) $\frac{(3bc - a^3)}{b^2c}$
45. If the roots of $ax^2 + bx + c = 0$ be equal, then the value of c is:
 (A) $\frac{b}{2a}$ (B) $\frac{-b}{2a}$

- (C) $\frac{b^2}{4a}$ (D) $\frac{-b^2}{4a}$
46. Value of k for which the roots of the equation $9x^2 + 2kx + 4 = 0$ are equal.
 (A) ± 4 (B) ± 6
 (C) ± 2 (D) ± 5
47. Find the equation whose roots are $2 + \sqrt{2}$ and $2 - \sqrt{2}$.
 (A) $x^2 + 4x + 2 = 0$ (B) $x^2 - 6x + 3 = 0$
 (C) $x^2 + 6x - 3 = 0$ (D) $x^2 - 4x + 2 = 0$
48. If one root of $3x^2 + 11x + k = 0$ be reciprocal of the other, what is the value of k ?
 (A) 3 (B) 4
 (C) 5 (D) 6
49. If α, β are the roots of the equation $x^2 - 5x + 6 = 0$ the value of $\alpha^2 - \beta^2$ is:
 (A) ± 4 (B) ± 5
 (C) ± 6 (D) 0
50. If the roots of the equation $x^2 - bx + c = 0$ differ by 2, then which of the following is true?
 (A) $c^2 = 4(c + 1)$ (B) $b^2 = 4c + 4$
 (C) $c^2 = b + 4$ (D) $b^2 = 4(c + 2)$
51. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal if
 (A) $a = b$ (B) $a = bc$
 (C) $c = a$ (D) $b = c$
52. If $a + b + c = 27$, then what is the value of $(a - 7)^3 + (b - 9)^3 + (c - 11)^3 - 3(a - 7)(b - 9)(c - 11)$?
 (A) 0 (B) 9
 (C) 27 (D) 81
53. If $a^3 + b^3 = 19$ and $ab = -6$, then what is the value of $a + b$?
 (A) 5 (B) 7
 (C) 1 (D) -5
54. If $a + b = 10$ and $ab = 24$, then what is the value of $a^3 + b^3$?
 (A) 280 (B) 152
 (C) 140 (D) 72
55. If $a + b + c = 6$ and $ab + bc + ca = 11$, then the value of $bc(b + c) + ca(c + a) + ab(a + b) + 3abc$ is:
 (A) 33 (B) 66
 (C) 55 (D) 32
56. If $x + y = 4$, then what is the value of $x^3 + y^3 + 12xy$?
 (A) 16 (B) 32
 (C) 64 (D) 256
57. If $x^4 + \frac{1}{x^4} = 198$ and $x > 0$, then what is the value of $x^2 - \frac{1}{x^2}$?
 (A) 14 (B) $2\sqrt{7}$
 (C) $10\sqrt{2}$ (D) 10
58. If $3x - \frac{1}{3x} = 9$, then what is the value of $x^2 + \frac{1}{81x^2}$?
 (A) 7 (B) $\frac{83}{9}$
 (C) 11 (D) $\frac{121}{9}$
59. If $x^3 - y^3 = 112$ and $x - y = 4$, then what is the value of $x^2 + y^2$?
 (A) 16 (B) 20
 (C) 24 (D) 28
60. If $x + y + z = 0$, then what is the value of $\frac{xy + yz + zx}{x^2 + y^2 + z^2}$?
 (A) 1 (B) -1
 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
61. If $x^2 + \frac{1}{x^2} = 79$, then find the value of $x + \frac{1}{x}$. [x' is real number]
 (A) 7 (B) 8
 (C) 11 (D) 9
62. What the value of y in the solution of the equation $2^{x+y} = 2^{x-y} = ?$
 (A) 0 (B) $\sqrt{8}$
 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

63. If $m + \frac{1}{m-2} = 4$, then what is the value of $(m-2)^2 + \frac{1}{(m-2)^2}$?
- (A) -2 (B) 0
(C) 2 (D) 4
64. If $x - y = \frac{x+y}{7} = \frac{xy}{4}$, then what is the numerical value of xy ?
- (A) $\frac{4}{3}$ (B) $\frac{3}{4}$
(C) $\frac{1}{4}$ (D) $\frac{1}{3}$
65. If $x = b + c - 2a$, $y = c + a - 2b$, $z = a + b - 2c$, then what is the value of $x^2 + y^2 - z^2 + 2xy$?
- (A) 0 (B) $a + b + c$
(C) $a - b + c$ (D) $a + b - c$
66. If $a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4$, then what is the value of $a^2 + b^2$?
- (A) 1 (B) $1\frac{1}{2}$
(C) 2 (D) $2\frac{1}{2}$
67. If $a = 23$ and $b = -29$, then what is the value of $25a^2 + 40ab + 16b^2$?
- (A) 1 (B) -1
(C) 0 (D) 2
68. If $ax^2 + bx + c = a(x - p)^2$, then what is the relation among a, b, c ?
- (A) $abc = 1$ (B) $b^2 = ac$
(C) $b^2 = 4ac$ (D) $2b = a + c$
69. If $a^2 - 4a - 1 = 0$, then what is the value of $a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$?
- (A) 25 (B) 30
(C) 35 (D) 40
70. If $ax + by = 3$, $bx - ay = 4$ and $x^2 + y^2 = 1$, then what is the value of $a^2 + b^2$?
- (A) 17 (B) 16
(C) 9 (D) 25
71. What is the term to be added to $121a^2 + 64b^2$ to make a perfect square?
- (A) $176ab$ (B) $276a^2b$
(C) $178ab$ (D) $188b^2a$
72. If $a^2 + b^2 + c^2 = ab + bc + ca$, then what is the value of $\frac{a+c}{b}$?
- (A) 3 (B) 2
(C) 0 (D) 1
73. If $2\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 7 = 0$, then what are the value of x ?
- (A) 1, 2 (B) $2, -\frac{1}{2}$
(C) 0, 1 (D) $\frac{1}{2}, 1$
74. If $x^2 + \frac{1}{x^2} = 66$, then what is the value of $\frac{x^2 - 1 + 2x}{x} = ?$
- (A) ± 8 (B) 10, -6
(C) 6, -10 (D) ± 4
75. If $\frac{5x}{2x^2 + 5x + 1} = \frac{1}{3}$, what is the value of $\left(x + \frac{1}{2x}\right)$?
- (A) 15 (B) 10
(C) 20 (D) 5
76. If $2x - \frac{1}{2x} = 6$, then what will be the value of $x^2 + \frac{1}{16x^2}$?
- (A) $\frac{19}{2}$ (B) $\frac{17}{2}$
(C) $\frac{18}{3}$ (D) $\frac{15}{2}$
77. If a, b, c are non-zero, $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then what is the value of abc ?
- (A) -1 (B) 3
(C) -3 (D) 1

78. If $x + \frac{1}{x} = 99$, then what is the value of $\frac{100x}{2x^2 + 102x + 2}$?
- (A) $\frac{1}{6}$ (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) $\frac{1}{4}$
79. If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$, then what is the value of $\frac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)}$?
- (A) 1 (B) 0
(C) 2 (D) $\frac{1}{2}$
80. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then what is the value of $c + \frac{1}{a}$?
- (A) $\frac{1}{2}$ (B) 2
(C) 1 (D) 0
81. If $(x-a)(x-b) = 1$ and $a-b+5 = 0$, then what is the value of $(x-a)^3 - \frac{1}{(x-a)^3}$?
- (A) -125 (B) 1
(C) 125 (D) 140
82. If $a = \frac{b^2}{b-a}$, then what is the value of $a^3 + b^3$?
- (A) 6ab (B) 0
(C) 1 (D) 2
83. If $x = y = 333$ and $z = 334$, then what is the value of $x^3 + y^3 + z^3 - 3xyz$?
- (A) 0 (B) 667
(C) 1000 (D) 2334
84. What is the value of $\frac{4x^3 - x}{(2x+1)(6x-3)}$ when $x = 9999$?
- (A) 1111 (B) 2222
(C) 3333 (D) 6666
85. If $a + b + c + d = 4$, then what is the value of $\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)} + \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}$?
- (A) 0 (B) 5
(C) 1 (D) 4
86. If $3x + \frac{3}{x} = 1$, then what is the value of $x^3 + \frac{1}{x^3} + 1$?
- (A) 0 (B) $\frac{1}{27}$
(C) $\frac{5}{27}$ (D) $\frac{28}{27}$
87. If $x + \frac{1}{x} = 2$, then what is the value of $2x$?
- (A) 2 (B) -4
(C) 0 (D) 4
88. If $x^2 + x = 5$, then what is the value of $(x+3)^3 + \frac{1}{(x+3)^3}$?
- (A) 140 (B) 110
(C) 130 (D) 120
89. If $x + \frac{1}{x} = 1$, then what is the value of $\frac{2}{x^2 - x + 2} = ?$
- (A) 2 (B) 4
(C) $\frac{2}{3}$ (D) 1
90. If $a - \frac{1}{a-3} = 5$, then what is the value of $a-3)^3 - \frac{1}{(a-3)^3}$?
- (A) 5 (B) 7
(C) 2 (D) 14
91. If $a = 4.965$, $b = 2.343$ and $c = 2.622$, then what is the value of $a^3 - b^3 - c^3 - 3abc$?
- (A) -2 (B) -1
(C) 0 (D) 9.93

92. If $P = 99$, then what is the value of the $P(P^2 + 3P + 3)$?
 (A) 989898 (B) 988899
 (C) 999999 (D) 998889
93. If average two numbers x and $\frac{1}{x}$ (where $x \neq 0$) is A , then what will be the average of x^3 and $\frac{1}{x^3}$?
 (A) $4A^3 - 2A$ (B) $4A^3 - 3A$
 (C) $4A^3 - 4A$ (D) $4A^3 - A$
94. If $x + \frac{1}{x} = 3$, then what is the value of $\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$?
 (A) $\frac{3}{2}$ (B) $\frac{5}{2}$
 (C) $\frac{7}{2}$ (D) $\frac{11}{2}$
95. If $x = \frac{1}{2 + \sqrt{3}}$ and $y = \frac{1}{2 - \sqrt{3}}$, then what is the value of $\left(\frac{1}{x+1} + \frac{1}{y+1}\right)$?
 (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$
 (C) 2 (D) 1
96. If $x + \frac{1}{x} = 2$, then what is the value of $\left(x^{100} + \frac{1}{x^{100}}\right)$?
 (A) 2 (B) 0
 (C) 1 (D) -2
97. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$, then what will be the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$?
 (A) 1 (B) 2
 (C) 3 (D) 4
98. If $x + \frac{1}{4x} = \frac{3}{2}$, then what will be the value of $8x^3 + \frac{1}{8x^3}$?
 (A) 18 (B) 36
- (C) 24 (D) 16
99. If x is real, $x + \frac{1}{x} \neq 0$ and $x^3 + \frac{1}{x^3} = 0$, then what is the value of $\left(x + \frac{1}{x}\right)^4$?
 (A) 4 (B) 9
 (C) 16 (D) 25
100. If $x > 1$ and $x^2 + \frac{1}{x^2} = 79$, then what is the value of $x^3 + \frac{1}{x^3}$?
 (A) 702 (B) 709
 (C) 756 (D) 711
101. If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $xy = 1$, then what will be the value of $\frac{2x^2 + 3xy + 2y^2}{2x^2 - 3xy + 2y^2}$?
 (A) $\frac{71}{65}$ (B) $3 + 2\sqrt{2}$
 (C) $\frac{81}{65}$ (D) $3 - 2\sqrt{2}$
102. If $\frac{1}{x^3} + \frac{1}{y^3} = \frac{1}{z^3}$, then what is the value of $\{(x+y-z)^3 + 27xyz\}$?
 (A) 0 (B) 1
 (C) -1 (D) 27
103. What will be the value of $x^4 - 17x^3 + 17x^2 - 17x + 17$ if $x = 16$?
 (A) 0 (B) 1
 (C) 2 (D) 3
104. If $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$ and a, b, c are rational numbers, then what is the value of $a + b + c$?
 (A) 0 (B) 1
 (C) 2 (D) 3
105. What is the value of $\sqrt{2^3} \sqrt[4]{4} \sqrt[2^3]{4} \sqrt[2^3]{4} \sqrt[2^3]{4} \dots$?
 (A) 2 (B) 2^2
 (C) 2^3 (D) 2^5

106. If $\sqrt{2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}}} = 32^a$, then what is the value of a ?
- (A) $\frac{41}{160}$ (B) $\frac{41}{80}$
(C) 1 (D) 2
107. What is the value of $p + \sqrt{p^2 + \sqrt{p^4 + \sqrt{p^8 + \sqrt{p^{16}}}}} \dots \infty$?
- (A) $p\left(\frac{\sqrt{5}+2}{2}\right)$ (B) $p\left(\frac{3+\sqrt{5}}{2}\right)$
(C) $\frac{p}{1+\sqrt{p}}$ (D) $p\left(\frac{\sqrt{5}+1}{2}\right)$
108. If $x = \sqrt[4]{4\sqrt[4]{4\sqrt[4]{4}}} \dots \infty = 32^a$, then what is the value of a ?
- (A) $\frac{2}{15}$ (B) $\frac{4}{15}$
(C) $\frac{2}{5}$ (D) $\frac{1}{5}$
109. What is the value of $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$?
- (A) $3abc$
(B) $(a-b)(b-c)(c-a)$
(C) $(a-b)(b-c)(c-a)(a+b+c)$
(D) $(a+b)(b+c)(c+a)(a+b+c)$
110. What is the value of $a^4(b^2-c^2) + b^4(c^2-a^2) + c^4(a^2-b^2)$?
- (A) $3a^2b^2c^2$
(B) $(a^2-b^2)(b^2-c^2)(c^2-a^2)$
(C) $-(a^2-b^2)(b^2-c^2)(c^2-a^2)$
(D) None of these
111. What is the value of $a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$?
- (A) $(a+b)(b+c)(c+a)$ (B) $(a-b)(b-c)(c-a)$
(C) 0 (D) abc
112. What is the value of $(bc+ca+ab)^3 - b^3c^3 - c^3a^3 - a^3b^3$?
- (A) $3abc(a+b)(b+c)(c+a)$
(B) $(a+b)(b+c)(c+a)$
(C) $(a-b)(b-c)(c-a)$
(D) $24abc$
113. What is the value of $(x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \cdot (x^{a+b})^{a-b}$ ($x \neq 0$)?
- (A) 1 (B) 2
(C) x^{abc} (D) $x^{(a+b+c)}$
114. What is the value of $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$?
- (A) 0 (B) 1
(C) x^{abc} (D) $x^{(a+b+c)}$
115. What is the value of $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$?
- (A) 0 (B) 1
(C) x^{a-b-c} (D) None of these
116. What is the next number in the arithmetic progression 2, 5, 8,?
- (A) 7 (B) 9
(C) 10 (D) 11
117. Find the ninth term of an AP with first term 5 and common difference 4.
- (A) 41 (B) 35
(C) 37 (D) None of these
118. If the ratio between 7th and 3rd terms of an A.P. is 12 : 5. Then ratio of 13th and 4th term is:
- (A) 13 : 7 (B) 4 : 3
(C) 10 : 3 (D) None of these
119. The sum of the first fifteen terms of an A.P. is 105 and sum of next fifteen terms is 780. Then the common difference is:
- (A) 4 (B) 3
(C) 6 (D) 5
120. Fourth term of an arithmetic progression is 8. What is the sum of the first 7 terms of the arithmetic progression?
- (A) 7 (B) 64
(C) 56 (D) Can't say
121. The first and last terms of an A.P. are -7 and 233 and sum of A.P. is 9153. Find the number of terms in A.P.?

- (A) 83 (B) 81
(C) 49 (D) 99
122. What is the sum of the first 12 terms of an arithmetic progression if the 3rd term is -13 and the 6th term is -4?
(A) 67 (B) 45
(C) -30 (D) -48
123. The 3rd and 6th term of an arithmetic progression are 13 and -5 respectively. What is the 11th term?
(A) -29 (B) -41
(C) -47 (D) -35
124. If 7 times the 7th term of an AP is equal to 11 times its 11th term, find the 18th term of the AP.
(A) 0 (B) 5
(C) 6 (D) 3
125. Find the three numbers in AP whose sum is 21, and sum of their squares is 179.
(A) 3, 5, 7 (B) 5, 9, 13
(C) 6, 13, 20 (D) 3, 7, 11
126. Which term of the AP series 3, 8, 13 ... is the term 78?
(A) 15 (B) 14
(C) 16 (D) 17
127. If the 4th term of an arithmetic progression is 14 and its 12th term is 70, then its first term is:
(A) -10 (B) -7
(C) 7 (D) 10
128. If the first term of a GP is 2 and the common ratio is 3, what is the fifth term of the GP?
(A) 81 (B) 243
(C) 162 (D) 324
129. If the ratio of the fourth and the sixth term of the GP is 1 : 4 and the first term is 5, what will be the second term of the GP?
(A) -10 (B) 10
(C) 20 (D) 10 or -10
130. The third term of a geometrical progression is 4. The product of first 5 terms is :
(A) 4³ (B) 4⁴
(C) 4⁵ (D) None of these
131. The fourth, seventh and tenth terms of a G.P. are p, q and r respectively, then:
(A) $p^2 = q^2 + r^2$ (B) $q^2 = pr$
(C) $p^2 = qr$ (D) $pqr + pq + 1 = 0$
132. Find the general term of the GP with the third term equal to 1 and the seventh term equal to 8.
(A) $(2^{3/4})^{n-3}$ (B) $(2^{3/2})^{n-3}$
(C) $(2^{3/4})^{3-n}$ (D) $(2^{3/4})^{2-n}$
133. What is the sum of all the two-digit numbers which when divided by 7 gives a remainder of 3?
(A) 94 (B) 676
(C) 696 (D) None of these
134. In a geometric progression the sum of the first and the last term, is 66 and the product of the second and the last but one term is 128. Determine the first term and the last term of the series.
(A) 64 (B) 64 or 2
(C) 2 or 32 (D) 32
135. What is the 10th term of the H.P.
 $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \dots$?
(A) $\frac{1}{17}$ (B) $\frac{1}{19}$
(C) $\frac{1}{21}$ (D) $\frac{1}{23}$

Practice exercise Level 2

1. If $x^2 - 1$ is a factor of $x^4 + ax^3 + 3x - b$, then
(A) $a = 3, b = -1$ (B) $a = -3, b = 1$
(C) $a = 3, b = 1$ (D) None of these
2. If $(a + 4)^3 = a^3 + 12a^2 + ka + 64$, then what is the value of k?
(A) 12 (B) 24
(C) 36 (D) 48

3. If the expression $(px^3 - 8x^2 - qx + 1)$ is completely divisible by the expression $(3x^2 - 4x + 1)$, then what will be the value of p and q respectively?
- (A) $\frac{21}{24}, \frac{15}{8}$ (B) 6, 1
(C) $\frac{33}{4}, \frac{5}{4}$ (D) 1, 6
4. The solution of the equations $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$ is given by:
- (A) $x = \frac{1}{2}, y = \frac{3}{2}$ (B) $x = \frac{5}{2}, y = \frac{1}{2}$
(C) $x = \frac{3}{2}, y = \frac{1}{2}$ (D) $x = \frac{1}{2}, y = \frac{5}{2}$
5. Let a and b be the roots of the equation, $x^2 + 7x + 12$. Then equation having roots $(a+b)^2$ and $(a-b)^2$ is:
- (A) $x^2 + 49x + 144 = 0$ (B) $x^2 - 33x + 84 = 0$
(C) $x^2 - 50x + 49 = 0$ (D) $x^2 - 49x + 144 = 0$
6. If m and n are the roots of the equation $x^2 - bx + c = 0$. Then equation having roots $\frac{m}{n}$ and $\frac{n}{m}$ is:
- (A) $bc(x)^2 - (b)^2x + c^2 = 0$
(B) $bx^2 - c^2x + bc = 0$
(C) $x^2 - (b^2 - 2c) + b = 0$
(D) $cx^2 - (b^2 - 2c)x + c = 0$
7. The roots of the quadratic equations $px^2 + qx + r = 0$ and $rx^2 - qx + p = 0$ are:
- (A) same
(B) same but opposite in sign
(C) reciprocal of each other
(D) reciprocal of each other but opposite in sign
8. If sum of the roots of the quadratic equation $px^2 + qx + r = 0$ is equal to the sum of the squares of their reciprocals then $\frac{q^2}{pr} + \frac{qr}{p^2}$ is equal to?
- (A) 2 (B) -2
(C) 1 (D) -1
9. If (α, β) are the roots of the equations $x^2 - 23x + 42 = 0$. Find the equation whose roots are $\frac{\alpha}{\beta^3}, \frac{\beta}{\alpha^3}$.
- (A) $x^2 - 2.6x + 5.6 \times 10^{-4} = 0$
(B) $x^2 + 2.6x - 5.6 \times 10^{-4} = 0$
(C) $x^2 - 2.6x - 5.6 \times 10^{-4} = 0$
(D) None of these
10. The quadratic equation whose roots are the same but opposite in sign to the roots of $2x^2 - 7x + 3 = 0$ is:
- (A) $3x^2 - 7x + 2 = 0$ (B) $2x^2 + 7x + 3 = 0$
(C) $2x^2 - 7x - 3 = 0$ (D) $3x^2 + 7x - 2 = 0$
11. What is the value of 'x', where $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \dots}}}}$ to infinity ?
- (A) 3 (B) $\left(\frac{\sqrt{13}-1}{2}\right)$
(C) $\left(\frac{\sqrt{13}+1}{2}\right)$ (D) $\sqrt{13}$
12. If $x^3 + 2x^2 + ax + b$ is exactly divisible by $x^2 - 1$, then the values of a and b are respectively.
- (A) 1 and 2 (B) 1 and 0
(C) -1 and -2 (D) 0 and 1
13. If α, β are the roots of $x^2 + x + 1 = 0$ and γ, δ are the roots of $x^2 + 3x + 1 = 0$, then $(\alpha - \gamma)(\beta + \delta)(\alpha + \delta)(\beta - \gamma) =$
- (A) 2 (B) 4
(C) 6 (D) 8
14. If α and β are the roots of the equation $x^2 + 5x - 5 = 0$, then what is the value of $\left(\frac{1}{\alpha+1}\right)^3 + \left(\frac{1}{\beta+1}\right)^3$?
- (A) -322 (B) 4
(C) -4 (D) $3 + \sqrt{5}$

15. If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are $\alpha^{2000}, \beta^{2000}$ is:
 (A) $x^2 + x - 1 = 0$ (B) $x^2 + x + 1 = 0$
 (C) $x^2 - x + 1 = 0$ (D) $x^2 - x - 1 = 0$
16. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are α^2, β^2 is:
 (A) $a^2x^2 + (b^2 - 2ac)x + c^2 = 0$
 (B) $a^2x^2 - (b^2 + 2ac)x + c^2 = 0$
 (C) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
 (D) None of these
17. The number of real solutions of the equation $x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$ is/are:
 (A) 0 (B) 1
 (C) 2 (D) Infinite
18. The equation $x^2 - 6x + 8 + \lambda(x^2 - 4x + 3) = 0$, $\lambda \in \mathbb{R}$, has
 (A) real and unequal roots for all λ
 (B) real roots for $\lambda < 0$ only
 (C) real roots for $\lambda > 0$ only
 (D) real and unequal roots for $\lambda = 0$ only
19. If a, b, c are non-zero, unequal rational numbers, then the roots of the equation $abc^2x^2 + (3a^2 + b^2)cx - 6a^2 - ab + 2b^2 = 0$ are
 (A) rational (B) imaginary
 (C) irrational (D) None of these
20. If α and β are the roots of the equation $(x - a)(x - b) = c$, $c \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are
 (A) a, c (B) b, c
 (C) a, b (D) $a + c, b + c$
21. If the roots of $4x^2 + 5k = (5k + 1)x$ differ by unity, then the negative value of k is:
 (A) -3 (B) $-\frac{1}{5}$
 (C) $-\frac{3}{5}$ (D) None of these
22. If the roots of $a_1x^2 + b_1x + c_1 = 0$ are α_1, β_1 , and those $a_2x^2 + b_2x + c_2 = 0$ are α_2, β_2 such that $\alpha_1\alpha_2 = \beta_1\beta_2 = 1$, then
 (A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (B) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$
 (C) $a_1a_2 = b_1b_2 = c_1c_2$ (D) None of these
23. If α and β are the roots of the equation $x^2 - px + q = 0$, then the product of the roots of the quadratic equation whose roots are $\alpha^2 - \beta^2$ and $\alpha^3 - \beta^3$ is:
 (A) $p(p^2 - q)^2$ (B) $p(p^2 - q)(p^2 - 4q)$
 (C) $p(p^2 - 4q)(p^2 + q)$ (D) None of these
24. If α and β are the roots of the equation $x^2 + x + 3 = 0$, then the equation $3x^2 + 5x + 3 = 0$ has a root
 (A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$
 (C) Both (A) and (B) (D) None of these
25. If $2x^2 + 12x + 18 = 0$, what is the value of x ?
 (A) -3 (B) -2
 (C) 2 (D) 3
26. If $a = 2017$, $b = 2016$ and $c = 2015$, then what is the value of $a^2 + b^2 + c^2 - ab - bc - ca$?
 (A) -2 (B) 0
 (C) 3 (D) 4
27. If $x = 5 - \frac{1}{x}$, then what is the value of $x^5 + \frac{1}{x^5}$?
 (A) 625 (B) 3125
 (C) 2525 (D) 2500
28. If $\left(x + \frac{1}{x}\right)^2 = 5$ and $x > 0$, then what is the value of $x^3 + \frac{1}{x^3}$?
 (A) $2\sqrt{5}$ (B) $3\sqrt{5}$
 (C) $4\sqrt{5}$ (D) $5\sqrt{5}$
29. If $\frac{x^2 + 1}{x} = 4\frac{1}{4}$, then what is the value of $x^3 + \frac{1}{x^3}$?
 (A) $\frac{529}{16}$ (B) $\frac{527}{64}$

- (C) $\frac{4913}{64}$ (D) $\frac{4097}{64}$
30. If $\frac{x}{xa+yb+zc} = \frac{y}{ya+zb+xc} = \frac{z}{za+xb+yc} = k$ and $x+y+z \neq 0$, then what is the value of ratio of k ?
- (A) $\frac{1}{a-b-c}$ (B) $\frac{1}{a+b-c}$
 (C) $\frac{1}{a-b+c}$ (D) $\frac{1}{a+b+c}$
31. If $\frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1$ and $\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 0$, where p, q, r and a, b, c are non-zero, then what is the value of $\frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2}$?
- (A) -1 (B) 0
 (C) 1 (D) 2
32. If $ab + bc + ca = 0$, then what is the value of $\left(\frac{1}{a^2-bc} + \frac{1}{b^2-ca} + \frac{1}{c^2-ab}\right)$?
- (A) 0 (B) 1
 (C) 3 (D) $a+b+c$
33. If $bc + ab + ca = abc$, then what is the value of $\frac{b+c}{bc(a-1)} + \frac{a+c}{ac(b-1)} + \frac{a+b}{ab(c-1)}$?
- (A) 0 (B) $-\frac{1}{2}$
 (C) $-\frac{3}{2}$ (D) 1
34. If $\frac{a^2-bc}{a^2+bc} + \frac{b^2-ca}{b^2+ca} + \frac{c^2-ab}{c^2+ab} = 1$, then what is the value of $\frac{a^2}{a^2+bc} + \frac{b^2}{b^2+ca} + \frac{c^2}{c^2+ab}$?
- (A) 0 (B) 1
 (C) -1 (D) 2
35. If $x + \frac{1}{x} = 1$, then what is the value of $\frac{x^2+3x+1}{x^2+7x+1}$?
- (A) 1 (B) $\frac{3}{7}$
- (C) $\frac{1}{2}$ (D) 2
36. If $a + b + c = 2s$, then what is the value of $\frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$?
- (A) $a^2 + b^2 + c^2$ (B) 0
 (C) 1 (D) 2
37. If $\left(a + \frac{1}{a}\right)^2 = 3$, then what is the value of $a^3 + \frac{1}{a^3}$?
- (A) $2\sqrt{3}$ (B) 2
 (C) $3\sqrt{3}$ (D) 0
38. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are all non-zero, then what is the value of 'abc'?
- (A) $\frac{2}{3}$ (B) $\frac{3}{2}$
 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$
39. If $x = a(b-c)$, $y = b(c-a)$, $z = c(a-b)$, then what is the value of $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$?
- (A) $\frac{2xyz}{abc}$ (B) $\frac{xyz}{abc}$
 (C) 0 (D) $\frac{3xyz}{abc}$
40. If $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$, $b \neq a$, then what is the value of x ?
- (A) $\frac{b}{a+b}$ (B) $\frac{ab}{a+b}$
 (C) 1 (D) $\frac{a}{a+b}$
41. If $x = (\sqrt{2}-1)^{-\frac{1}{2}}$, then what is the value of $\left(x^2 - \frac{1}{x^2}\right)$?
- (A) 2 (B) $-2\sqrt{2}$
 (C) $2\sqrt{2}$ (D) $-\sqrt{2}$

42. If $x + \frac{1}{x} = 5$, then what is the value of $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$?
- (A) $\frac{43}{23}$ (B) $\frac{47}{21}$
(C) $\frac{41}{23}$ (D) $\frac{45}{21}$
43. If $x \neq 0$, $y \neq 0$ and $z \neq 0$ and $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$, then what is the value of x, y, z ?
- (A) $x + y + z = 0$ (B) $x + y = z$
(C) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ (D) $x = y = z$
44. If $a + b + c = 0$, then what is the value of $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$?
- (A) 8 (B) -3
(C) 9 (D) 0
45. If $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{4}} \div \left(\frac{p^6q^{-3}}{p^{-2}q^3}\right) = p^a q^b$, then what is the value of $a + b$, where p and q are different positive primes?
- (A) -1 (B) 2
(C) 1 (D) -2
46. If $x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}$, $y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$, then what is the value of $(x^4 - x^2y^2 - 1) + (y^4 + x^2y^2 + 1)$?
- (A) 16 (B) 13
(C) 12 (D) 14
47. If $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$, then what is the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$?
- (A) 9 (B) 3
(C) 4 (D) 6
48. If $x = \frac{4ab}{a+b}$ ($a \neq b$), then what is the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$?
- (A) a (B) b
(C) $2ab$ (D) 2
49. If $xy + yz + zx = 0$, then what is the value of $\left(\frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}\right)(x, y, z \neq 0)$?
- (A) 3 (B) 1
(C) $x + y + z$ (D) 0
50. If $x = \frac{4\sqrt{15}}{\sqrt{5} + \sqrt{3}}$, then what is the value of $\frac{x + \sqrt{20}}{x - \sqrt{20}} + \frac{x + \sqrt{12}}{x - \sqrt{12}}$?
- (A) 1 (B) 2
(C) $\sqrt{3}$ (D) $\sqrt{5}$
51. What is the value of $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$?
- (A) 0 (B) 3
(C) (D) 2
52. If $\sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$, then what is the simplest value of $(a + b - c)^3 + 27abc$?
- (A) -1 (B) 3
(C) -3 (D) 0
53. If $\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = 2$, then what is the value of x ?
- (A) 2.4 (B) 3.2
(C) 4 (D) 5
54. If $\left(x + \frac{1}{x}\right)^2 = 3$, then what is the value of $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$?
- (A) 0 (B) 1
(C) 84 (D) 206
55. If $x = -1$, then what is the value of $\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x}$?

- (A) 1 (B) 0
(C) -2 (D) -1
56. If $x = 3 + 2\sqrt{2}$, then what is the value of $\frac{x^6 + x^4 + x^2 + 1}{x^3}$?
- (A) 216 (B) 192
(C) 198 (D) 204
57. If $ab = 21$ and $\frac{(a+b)^2}{(a-b)^2} = \frac{25}{4}$, then what is the value of $a^2 + b^2 + 3ab$?
- (A) 115 (B) 121
(C) 125 (D) 127
58. What is the value of $\frac{p^2 - p}{2p^3 + 6p^2} \div \frac{p^2 - 1}{p^2 + 3p} \div \frac{p^2}{p + 1}$?
- (A) $2p^2$ (B) $\frac{1}{2p^2}$
(C) $p + 3$ (D) $\frac{1}{p + 3}$
59. What is the value of $\left(\frac{1}{(p-n)(n-q)} + \frac{1}{(n-q)(q-p)} + \frac{1}{(q-p)(p-n)} \right)$?
- (A) 1 (B) 0
(C) $p + q + n$ (D) $\frac{2n}{p + q}$
60. If $u_n = \frac{1}{n} - \frac{1}{n+1}$, then what is the value of $u_1 + u_2 + u_3 + u_4 + u_5$?
- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{5}{6}$
61. How many terms of the series $1 + 3 + 5 + 7 + \dots$ amount to 123454321?
- (A) 11101 (B) 11011
(C) 10111 (D) 11111
62. A square S_1 had dimensions 6 cm $\times$ 6 cm. Another square S_2 is drawn by joining the mid-points of the sides of S_1 . Square S_3 is drawn joining the mid-point of S_2 and so on. The total area of all squares equals to
- (A) 72 cm^2 (B) $36\sqrt{2}(\sqrt{2} - 1) \text{ cm}^2$
(C) 14.4 cm^2 (D) $36\sqrt{2} \text{ cm}^2$
63. A man starts moving in north direction and covers a distance of 100 m, followed by 50 m in the east direction, 25 m in the south direction, 12.5 m in the west direction and so on. What is the distance between his initial and final position?
- (A) $30\sqrt{5} \text{ m}$ (B) $40\sqrt{5} \text{ m}$
(C) $50\sqrt{5} \text{ m}$ (D) $60\sqrt{5} \text{ m}$
64. What is the sum of $1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} + \dots + \infty$?
- (A) $\frac{2}{3}$ (B) $\frac{5}{2}$
(C) $\frac{19}{45}$ (D) $\frac{81}{17}$

Solution

Practice Exercise Level 1

- 1.(A) For finding the remainder put $x = -1$ i.e.

$$f(-1) = (-1)^3 + 3(-1) + 7$$

$$= -1 - 3 + 7 = 3$$

- 2.(D) For finding the remainder put $x = 2$ i.e.

$$f(2) = (2)^4 + 2(2)^3 - 3(2)^2 + 2 - 1$$

$$= 16 + 16 - 12 + 2 - 1$$

$$= 21$$

- 3.(B) $(x - 5)$ is factor, so put $x = 5$

$$2(5)^2 + 2p(5) - 2p = 0$$

$$50 + 10p - 2p = 0$$

$$50 = -8p$$

$$p = \frac{-50}{8} = \frac{-25}{4}$$

- 4.(A) $x = -1$ satisfies the equation

$$2x^3 - ax^2 - (2a - 3)x + 2 = 0$$

$$\Rightarrow -2 - a + (2a - 3) + 2 = 0$$

$$\therefore a = 3$$

- 5.(D) $3(-4)^2 + k(-4) + 8 = 0$

$$48 - 4k + 8 = 0$$

$$k = 14$$

- 6.(D) Let root be a and b .

So, Sum of the roots

$$a + b = \frac{-8}{2} = -4$$

$$2a = -4$$

$$a = -2 \text{ (roots are equal)}$$

Now, Product of the roots

$$a \times b = \frac{-k^3}{2}$$

$$-2 \times -2 = \frac{-k^3}{2}$$

$$k = -2$$

For equal root; $b^2 - 4ac = 0$

$$64 - 4 \times 2 \times -k^3 = 0$$

$$k^3 = -8$$

$$k = -2$$

- 7.(B) Let the number = x

$$\text{So, } \frac{x}{3} - \frac{x}{6} = 4$$

$$\text{So, } \frac{x}{3} - \frac{x}{6} = 4$$

$$\text{So, } \frac{x}{3} - \frac{x}{6} = 4$$

$$\Rightarrow x = 24$$

- 8.(C) Let the two numbers be x and y where $x > y$.

$$x + y = 18 \text{ and } x - y = 4$$

Adding the two equations we get $x = 11$ and $y = 7$.

Use options

- 9.(B) $4x - y = 3$ (1)

$$3x + 4y = 7$$
..... (2)

Multiplying equation (1) by 4,

$$16x - 4y = 12$$
..... (3)

Adding (3) and (2)

$$19x = 19$$

$$x = 1$$

Putting $x = 1$, in equation (1)

$$4(1) - y = 3$$

$$y = 4 - 3 = 1$$

$$(x, y) = (1, 1)$$

Use options

- 10.(B) Let the units digit = x

$$\text{Tens digit} = 2x$$

$$\text{Number} = (2x \times 10) + x$$

$$= 20x + x = 21x$$

When 18, is subtracted,

$$\text{Units digit} = 2x$$

$$\text{Tens digit} = x$$

$$\text{New number} = (2x) + (10 \times x)$$

$$= 12x$$

$$\text{So, } 21x - 18 = 12x$$

$$9x = 18$$

$$x = 2$$

$$\text{Number} = 42$$

- 11.(A)** Let the present age of father = f
 Let the present age of son = s
 10 years ago:
 $f - 10 = 4(s - 10)$
 $4s - f = 30$ (1)
 10 years later,
 $f + 10 = 2[s + 10]$
 $f - 2s = 10$ (2)
 Solving equation (1) and equation (2), by adding (1) and (2)
 $2s = 40$
 $s = 20$ and $f = 50$
- 12.(C)** Four years ago, let the ages of the father and son be $9x$ and $4x$, respectively. Their combined age was 52.
 Age of the father will be
 $\left(\frac{9}{13}\right) \times 52 = 36$ years
 Age of the son will be
 $\left(\frac{4}{13}\right) \times 52 = 16$ years
 Today their ages will be 40 and 20 respectively.
 After another 3 years, their ages will be 43 and 23, respectively.
 Therefore, required ratio = 43 : 23.
- 13.(A)** $x + y = 100$ (1)
 Where, $x = \text{boy}$; $y = \text{Girls}$
 $3.60x + 2.40y = 312$ (2)
 From equation (1) and (2), we get
 $y = 40$
- 14.(B)** Let the price of each apple be A and that of banana be B .
 then, $3A + 4B = 5A + 3B$
 $\Rightarrow B = 2A$
 Therefore, one banana can be purchased in the same amount as that used to purchase two apples.
 Therefore, instead of 18 bananas one will be able to purchase 36 apples.
- 15.(B)** Let Anand's and Vishal's present age be x and y . Then
 $x - y = 20$ (1)
 $(x - 5) = 5(y - 5)$ (2)
 On solving equations (1) and (2), we get
 $x = 30$ and $y = 10$
- 16.(B)** $2x + 4y = 6$ (1)
 $3x + 15y = 25$ (2)
 Multiplying the equation (1) by 1.5, we get
 $3x + 6y = 9$ (3)
 Subtracting (3) from (2),
 $9y = 16$
 $y = \frac{16}{9}$
- 17.(B)** Let cost price of a book = ₹ x
 Cost price of a pencil = ₹ y
 then, $6x + 4y = 34$ (1)
 $5x + 5y = 30$ (2)
 Solving these equations, we get
 $x = 5$ and $y = 1$
- 18.(B)** Let the numbers be x and y then,
 $x + y = 80$ (1)
 Let $x > y$
 $x - 4y = 45$ (2)
 $y = 7$
- 19.(A)** Let there be x hens and y cows.
 $x + y = 48$ (1)
 $2x + 4y = 140$ (2)
 Solving (1) and (2) we get
 $x = 26$, $y = 22$
- 20.(A)** Let the number be x . Then,
 $x + \frac{1}{x} = \frac{13}{6} \Rightarrow \frac{x^2 + 1}{x} = \frac{13}{6}$
 $\Rightarrow 6x^2 - 13x + 6 = 0$
 $\Rightarrow 6x^2 - 9x - 4x + 6 = 0$
 $\Rightarrow (3x - 2)(2x - 3) = 0$
 $\Rightarrow x = \frac{2}{3}$ or $x = \frac{3}{2}$
 Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.
- 21.(D)** $Kx - y = 2$

$$6x - 2y = 3$$

For unique solution,

$$\frac{k}{6} \neq \frac{-1}{-2}$$

$$k \neq 3$$

22.(D) From the given values:

$$\frac{3}{6} = \frac{-5}{-10} = \frac{2}{4}$$

So, infinite solutions.

$$\begin{aligned} 23.(B) \quad x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{+9 \pm \sqrt{81 - 4 \times 1 \times 20}}{2 \times 1} \\ &= \frac{+9 \pm 1}{2}, x = 4, 5 \end{aligned}$$

$$\begin{aligned} 24.(B) \quad x^2 - 8x + 15 &= 0 \\ \Rightarrow x^2 - 5x - 3x + 15 &= 0 \\ \Rightarrow x(x - 5) - 3(x - 5) &= 0 \\ \Rightarrow (x - 5)(x - 3) &= 0 \\ \Rightarrow x = 5 \text{ or } x = 3 \end{aligned}$$

$$\begin{aligned} 25.(C) \quad \text{Clearly, } x = 5 \text{ satisfies } x^2 - 6kx + 5 &= 0 \\ \therefore 5^2 - 6 \times k \times 5 + 5 &= 0 \\ \Rightarrow 30k &= 30 \\ \Rightarrow k &= 1 \end{aligned}$$

$$\begin{aligned} 26.(A) \quad D &= b^2 - 4ac = (-7)^2 - 4(1)(11) \\ &= 49 - 44 = 5 \end{aligned}$$

As $D > 0$, the roots of the equation are real and unequal.

$$\begin{aligned} 27.(B) \quad (x - 9)(x - 5) &= 0 \\ x^2 - 5x - 9x + 45 &= 0 \\ x^2 - 14x + 45 &= 0 \\ x^2 - 3x + 1 &= 0 \\ \alpha + \beta &= 3 \text{ and } \alpha\beta = 1 \end{aligned}$$

$$\begin{aligned} 28.(A) \quad \text{Sum of the roots} &= \frac{4}{\alpha} + \frac{4}{\beta} \\ &= \frac{4\beta + 4\alpha}{\alpha\beta} \\ &= \frac{4(\alpha + \beta)}{\alpha\beta} \\ &= \frac{4(3)}{1} = 12 \end{aligned}$$

Product of the roots

$$\begin{aligned} &= \frac{4}{\alpha} \times \frac{4}{\beta} = \frac{16}{\alpha\beta} \\ &= \frac{16}{1} = 16 \end{aligned}$$

Required equation:

$$x^2 - 12x + 16 = 0$$

29.(B) Sum of the roots

$$\begin{aligned} &= \alpha^2 + \beta + \beta^2 + \alpha \\ &= \alpha^2 + \beta^2 + \alpha + \beta \\ &= (\alpha + \beta)^2 - 2\alpha\beta + \alpha + \beta \\ &= (3)^2 - 2(1) + 3 \\ &= 10 \\ \text{Product of the roots} \\ &= (\alpha^2 + \beta)(\beta^2 + \alpha) \\ &= \alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta \\ &= (\alpha\beta)^2 + (\alpha\beta) + [(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)] \\ &= (\alpha\beta)^2 + (\alpha\beta) + [(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)] \\ &= 1^2 + 1 + [3(3^2 - (3 \times 1))] \\ &= 2 + [3(9 - 3)] \\ &= 20 \end{aligned}$$

Required equation:

$$x^2 - 10x + 20 = 0$$

$$30.(D) \quad x^2 - x - 1 = 0$$

$$\text{Sum of the roots} = \alpha + \beta = 1$$

$$\text{Product of the roots} = \alpha\beta = -1$$

$$\begin{aligned} \frac{\alpha^3 - \beta^3}{\alpha\beta} &= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 + \alpha\beta)}{\alpha\beta} \\ \frac{\alpha^3 - \beta^3}{\alpha\beta} &= \frac{(\alpha + \beta)(\alpha^2 + \beta^2 + \alpha\beta)}{\alpha\beta} \\ &= \frac{\sqrt{(1)^2 - 4(-1)}(1^2 - (-1))}{-1} = -2\sqrt{5} \end{aligned}$$

31.(C) We have to replace x by $(x - 6)$ to form the equation whose roots are 6 more than that of $x^2 + 5x + 18 = 0$.

$\therefore$ The equation will be

$$\begin{aligned} (x - 6)^2 + 5(x - 6) + 18 &= 0 \\ \Rightarrow x^2 + 36 - 12x + 5x - 30 + 18 &= 0 \\ \Rightarrow x^2 - 7x + 24 &= 0 \end{aligned}$$

- 32.(A)** We have to replace 'x' by '3x' to get the new equation.

∴ The equation will be

$$3(3x)^2 + 8(3x) + 7 = 0$$

$$\Rightarrow 27x^2 + 24x + 7 = 0$$

- 33.(D)** Let the root of the equation be x and x + 4 sum of the roots.

$$x + x + 4 = 8$$

$$2x + 4$$

$$\therefore x = 2$$

So, roots are 2, 6.

Product of the roots

$$(x)(x + 4) = m$$

$$(2)(6) = m$$

$$m = 12$$

- 34.(A)** $2x^2 + (k + 1)x + 8 = 0$.

$$b^2 - 4ac = 0$$

$$\Rightarrow (k + 1)^2 - 4 \times 2 \times 8$$

$$\Rightarrow k^2 + 2k + 1 - 64$$

$$\Rightarrow k^2 + 2k - 63 = 0$$

$$\Rightarrow k^2 + 2k - 63 = 0$$

$$\Rightarrow k = 7, -9$$

- 35.(B)** Sum of the roots = $a + b = 6$

$$\text{Product of the roots} = a \times b = 6$$

$$\text{Now, } a^2 + b^2 = (a + b)^2 - 2ab$$

$$= 36 - 12 = 24$$

- 36.(A)** If one root of $x^2 + px + 12 = 0$ is 4, then $4^2 +$

$$4p + 12 = 0, \text{ i.e. } p = -7.$$

$$x^2 - 7x + q = 0 \text{ has equal roots.}$$

For equal root

$$\text{Then, } b^2 - 4ac = 0$$

$$49 - 4 \times 1 \times q = 0$$

$$q = \frac{49}{4}$$

$$2\alpha = -\left(\frac{7}{1}\right) = 7, \text{ i.e. } \alpha = \frac{7}{2} \text{ and}$$

$$\alpha^2 = \frac{q}{1} \Rightarrow q = \frac{49}{4}$$

- 37.(C)** Sum of the roots = -13

$$\text{Product of roots} = -30$$

∴ Equation

$$x^2 - x (\text{Sum of the roots}) + \text{Product of the roots} = 0$$

$$x^2 + 13x - 30 = 0$$

- 38.(C)** $\alpha + \beta = \frac{7}{2}$

$$\alpha\beta = \frac{-3}{2}$$

$$(\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$$

$$= \alpha\beta + 2(\alpha + \beta) + 4$$

$$= \frac{-3}{2} + 2 \times \frac{7}{2} + 4$$

$$= \frac{-3}{2} + 11 = \frac{19}{2} = 9.5.$$

- 39.(B)** Equation $x^2 + 3x + k = 0$ putting $x = 4$

$$16 + 12 + k = 0$$

$$k = -28$$

By option method, put $x = -7$

$$x^2 + 3x + k = (-7)^2 + 3(-7) - 28$$

$$= 49 - 21 - 28$$

$$= 0 \text{ satisfy equation.}$$

$$x = -7$$

- 40.(C)** Let roots are a and b.

Sum of the roots

$$a + b = (k + 4)$$

$$2a = k + 4$$

$$a = \frac{k + 4}{2} \quad \dots (1)$$

Product of the roots

$$a^2 = 2k + 5 \quad \dots (2)$$

Squaring equation (1) and substituting in (2),

$$\frac{k^2 + 16 + 8k}{4} = 2k + 5$$

$$k^2 + 16 + 8k = 8k + 20$$

$$k^2 = 4$$

$$k = \pm 2$$

- 41.(D)** The required equation is $(x-2)^2 - 2(x-2) - 195 = 0$

$$x^2 - 4x + 4 - 2x + 4 - 195 = 0$$

$$x^2 - 6x - 187 = 0$$

42.(B) The original equation is $x^2 - 5x + 8 = 0$.

Then putting $x = x - 1$, we have

$$(x-1)^2 - 5(x-1) + 8 = 0$$

$$x^2 - 2x + 1 - 5x + 5 + 8 = 0$$

$$x^2 - 7x + 14 = 0$$

43.(A) In the given quadratic equation, sum of the roots = 7.

That is,

$$x + 2x = 7$$

$$3x = 7 \Rightarrow x = \frac{7}{3}$$

One root is $\frac{7}{3}$ while the other is $\frac{14}{3}$.

Product of the roots is $\frac{98}{9} = S$.

44.(C) $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

$$\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \right) = \frac{(\alpha^3 + \beta^3)}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{-b}{a} \right)^3 - 3\left(\frac{-b}{a} \right) \left(\frac{-b}{a} \right)}{\left(\frac{c}{a} \right)} = \frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\left(\frac{c}{a} \right)}$$

$$= \frac{(3abc - b^3)}{a^3} \times \frac{a}{c} = \frac{(3abc - b^3)}{a^2c}$$

45.(C) For equal roots, we have $D = 0$

$$\Rightarrow D^2 = 0 \Leftrightarrow b^2 - 4ac = 0$$

$$c = \frac{b^2}{4a}$$

46.(B) Equal roots $\Rightarrow b^2 - 4ac = 0$

$$\Rightarrow (2k)^2 - (4 \times 4 \times 9) = 0$$

$$\Rightarrow 4k^2 = 144$$

$$\Rightarrow k = \pm 6$$

47.(D) Sum of the roots:

$$\alpha + \beta = 2 + \sqrt{2} + 2 - \sqrt{2}$$

$$\alpha + \beta = 4$$

Product of root,

$$\alpha\beta = (2 + \sqrt{2})(2 - \sqrt{2})$$

$$= 4 - 2 = 2$$

So, the equation is $x^2 - (\alpha + \beta)x + \alpha\beta$
 $= x^2 - 4x + 2 = 0$

48.(A) Let one root be α , then another root = $\left(\frac{1}{\alpha} \right)$

Product of root = $\alpha \times \frac{1}{\alpha} = 1$

$$\text{i.e. } \frac{K}{3} = 1$$

Therefore, $K = 3$

49.(B) $\alpha + \beta = 5$ and $\alpha\beta = 6$

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$\alpha - \beta = \sqrt{25 - 24} = \pm 1$$

$$(\alpha + \beta) \times (\alpha - \beta) = 5 \times (\pm 1)$$

$$\alpha^2 - \beta^2 = 5 \text{ or } -5$$

50.(B) Let the roots be α and $\alpha + 2$.

Then, $\alpha + \alpha + 2 = b$

$$\Rightarrow \alpha = \frac{(b-2)}{2} \quad \dots (1)$$

and $\alpha(\alpha + 2) = c$

$$\Rightarrow \alpha^2 + 2\alpha = c \quad \dots (2)$$

Putting the value of α from (1) in (2),

$$\left[\left(\frac{(b-2)}{2} \right)^2 + \left(\frac{2(b-2)}{2} \right) \right] = c$$

$$\Rightarrow \frac{b^2 + 4 - 4b + 4b - 8}{4} = c$$

$$\Rightarrow b^2 + 4 - 8 = 4c$$

$$\Rightarrow b^2 = 4c + 4$$

51.(C) Since roots are reciprocal, product of the roots = 1

$$\Rightarrow \frac{c}{a} = 1$$

$$\Rightarrow c = a$$

52.(A) $(a-7) + (b-9) + (c-11) = (a+b+c) - 27 = 0$

$$\text{Hence, } (a-7)^3 + (b-9)^3 + (c-11)^3 - 3(a-7)(b-9)(c-11) = 0$$

(using $x^3 + y^3 + z^3 - 3xyz = 0$, if $x + y + z = 0$)

53.(C) $a = 3, b = -2$

$$\Rightarrow a + b = 3 - 2 = 1$$

$$\mathbf{54.(A)} (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$1000 = a^3 + b^3 + 720$$

$$a^3 + b^3 = 280$$

$$\begin{aligned} 55.(B) \quad & bc(b+c) + ca(c+a) + ab(a+b) + 3abc \\ & = b^2c + c^2b + c^2a + a^2c + a^2b + b^2a + abc + abc \\ & + abc \end{aligned}$$

$$= bc(a+b+c) + ac(a+b+c) + ab(a+b+c)$$

$$= (a+b+c)(ab+bc+ac) = 6 \times 11 = 66$$

If $a = 1$, $b = 2$ and $c = 3$ then,

$$bc(b+c) + ca(c+a) + ab(a+b) + 3abc$$

$$= 6 \times 5 + 3 \times 4 + 2 \times 3 + 3 \times 6$$

$$= 30 + 12 + 6 + 18$$

$$= 66$$

$$56.(C) \quad (x+y)^3 = x^3 + y^3 + 3(x+y)xy$$

$$64 = x^3 + y^3 + 12xy$$

$$57.(A) \quad x^4 + \frac{1}{x^4} - 2 = 198 - 2$$

$$\left(x^2 - \frac{1}{x^2}\right)^2 = 196$$

$$x^2 - \frac{1}{x^2} = 14$$

$$58.(B) \quad 3x - \frac{1}{3x} = 9$$

$$x - \frac{1}{9x} = 3$$

$$x^2 + \frac{1}{81x^2} = 3^2 + \frac{2}{9} = \frac{83}{9}$$

$$59.(C) \quad x^3 - y^3 = (x-y)(x^2 + y^2 + xy)$$

$$xy = 28 - x^2 - y^2$$

$$(x-y)^2 = x^2 + y^2 - 2xy$$

$$16 = x^2 + y^2 + 2x^2 + 2y^2 - 56$$

$$3(x^2 + y^2) = 72$$

$$x^2 + y^2 = 24$$

$$60.(D) \quad (x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy+yz+zx)$$

$$\text{So, } \frac{xy+yz+zx}{x^2+y^2+z^2} = \frac{-1}{2}$$

$$61.(D) \quad \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \left(\frac{1}{x}\right) + \frac{1}{x^2}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 79 + 2 = 81$$

$$\therefore x + \frac{1}{x} = \sqrt{81} = 9$$

$$62.(A) \quad 2^{x+y} = 2^{(x-y)} = \sqrt{8} = (2)^{\frac{3}{2}}$$

$$x+y = \frac{3}{2} \quad \dots (1)$$

$$x-y = \frac{3}{2} \quad \dots (2)$$

$$\text{Solving, } x = \frac{3}{2} \text{ and } y = 0$$

$$63.(C) \quad m + \frac{1}{m-2} = 4$$

$$(m-2) + \frac{1}{(m-2)} = 2$$

Squaring both the sides,

$$(m-2)^2 + \frac{1}{(m-2)^2} + 2 \times \frac{1}{(m-2)}$$

$$\times (m-2) = 4$$

$$\text{Now, } (m-2)^2 + \frac{1}{(m-2)^2} + 2 = 4$$

$$\text{Now, } (m-2)^2 + \frac{1}{(m-2)^2} = 2$$

$$64.(A) \quad x-y = \frac{x+y}{7} = \frac{xy}{4} = k$$

$$\Rightarrow x-y = k$$

$$x+y = 7k$$

$$\therefore (x+y)^2 - (x-y)^2 = 49k^2 - k^2$$

$$16k = 48k^2$$

$$\Rightarrow k = \frac{1}{3}$$

$$\therefore xy = 4k$$

$$= 4 \times \frac{1}{3} = \frac{4}{3}$$

$$65.(A) \quad x^2 + y^2 - z^2 + 2xy = x^2 + y^2 + 2xy - z^2$$

$$= (x+y)^2 - z^2 = (x+y+z)(x+y-z)$$

$$= (b+c-2a+c+a-2b+a+b-2c)(x+y-z) = 0$$

$$66.(C) \quad a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4$$

$$\Rightarrow a^2 + \frac{1}{a^2} + b^2 + \frac{1}{b^2} = 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 + 2 + \left(b - \frac{1}{b}\right)^2 + 2 = 4$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^2 + \left(b - \frac{1}{b}\right)^2 = 0$$

$$\Rightarrow a = \frac{1}{a}$$

$$\Rightarrow a^2 = 1$$

$$\Rightarrow a = \pm 1$$

$$\Rightarrow a - \frac{1}{a} = 0; b - \frac{1}{b} = 0$$

Similarly,

$$\Rightarrow a = b = \pm 1$$

$$\therefore a^2 + b^2 = 1 + 1 = 2$$

67.(A) $25a^2 + 40ab + 16b^2 = (5a + 4b)^2$
 $= (5 \times 23 - 29 \times 4)^2 = (115 - 116)^2 = 1$

68.(C) $ax^2 + bx + c = a(x - p)^2$
 $\Rightarrow ax^2 + bx + c = a(x^2 - 2px + p^2)$
 $\Rightarrow ax^2 + bx + c = ax^2 - 2apx + ap^2$

Comparing the corresponding coefficients.

$$b = -2ap \text{ and } c = ap^2$$

$$b^2 = 4a^2p^2 \text{ and}$$

$$p^2 = \frac{c}{a} \Rightarrow p^2 = \frac{b^2}{4a^2};$$

$$\therefore \frac{b^2}{4a^2} = \frac{c}{a}$$

$$\Rightarrow b^2 = 4ac$$

69.(B) $a^2 - 4a - 1 = 0$

$$\Rightarrow a^2 - 1 = 4a$$

$$a - \frac{1}{a} = 4$$

$$\therefore a^2 + \frac{1}{a^2} = 18$$

$$= a^2 + \frac{1}{a^2} + 3 \left(a - \frac{1}{a}\right)$$

$$= 18 + 12 = 30$$

70.(D) $ax + by = 3 \dots\dots (1)$

$$bx - ay = 4 \dots\dots (2)$$

On squaring and adding both the equation.

$$a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2 + 2abxy - 2abxy =$$

$$9 + 16$$

$$\Rightarrow a^2x^2 + b^2x^2 + a^2y^2 + b^2y^2 = 25$$

$$\Rightarrow x^2(a^2 + b^2) + y^2(a^2 + b^2) = 25$$

$$\Rightarrow (a^2 + b^2)(x^2 + y^2) = 25$$

$$\Rightarrow a^2 + b^2 = 25$$

71.(A) $121a^2 + 64b^2 = (11a)^2 + (8b)^2$

$$\ominus (x + y)^2 = x^2 + y^2 + 2xy$$

$\therefore$ Required expression

$$= 2 \times 11a \times 8b = 176ab$$

72.(B) $a^2 + b^2 + c^2 = ab + bc + ca$

$$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$$

$$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca = 0$$

$$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

$$\Rightarrow a - b = 0, b - c = 0, c - a = 0$$

$$\Rightarrow a = b, b = c, c = a$$

$$\Rightarrow a = b = c$$

$$\therefore \frac{a+c}{b} = \frac{2a}{a} = 2$$

73.(B) By using options.

74.(B) $x^2 + \frac{1}{x^2} = 66$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 66$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 66 - 2 = 64$$

$$\Rightarrow x - \frac{1}{x} = \pm 8$$

$$\therefore \text{Expression} = \frac{x^2 - 1 + 2x}{x}$$

$$= \frac{x^2}{x} - \frac{1}{x} + 2 = x - \frac{1}{x} + 2$$

Putting the value of $x - \frac{1}{x}$

$$= 8 + 2 \text{ or } -8 + 2$$

$$= 10, 6$$

75.(D) $\frac{5x}{2x^2 + 5x + 1} = \frac{1}{3}$

Dividing Numerator and Denominator by x.

$$\frac{5}{2x + 5 + \frac{1}{x}} = \frac{1}{3}$$

On dividing N^f and D^f by 2

$$\frac{\frac{5}{2}}{x + \frac{5}{2} + \frac{1}{2x}} = \frac{1}{3}$$

$$\Rightarrow \left(x + \frac{1}{2x}\right) + \frac{5}{2} = \frac{15}{2}$$

$$\Rightarrow x + \frac{1}{2x} = \frac{15}{2} - \frac{5}{2} = \frac{10}{2} = 5$$

76.(A) $2x - \frac{1}{2x} = 6$

$$\Rightarrow x - \frac{1}{4x} = 3$$

$$\Rightarrow x^2 + \frac{1}{16x^2} = 9 + \frac{1}{2} = \frac{19}{2}$$

77.(A) $a + \frac{1}{b} = 1$

$$\Rightarrow ab + 1 = b$$

$$\Rightarrow ab = b - 1 \quad \dots (1)$$

$$b + \frac{1}{c} = 1$$

$$\frac{1}{c} = 1 - b \Rightarrow c = \frac{1}{1-b} \quad \dots (2)$$

On multiplying (1) and (2),

$$abc = \frac{b-1}{1-b} = -1$$

78.(C) $x + \frac{1}{x} = 99$

$$\therefore \frac{100x}{2x^2 + 102x + 2} = \frac{100x}{2x^2 + 2 + 102x}$$

$$= \frac{100}{2x + \frac{2}{x} + 102} = \frac{100}{2\left(x + \frac{1}{x}\right) + 102}$$

$$= \frac{100}{2 \times 99 + 102} = \frac{100}{300} = \frac{1}{3}$$

79.(A) $\frac{x}{1} = \frac{a-b}{a+b}$

By componendo and dividendo,

$$\frac{1-x}{1+x} = \frac{1 - \frac{a-b}{a+b}}{1 + \frac{a-b}{a+b}} = \frac{a+b-a+b}{a+b+a-b} = \frac{b}{a}$$

$$\text{Similarly, } \frac{1-y}{1+y} = \frac{c}{b}; \frac{1-z}{1+z} = \frac{a}{c}$$

$\therefore$ Expression

$$= \frac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)} = 1$$

80.(C) $a + \frac{1}{b} = 1$

$$\Rightarrow a = \frac{1}{2}; b = 2$$

$$b + \frac{1}{c} = 1$$

$$\Rightarrow b = 2, c = -1$$

$$\therefore c + \frac{1}{a} = -1 + 2 = 1$$

81.(D) $(x-a)^3 - \frac{1}{(x-a)^3}$

$$= \left(x - a - \frac{1}{x-a}\right)^3 + 3\left(x - a - \frac{1}{x-a}\right)$$

$$= (x-a-x+b)^3 + 3(x-a-x+b)$$

$$= (b-a)^3 + 3(b-a)$$

$$= 5^3 + 3 \times 5$$

$$= 125 + 15 = 140$$

82.(B) $a = \frac{b^2}{b-a}$

$$\Rightarrow ab - a^2 = b^2$$

$$\Rightarrow a^2 + b^2 - ab = 0$$

$$\therefore a^3 + b^3 = (a+b)(a^2 + b^2 - ab) = (a+b) \times 0$$

$$= 0$$

83.(C) $x^3 + y^3 + z^3 - 3xyz$

$$= \frac{1}{2}(x+y+z)$$

$$[(x-y)^2 + (y-z)^2 + (z-x)^2]$$

$$= \frac{1}{2}(333+333+334)(0+1+1)$$

$$= 1000$$

84.(C) $\frac{4x^3 - x}{(2x+1)(6x-3)}$

$$= \frac{x(4x^2-1)}{(2x+1) \times 3(2x-1)} = \frac{x(2x+1)(2x-1)}{3(2x+1)(2x-1)}$$

$$= \frac{x}{3} = \frac{9999}{3} = 3333$$

85.(A) Given, $a + b + c + d = 4$

Expression;

$$= \frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)}$$

$$\begin{aligned}
 & + \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)} \\
 & = \frac{1-d+1-a+1-b+1-c}{(1-a)(1-b)(1-c)(1-d)} \\
 & = \frac{4-(a+b+c+d)}{(1-a)(1-b)(1-c)(1-d)} \\
 & = \frac{4-4}{(1-a)(1-b)(1-c)(1-d)} = 0
 \end{aligned}$$

86.(B) $3\left(x + \frac{1}{x}\right) = 1$

$$\Rightarrow x + \frac{1}{x} = \frac{1}{3}$$

On cubing both sides,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{1}{3} = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 1 = \frac{1}{27}$$

87.(A) $x + \frac{1}{x} = 2$

$$\Rightarrow \frac{x^2 + 1}{x} = 2$$

$$\Rightarrow x^2 + 1 = 2x$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 = 0$$

$$\Rightarrow x-1 = 0$$

$$\Rightarrow x = 1$$

$$\text{So, } 2x = 2$$

88.(B) $x + 3 = a$

$$\frac{1}{x+3} = \frac{1}{a}$$

$$a + \frac{1}{a} = (x+3) + \frac{1}{(x+3)}$$

$$\Rightarrow \frac{(x+3)^2 + 1}{x+3}$$

$$\Rightarrow \frac{x^2 + 6x + 9 + 1}{x+3}$$

$$\Rightarrow \frac{x^2 + x + 5x + 10}{x+3}$$

$$\text{Put, } x^2 + x = 5$$

$$\Rightarrow \frac{5x + 10 + 5}{x+3} = \frac{5x + 15}{x+3}$$

$$\Rightarrow 5\left(\frac{x+3}{x+3}\right)$$

$$\Rightarrow a + \frac{1}{a} = 5$$

$$(x+3) + \frac{1}{(x+3)} = 5$$

$$a^3 + \frac{1}{a^3} = (5)^3 - 3(5)$$

$$= 125 - 15 = 110$$

89.(A)

$$x + \frac{1}{x} = 1$$

$$\Rightarrow x^2 + 1 = x$$

$$\Rightarrow x^2 - x + 1 = 0$$

$$\therefore \frac{2}{x^2 - x + 2} = \frac{2}{x^2 - x + 1 + 1}$$

$$= \frac{2}{0+1} = 2$$

90.(D)

$$(a-3) - \frac{1}{(a-3)}$$

$$= 5 - 3 = 2$$

$$(a-3)^3 - \frac{1}{(a-3)^3}$$

$$= \left((a-3) - \frac{1}{a-3}\right)^3 + 3\left((a-3) - \frac{1}{a-3}\right)$$

$$2^3 + 3 \times 2$$

$$= 14$$

91.(C)

$$a = 4.965, b = 2.343,$$

$$c = 2.622$$

$$a + (-b) + (-c) = 4.965 - 2.343 - 2.622 = 0$$

$$a^3 - b^3 - c^3 - 3abc = a^3 + (-b)^3 + (-c)^3 - 3abc = 0$$

92.(C)

$$\text{Given, } p = 99$$

$$\therefore \text{Expression}$$

$$= p(p^2 + 3p + 3) = p^3 + 3p^2 + 3p$$

$$= p^3 + 3p^2 + 3p + 1 - 1$$

$$= (p+1)^3 - 1$$

$$= (99+1)^3 - 1 = (100)^3 - 1$$

$$= 1000000 - 1 = 999999$$

93.(B)

$$x + \frac{1}{x} = 2A$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$= 8A^3 - 3 \times 2A$$

$$x^3 + \frac{1}{x^3} = 8A^3 - 6A$$

$$\text{So, average} = 4A^3 - 3A$$

$$\begin{aligned} 94.(C) \quad \frac{x^3 + \frac{1}{x}}{x^2 - x + 1} &= \frac{x^2 + \frac{1}{x^2}}{x - 1 + \frac{1}{x}} \\ &= \frac{\left(x + \frac{1}{x}\right)^2 - 2}{\left(x + \frac{1}{x}\right) - 1} = \frac{9 - 2}{3 - 1} = \frac{7}{2} \end{aligned}$$

$$\begin{aligned} 95.(D) \quad x &= \frac{1}{2 + \sqrt{3}} \\ &= \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})} = 2 - \sqrt{3} \end{aligned}$$

Similarly,

$$\begin{aligned} \therefore \frac{1}{x+1} + \frac{1}{y+1} &= \frac{1}{3-\sqrt{3}} + \frac{1}{3+\sqrt{3}} \\ &= \frac{3+\sqrt{3}+3-\sqrt{3}}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{6}{6} = 1 \end{aligned}$$

$$\begin{aligned} 96.(A) \quad x + \frac{1}{x} &= 2 \\ \Rightarrow \frac{x^2 + 1}{x} &= 2 \\ \Rightarrow x^2 - 2x + 1 + 0 &= 0 \\ (x - 1)^2 &= 0 \Rightarrow x = 1 \\ \therefore x^{100} + \frac{1}{x^{100}} &= 1 + 1 = 2 \\ x &= 1 \end{aligned}$$

$$\therefore x^{100} + \frac{1}{x^{100}} = 1 + 1 = 2$$

$$\begin{aligned} 97.(D) \quad \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} &= 1 \\ \Rightarrow \left(\frac{a}{1-a} + 1\right) + \left(\frac{b}{1-b} + 1\right) + \left(\frac{c}{1-c} + 1\right) & \\ = 3 + 1 &= 4 \end{aligned}$$

$$\Rightarrow \frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

$$98.(A) \quad x + \frac{1}{4x} = \frac{3}{2}$$

$$\Rightarrow 2x + \frac{1}{2x} = 3$$

On cubing both sides,

$$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}$$

$$\left(2x + \frac{1}{2x}\right)^3 = 27$$

$$\Rightarrow 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$$

$$\Rightarrow 8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$$

$$\begin{aligned} 99.(B) \quad \left(x + \frac{1}{x}\right)^3 &= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\left(x + \frac{1}{x}\right) \\ \Rightarrow \left(x + \frac{1}{x}\right)^2 &= 3 \left[\because x + \frac{1}{x} \neq 0\right] \\ \therefore \left(x + \frac{1}{x}\right)^4 &= 3 \times 3 = 9 \end{aligned}$$

$$\begin{aligned} 100.(A) \quad \left(x + \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} + 2\left(x\right)\left(\frac{1}{x}\right) \\ &= x^2 + \frac{1}{x^2} + 2 = 81 \end{aligned}$$

$$\left(x + \frac{1}{x}\right)^2 = 79 + 2 = 81$$

$$\left(x + \frac{1}{x}\right) = 9$$

Cubing both sides

$$\left(x + \frac{1}{x}\right)^3 = 729$$

$$x^3 + \frac{1}{x^3} + 3 \times \left(x\right)\left(\frac{1}{x}\right) = 729$$

$$x^3 + \frac{1}{x^3} + 3 \times 9 = 729$$

$$x^3 + \frac{1}{x^3} = 702$$

$$101.(A) \quad y = \frac{\sqrt{2}-1}{\sqrt{2}+1},$$

$$x + y = \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$x + y = 2 + 1 + 2 + 1$$

$$x + y = 6$$

Similarly, $x - y = 4\sqrt{2}$

$$\begin{aligned} \frac{2x^2 + 3xy + 2y^2}{2x^2 - 3xy + 2y^2} &= \frac{2x^2 + 4xy + 2y^2 - xy}{2x^2 - 4xy + 2y^2 + xy} \\ &= \frac{2(x+y)^2 - xy}{2(x-y)^2 + xy} = 6 \\ &= \frac{2 \times 36 - 1}{2 \times 32 + 1} = \frac{71}{65} \end{aligned}$$

102.(A) $x^{1/3} + y^{1/3} = z^{1/3}$

$$x + y + 3(x^{1/3} + y^{1/3})x^{1/3}y^{1/3} = z$$

$$(x + y - z) = -3x^{1/3}y^{1/3}z^{1/3}$$

$$(x + y - z)^3 = -27xyz$$

$$(x + y - z)^3 + 27xyz = 0$$

103.(B) $x^4 - 17x^3 + 17x^2 - 17x + 17$

$$= x^4 - 16x^3 + 16x^2 - 16x - x^3 + x^2 - x + 17$$

When, $x = 16$

Expression,

$$= 16^4 - 16^4 + 16^3 - 16^2 - 16^3 + 16^2 - 16 + 17 = 1$$

104.(A) $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

Let, $t = \sqrt[3]{2}$

$$\Rightarrow t^2 = \sqrt[3]{4} \text{ and } t^3 = 2$$

$$\frac{1}{t^2 + t + 1} = at^2 + bt + c$$

$$\Rightarrow \frac{t-1}{(t-1)(t^2+t+1)} = at^2 + bt + c$$

$$\Rightarrow \frac{t-1}{(t^3-1)} = at^2 + bt + c$$

$$(t^3 - 1 = 2 - 1 = 1)$$

$$t - 1 = at^2 + bt + c$$

Comparing coefficient

$$a = 0, b = 1, c = -1$$

$$a + b + c = 0 + 1 - 1 = 0$$

105.(A) Let, $x = \sqrt{2^3 \sqrt{4} \sqrt{2^3 \sqrt{4} \sqrt{2^3 \sqrt{4} \dots \infty}}}$

$$x = \sqrt{2^3 \sqrt{4x}} \quad (\text{squaring both sides})$$

$$x^2 = 2^3 \sqrt{4x} \quad (\text{cubing both sides})$$

$$x^6 = 2^3 \cdot 4x = 2^5 \cdot x \Rightarrow x^5 = 2^5$$

Hence, $x = 2$

106.(A) $2^{16} \times 4^8 \times 2^4 \times 4^2 \times 2^1 = (32^a)^{32}$

$$2^{41} = 2^{160a}$$

$$\text{So, } a = \frac{41}{160}$$

107.(B) $p\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{1} \dots \infty}}\right)$

$$\text{Let } 1 + \sqrt{1 + \sqrt{1 + \sqrt{1} \dots \infty}} = x$$

$$1 + \sqrt{x} = x$$

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$\text{So, } p\left(\frac{3 + \sqrt{5}}{2}\right)$$

108.(A) $x = \sqrt[4]{4^4 \sqrt[4]{4^4 \sqrt[4]{4^4 \dots \infty}}}$

$$\Rightarrow x = \sqrt[4]{4x}$$

$$\Rightarrow x^4 = 4x \Rightarrow x^3 = 4 = 2^2$$

$$\text{then, } x = 2^{\frac{2}{3}} = 32^{\frac{1}{3}}$$

$$\Rightarrow 2^{\frac{2}{3}} = (2^5)^{\frac{1}{3}} 2^{5a} = 2^{5a}$$

$$\Rightarrow 5a = \frac{2}{3}$$

$$\Rightarrow a = \frac{2}{15}$$

109.(C) Put, $a = 1, b = 2, c = 3$

$$\Rightarrow a(b-c)^3 + b(c-a)^3 + c(a-b)^3$$

$$= 1(2-3)^3 + 2(3-1)^3 + 3(1-2)^3$$

$$= -1 + 16 - 3 = 12$$

Put there value in all option-

$$a) \Rightarrow 3abc = 18$$

$$b) \Rightarrow (a-b)(b-c)(c-a) = 2$$

$$c) \Rightarrow (a-b)(b-c)(c-a)(a+b+c) = 12$$

$$d) \Rightarrow (a+b)(b+c)(c+a)(a+b+c) = 360$$

110.(C) Put, $a = 0, b = 1, c = 2$

$$a^4(b^2 - c^2) + b^4(c^2 - a^2) + c^4(a^2 - b^2)$$

$$= 0 + 1(4) + 16(-1) = -12$$

Put there value in all option:

$$a) \Rightarrow 3a^2b^2c^2 = 0$$

$$b) \Rightarrow (a^2 - b^2)(b^2 - c^2)(c^2 - a^2) = 12$$

$$c) \Rightarrow -(a^2 - b^2)(b^2 - c^2)(c^2 - a^2) = -12$$

$$d) \Rightarrow (a^2 + b^2)(b^2 + c^2)(c^2 + a^2) = 20$$

111.(A) Put, $a = 1, b = 2, c = 3$

$$\begin{aligned} & a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc \\ &= 1(-1)^2 + 2(2)^2 + 3(-1)^2 + 8(1)(2)(3) \\ &= 1 + 8 + 3 + 48 = 60 \end{aligned}$$

Put these value in all option.

$$a) \Rightarrow (a+b)(b+c)(c+a) = 60$$

$$b) \Rightarrow (a-b)(b-c)(c-a) = 2$$

$$c) \Rightarrow 0$$

$$d) \Rightarrow abc = 6$$

112.(A) Put, $a = 1, b = -2, c = 3$

$$\begin{aligned} & (bc + ca + ab)^3 - b^3c^3 - c^3a^3 - a^3b^3 \\ &= (-6 + 3 - 2)^3 - (-8)(27) - (27)(1) - (1)(-8) \\ &= -125 + 216 - 27 + 8 = 72 \end{aligned}$$

Put these value in all option-

$$a) \Rightarrow 3abc(a+b)(b+c)(c+a) = 72$$

$$b) \Rightarrow (a+b)(b+c)(c+a) = -4$$

$$c) \Rightarrow (a-b)(b-c)(c-a) = -30$$

$$d) \Rightarrow 24abc = -144$$

113.(A) $(x^b + c)^{b-c} \cdot (x^c + a)^{c-a} \cdot (x^a + b)^{a-b}$
 $= x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2}$

$$= x^{b^2-c^2+c^2-a^2+a^2-b^2} x^0 = 1$$

114.(B) $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$
 $= (x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c} \cdot (x^{c-a})^{c+a}$
 $= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} = 1$

(answer is not depending upon a, b, c so, we can put a, b, c equal to zero.)

$$\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

115.(B) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$
 $= \frac{1}{x^{-a}(x^a+x^b+x^c)} + \frac{1}{x^{-b}(x^a+x^b+x^c)} + \frac{1}{x^{-c}(x^a+x^b+x^c)}$

$$\begin{aligned} &= \frac{x^a}{(x^a+x^b+x^c)} + \frac{x^b}{(x^a+x^b+x^c)} + \frac{x^c}{(x^a+x^b+x^c)} \\ &= \frac{x^a+x^b+x^c}{(x^a+x^b+x^c)} = 1 \end{aligned}$$

(Put $a = 0, b = 0, c = 0$ or $a = b = c$)

$$\begin{aligned} &= \frac{1}{(1+x^0+x^0)} + \frac{1}{(1+x^0+x^0)} + \frac{1}{(1+x^0+x^0)} \\ &= \frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1} \\ &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \end{aligned}$$

116.(D) This is an A.P. in which $a = 2$ and $d = (5 - 2) = 3$

$$\therefore \text{Next number} = (8 + 3) = 11$$

117.(C) Given that the first term is 5 and common difference is 4, so the 9th term can be written as

$$T_9 = a + 8d = 5 + 8 \times 4 = 5 + 32 = 37$$

118.(C) $\frac{T_7}{T_3} = \frac{a+6d}{a+2d} = \frac{12}{5}$

$$\Rightarrow 5(a + 6d) = 12(a + 2d)$$

$$\Rightarrow 5a + 30d = 12a + 24d$$

$$\Rightarrow 7a = 6d$$

$$\therefore \frac{a}{d} = \frac{6}{7}$$

Now, $\frac{T_{13}}{T_4} = \frac{a+12d}{a+3d}$

$$= \frac{\frac{a}{d} + 12}{\frac{a}{d} + 3} = \frac{\frac{6}{7} + 12}{\frac{6}{7} + 3} = \frac{10}{3}$$

119.(B) $S_{15} = \frac{15}{2}[2a + 14d] = 105$

$$\Rightarrow 2a + 14d = 14 \quad \dots (1)$$

and

$$S_{30} = \frac{30}{2}[2a + 29d] = 105 + 780 = 885$$

$$\Rightarrow 2a + 29d = 59 \quad \dots (2)$$

$\therefore$ From equation (1) and (2)

$$\Rightarrow 15d = 45$$

$$\Rightarrow d = 3$$

120.(C) $T_4 = a + 3d = 8$

$$s_7 = \frac{7}{2}[2a + 6d]$$

$$= 7(a + 3d)$$

$$= 7 \times 8$$

$$= 56$$

$$121.(B) \quad S_n = \left(\frac{a+1}{2}\right) \times n$$

$$9153 = \left(\frac{-7+233}{2}\right) \times n$$

$$9153 = 113 \times n$$

$$n = 81$$

122.(C) Let the first term be a and d be the common difference

$$3^{\text{rd}} \text{ term} = a + 2d \text{ and } 6^{\text{th}} \text{ term} = a + 5d$$

$$(a + 5d) - (a + 2d) = -4 - (-13)$$

$$d = 3, a = -19$$

$$\text{Sum of 12 terms} = \frac{12}{2} [2 \times (-19) + (12 - 1)3]$$

$$= -30$$

123.(D) Let first term be ' a ' and ' d ' be the common difference.

$$3^{\text{rd}} \text{ term} = a + 2d = 13$$

$$6^{\text{th}} \text{ term} = a + 5d = -5$$

$$3d = -5 - 13 = -18$$

$$d = -6, a = 25$$

$$11^{\text{th}} \text{ term} = a + 10d = 25 - 60 = -35$$

124.(A) Given that 7 times the 7th term of an AP is equal to 11 times its 11th term. So the 18th term can be calculated as

$$7(A + 6D) = 11(A + 10D)$$

$$\Rightarrow 7A + 42D = 11A + 110D$$

$$\Rightarrow 4A + 68D = 0$$

$$\Rightarrow A + 17D = 0$$

Thus, the 18th term of this AP is 0.

125.(D) Let the three numbers in AP be $a - d, a, a + d$.

$$\text{Thus, } a - d + a + a + d = 21$$

$$\Rightarrow a = 7$$

$$\text{Also, } (7 - d)^2 + 7^2 + (7 + d)^2 = 179$$

$$49 + d^2 - 14d + 49 + 49 + d^2 + 14d = 179$$

$$2d^2 = 179 - 147$$

$$d = 4 \text{ or } -4$$

126.(C) In the AP series,

$$a_n = a + (n - 1)d = 78$$

We have $a = 3, d = 8 - 3 = 5$. Therefore,

$$3 + (n - 1) \times 5 = 78$$

$$(n - 1) \times 5 = 78 - 3 = 75$$

$$n - 1 = \frac{75}{5} = 15$$

$$n = 15 + 1 = 16$$

Hence, a_{16} or the 16th term is 78.

127.(B) Let the first term of the A.P. be a and common difference be d . Then,

$$a + 3d = 14 \quad \dots (1)$$

$$a + 11d = 70 \quad \dots (2)$$

On subtracting (1) from (2), we get

$$8d = 56 \Rightarrow d = 7$$

Putting $d = 7$ in (i), we get $a + 3 \times 7 = 14$

$$a = (14 - 21) = -7$$

$$\therefore \text{First term} = -7$$

128.(C) Given that the first term of GP is 2 and the common ratio is 3, so the fifth term is

$$T_5 = a \times r^4$$

$$T_5 = 2 \times 3^4 = 162$$

129.(D) Given that the first term of GP is 5. Let the common ratio be r . Then,

$$T_4 = ar^3 = 5 \times r^3$$

$$T_6 = ar^5 = 5 \times r^5$$

$$\text{Also, } \frac{T_4}{T_6} = \frac{1}{4} \Rightarrow \frac{5r^3}{5r^5} = \frac{1}{4}$$

$$\Rightarrow \frac{5 \times r^3}{5 \times r^5} = 4$$

$$\therefore r^2 = 4$$

$$\therefore r = 2 \text{ and } -2$$

Second term of the GP will be

$$a \times r = 5 \times 2 = 10 \text{ or } 5 \times (-2) = -10$$

130.(C) Let the first term be a and common ratio r .

$$\text{Then, } ar^2 = 4.$$

$$\text{Product of first 5 terms} = a \times ar \times ar^2 \times ar^3 \times ar^4$$

$$= a^5 \times r^{10} = (ar^2)^5 = 4^5$$

- 131.(B)** Let a be the first term and r be common ratio.

i.e., given conditions

Fourth term of G.P.:

$$p = T_4 = ar^3 \quad \dots (1)$$

Seventh term of G.P.:

$$q = T_7 = ar^6 \quad \dots (2)$$

Tenth term of G.P.:

$$r = T_{10} = ar^9 \quad \dots (3)$$

Equation (1) $\times$ equation (3),

$$pr = ar^3 \times ar^9 \Rightarrow pr = a^2 r^{12} \Rightarrow pr = (ar^6)^2 \Rightarrow pr = q^2$$

- 132.(A)** Go through the options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 8$.

Only option (a) satisfies both conditions.

- 133.(B)** This series is like 10, 17, 24, 31, 94.

Here $n = 13$, $d = 7$ and $a = 10$

Using the formula for the sum

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Sum} = 676$$

Using the average method,

$$\text{Average} = \frac{(1^{\text{st}} \text{ number} + \text{Last number})}{2}$$

$$\text{Average} = \frac{10 + 94}{2} = 52$$

$$\text{So, the sum} = \text{average} \times \text{number of numbers} \\ = 52 \times 13 = 676$$

- 134.(B)** Let a be the first term and r be the common ratio of the GP.

Also, assume that n^{th} term is the last term of the GP.

$$\text{Then, } a + ar^{n-1} = 66 \quad \dots (1)$$

$$\text{and } ar \cdot ar^{n-2} = 128$$

$$\text{or } a^2 r^{n-1} = 128 \quad \dots (2)$$

From (1) and (2),

$$a + \frac{128}{a} = 66$$

$$\text{or } a^2 - 66a + 128 = 0$$

$$a^2 - 64a - 2a + 128 = 0$$

$$a(a - 64) - 2(a - 64) = 0$$

$$(a - 2)(a - 64) = 0$$

$$\therefore a = 64 \text{ or } 2.$$

- 135.(B)** Since $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$ are in H.P.

$$\therefore 1, 3, 5, 7, \dots \text{ are in A.P.}$$

$$\therefore T_{10} = 1 + 9 \times 2$$

$$T_{10} = 19$$

Since 10^{th} term of the A.P. is 19

Therefore, 10^{th} term of the H.P. is $\frac{1}{19}$.

Practice Exercise Level 2

1.(B) $x^2 - 1 = 0$

$$x = \pm 1$$

$$\text{Put } x = 1$$

$$x^4 + ax^3 + 3x - b = 0$$

$$1 + a + 3 - b = 0$$

$$a - b = -4$$

$$\text{and } x = -1$$

$$1 - a - 3 - b = 0$$

$$a + b = -2$$

$$a = -3, b = 1$$

2.(D) $(a + 4)^3 = a^3 + 12a^2 + 48a + 64$

So, by comparing the coefficients

$$k = 48$$

3.(C) $3x^2 - 4x + 1 = 0$

$$x = \frac{1}{3} \text{ and } x = 1$$

$$\text{put } x = 1$$

$$p - 8 - q + 1 = 0$$

$$p - q = 7$$

$$\text{put } x = \frac{1}{3}$$

$$\frac{p}{27} - \frac{8}{9} - \frac{q}{3} + 1 = 0$$

$$p - 9q = -3$$

$$8q = 10, q = \frac{5}{4}$$

$$p = \frac{33}{4}$$

4.(B) The solution of the equation

$$\frac{3}{x+y} + \frac{2}{x-y} = 2$$

$$\text{and } \frac{9}{x+y} - \frac{4}{x-y} = 1$$

$$\text{Put } \frac{1}{x+y} = u$$

$$\text{and } \frac{1}{x-y} = v$$

$$3u + 2v = 2 \quad \dots (1)$$

$$9u - 4v = 1 \quad \dots (2)$$

On solving (1) and (2), we get

$$v = \frac{1}{2} \text{ and } u = \frac{1}{3}$$

$$\text{So, } x + y = 3 \text{ and } x - y = 2$$

$$\text{Therefore, } x = \frac{5}{2} \text{ and } y = \frac{1}{2}$$

5.(C) $x^2 + 7x + 12$

$$a + b = -7$$

$$ab = 12$$

roots are -3 and -4 ,

$$(a + b)^2 = ((-3) + (-4))^2 = 49$$

$$(a - b)^2 = (4 - 3)^2 = 1$$

Sum of the roots for new equation

$$(a + b)^2 + (a - b)^2 = 49 + 1 = 50$$

Product of the roots of new equation

$$(a + b)^2 (a - b)^2 = (49) \times (1) = 49$$

So, our new equation is,

$$x^2 - 50x + 49 = 0$$

6.(D) As m and n are the roots of the equation

$$x^2 - bx + c = 0$$

$$\text{then, } m + n = -(-b) = b$$

$$m \times n = c$$

$$\text{Now, } \frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn}$$

$$= \frac{(m + n)^2 - 2mn}{mn}$$

$$= \frac{b^2 - 2c}{c}$$

Product of the roots

$$= \frac{m}{n} \times \frac{n}{m} = 1$$

So, required equation is,

$$x^2 - \left(\frac{b^2 - 2c}{c} \right) x + 1 = 0$$

$$cx^2 - (b^2 - 2c)x + c = 0$$

7.(D)

Given equation,

$$px^2 + qx + r = 0 \quad \dots (1)$$

$$\text{and } rx^2 - qx + p = 0 \quad \dots (2)$$

Putting $x = \frac{-1}{x}$ in (1) we get,

$$p\left(\frac{-1}{x}\right)^2 + q\left(\frac{-1}{x}\right) + r = 0$$

$$rx^2 - qx + p = 0 \quad \dots (3)$$

Hence, roots are negative reciprocal of each other.

8.(A)

Let α and β are the roots of equation $px^2 + qx + r = 0$

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

Now, Sum of the roots

$$\alpha + \beta = \frac{-q}{p} \quad \dots (1)$$

Product of the roots

$$\alpha\beta = \frac{r}{p} \quad \dots (2)$$

Dividing (1) by (2),

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{-q}{r}$$

$$= \frac{1}{\beta} + \frac{1}{\alpha} = \frac{-q}{r}$$

Squaring both side

$$\left(\frac{1}{\beta} + \frac{1}{\alpha} \right)^2 = \frac{q^2}{r^2}$$

$$\frac{1}{\beta^2} + \frac{1}{\alpha^2} + \frac{2}{\alpha\beta} = \frac{q^2}{r^2}$$

$$\text{Given, } \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \alpha + \beta$$

$$\alpha + \beta + \frac{2}{\alpha\beta} = \frac{q^2}{r^2}$$

$$\frac{-q}{p} + \frac{2p}{r} = \frac{q^2}{r^2}$$

$$\frac{2p}{r} = \frac{q^2}{r^2} + \frac{q}{p}$$

$$2 = \frac{q^2}{pr} + \frac{qr}{p^2}$$

- 9.(A)** For equation $x^2 - 23x + 42 = 0$, $S = (\alpha + \beta) = 23$, $P = \alpha\beta = 42$
 $\therefore \alpha = 21, \beta = 2$.

The required equation has the roots as

$$\frac{\alpha}{(\beta)^3}, \frac{\beta}{(\alpha)^3}.$$

Therefore, the sum of the roots of the given equation is

$$s = \frac{\alpha}{\beta^3} + \frac{\beta}{\alpha^3} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^3}$$

$$= \frac{[(21)^4 + (2)^4]}{(42)^3} = 2.6$$

$$P = \frac{1}{(\alpha\beta)^2} = 5.6 \times 10^{-4}$$

So, required equation is $x^2 - 2.6x + 5.6 \times 10^{-4} = 0$.

- 10.(B)** Roots of the given equation are $\frac{1}{2}$ and 3.

Roots of new equation are $-\frac{1}{2}$ and -3 . So,

required equation is

$$2x^2 + 7x + 3 = 0$$

- 11.(C)** $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \dots}}}}$

$$\Rightarrow x < \sqrt{4 + 2} \Rightarrow x < \sqrt{6} \approx 2.4$$

Hence, option (c) is correct.

- 12.(C)** Using factor theorem

$$f(-1) = 0 \text{ and } f(1) = 0$$

We get equations,

$$a + b = -3$$

$$\text{and } a - b = 1$$

On solving, we get

$$a = -1 \text{ and } b = -2$$

- 13.(D)** $\alpha + \beta = -1, \alpha\beta = 1, \gamma + \delta = -3, \gamma\delta = 1$

$$\text{Also } \gamma^2 + 3\gamma + 1 = 0; \delta^2 + 3\delta + 1 = 0$$

$$(\alpha - \gamma)(\beta + \delta)(\alpha + \delta)(\beta - \gamma)$$

$$= [\alpha\beta - \gamma(\alpha + \beta) + \gamma^2] [\alpha\beta + \delta(\alpha + \beta) + \delta^2]$$

$$= (1 + \gamma + \gamma^2)(1 - \delta + \delta^2) = (-2\gamma)(-4\delta)$$

$$= 8\gamma\delta = 8$$

- 14.(B)** Let $\alpha' = 1 + \alpha$ and $\beta' = 1 + \beta$

$$\text{then, } \left(\frac{1}{\alpha+1}\right)^3 + \left(\frac{1}{\beta+1}\right)^3$$

$$= \left(\frac{1}{\alpha'}\right)^3 + \left(\frac{1}{\beta'}\right)^3$$

$$= \frac{\alpha'^3 + \beta'^3}{(\alpha'\beta')^3}$$

$$= \frac{(\alpha' + \beta')^3 - 3\alpha'\beta'(\alpha'\beta')}{(\alpha'\beta')^3} \dots (1)$$

From equation, $x^2 + 5x - 5 = 0$

$$\alpha + \beta = -5; \alpha\beta = -5$$

$$(\alpha + 1) + (\beta + 1) = -5 + 1 + 1$$

$$\alpha' + \beta' = -3$$

$$\alpha'\beta' = (\alpha + 1)(\beta + 1)$$

$$\alpha'\beta' = \alpha\beta + \alpha + \beta + 1$$

$$= -5 - 5 + 1 = -9$$

Putting these in equation (1)

Required value

$$= \frac{(-3)^3 - 3(-9)(-9)}{(-9)^3} = 4$$

- 15.(B)** $x^2 + x + 1 = 0$

$$\alpha + \beta = -1$$

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = -1$$

$$\text{So, } \alpha^{2000} + \beta^{2000} = -1$$

$$= x^2 + x + 1 = 0$$

- 16.(C)** $\alpha + \beta = -\frac{b}{a}$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$$

$$\alpha^2\beta^2 = \frac{c^2}{a^2}$$

$$\text{So, } a^2x^2 - (b^2 - 2ac)x + c^2 = 0$$

- 17.(C)** $x(x^2 - 4) - 1 = 2(x^2 - 4) - 1$

$$x^3 - 4x = 2x^2 - 8$$

$$x^3 - 4x - 2x^2 + 8 = 0$$

$$x = 2, -2$$

- 18.(D)** $x^2 - 6x + 8 + \lambda(x^2 - 4x + 3) = 0$

Using option method,

Put $\lambda = 0$

$$x^2 - 6x + 8 = 0$$

$$x = 4, 2$$

$\lambda = 1$ (No roots)

$\lambda = -1$ (Imaginary roots)

Option (a) is not possible for all λ .

19.(A) $abc^2x^2 + (3a^2 + b^2)cx - 6a^2 - ab + 2b^2 = 0$
 Discriminant, $D = [(3a^2 + b^2)c]^2 - 4(abc^2)(-6a^2 - ab + 2b^2)$
 $= c^2 [9a^4 + b^4 + 6a^2b^2 + 24a^3b + 4a^2b^2 - 8ab^3]$
 $= c^2 [(3a^2)^2 + (-b^2)^2 + (4ab)^2 + 2(3a^2)(-b^2) + 2(-b^2)(4ab) + 2(4ab)(3a^2)]$
 $= c^2 [3a^2 + 4ab - b^2]^2$
 $= [c(3a^2 + 4ab - b^2)]^2$

As determinant is a perfect square, the roots of equation will be rational

20.(C) $(x - a)(x - b) = c$
 $x^2 - (a + b)x + (ab - c) = 0$
 Sum of the roots, $\alpha + \beta = a + b$
 Product of the roots, $\alpha\beta = ab - c$
 Now, $(x - \alpha)(x - \beta) + c = 0$
 $x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$
 $x^2 - (a + b)x + ab - c + c = 0$
 $x^2 - (a + b)x + ab = 0$
 $x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$

$\therefore$ The required roots are a and b

21.(B) $4x^2 + 5k - (5k + 1)x = 0$

Let root, $x, x - 1$

$$\text{So, } x + x - 1 = \frac{5k}{4} + \frac{1}{4} \quad \dots (1)$$

$$x(x - 1) = \frac{5k}{4} \quad \dots (2)$$

Put value of $\frac{5k}{4}$ in equation (1)

$$\text{Now, } x^2 - 3x + \frac{5}{4} = 0$$

$$x = \frac{5}{2}, \frac{1}{2}$$

So, $k = 3$

$$k = -\frac{1}{5}$$

22.(B) $\alpha_1 \alpha_2 = \beta_1 \beta_2 = 1$

$$\alpha_2 = \frac{1}{\alpha_1} \text{ and } \beta_2 = \frac{1}{\beta_1}$$

$$\alpha_1 + \beta_1 = \frac{-b_1}{a_1}$$

$$\alpha_1 \beta_1 = \frac{c_1}{a_1}$$

$$\alpha_2 + \beta_2 = \frac{-b_2}{a_2}; \alpha_2 \beta_2 = \frac{c_2}{a_2}$$

$$\frac{1}{\alpha_1} \frac{1}{\beta_1} = \frac{c_2}{a_2}$$

$$\frac{1}{\alpha_1 \beta_1} = \frac{c_2}{a_2}$$

$$\frac{1}{\alpha_1} + \frac{1}{\beta_1} = \frac{-b_2}{a_2}$$

$$\frac{-b_2}{a_1} \frac{a_1}{c_1} = \frac{-b_2}{a_2} \frac{a_1}{c_1} = \frac{c_2}{a_2}$$

$$\frac{\alpha_1 + \beta_1}{\alpha_1 \beta_1} = \frac{-b_2}{a_2}$$

$$\Rightarrow \frac{c_1}{a_2} = \frac{b_1}{b_2} = \frac{a_1}{c_2}$$

23.(B) $\alpha + \beta = p; \alpha\beta = q$
 $(\alpha^2 - \beta^2) \times (\alpha^3 - \beta^3)$
 $\alpha^5 - \beta^2 \alpha^3 - \alpha^2 \beta^3 - \beta^5$
 $\alpha^5 + \beta^5 - \alpha^2 \beta^2 (\alpha + \beta)$
 $(\alpha^2 + \beta^2)(\alpha^3 + \beta^3) - 2\alpha^2 \beta^2 (\alpha + \beta)$
 $(p^2 - 2q)(p^3 - 3pq) - 2pq^2$
 $p(p^2 - q)(p^2 - 4q)$

24.(C) $x^2 + x + 3 = 0$

$$\alpha + \beta = -1$$

$$\alpha\beta = 3$$

$$3x^2 + 5x + 3 = 0$$

$$x^2 + \frac{5x}{3} + 1 = 0$$

$$x^2 - \left[\frac{\alpha^2 + \beta^2}{\alpha\beta} \right] x + \left[\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} \right] = 1$$

$$x^2 - \left[\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right] x + \left[\left(\frac{\alpha}{\beta} \right) \left(\frac{\beta}{\alpha} \right) \right] = 1$$

$\therefore$ Roots of the required equation

$$= \frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha}$$

$$25.(A) \quad 2x^2 + 12x + 18 = 0$$

$$\Rightarrow x^2 + 6x + 9 = 0$$

$$\Rightarrow (x+3)^2 = 0$$

$$\Rightarrow x+3=0$$

$$\Rightarrow x = -3$$

$$26.(C) \quad (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$= 2(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\frac{1^2 + 1^2 + (-2)^2}{2} = \frac{1+1+4}{2} = 3$$

$$27.(C) \quad x + \frac{1}{x} = 5$$

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right)$$

$$- \left(x + \frac{1}{x}\right)$$

$$= (5^3 - 3 \times 5)(5^2 - 2) - (5)$$

$$= 2525$$

$$28.(A) \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$= 5\sqrt{5} - 3\sqrt{5} = 2\sqrt{5}$$

$$29.(D) \quad x + \frac{1}{x} = \frac{17}{4}$$

$$x^3 + \frac{1}{x^3} = \frac{4913}{64} - \frac{51}{4} = \frac{4097}{64}$$

$$30.(D) \quad \frac{x}{xa+yb+zc} = \frac{y}{ya+zb+xc}$$

$$= \frac{z}{za+xb+yc}$$

$$= \frac{x+y+z}{xa+yb+zc+ya+zb+xc+za+xb+yc}$$

$$= \frac{x+y+z}{xa+ya+za+yb+ya+yc+zc+zb+za}$$

$$= \frac{x+y+z}{a(x+y+z)+b(x+y+z)+c(x+y+z)}$$

$$= \frac{x+y+z}{(x+y+z)(a+b+c)}$$

$$= \frac{1}{(a+b+c)}$$

$$31.(C) \quad \text{Let, } \frac{p}{a} = x, \frac{q}{b} = y, \frac{r}{c} = z$$

$$x + y + z = 1$$

$$\text{and } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$\Rightarrow \frac{yz+xz+xy}{xyz} = 0$$

$$x + y + z = 1$$

On squaring both sides,

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 1$$

$$\Rightarrow x^2 + y^2 + z^2 + 0 = 1$$

$$\Rightarrow x^2 + y^2 + z^2 = 1$$

$$\text{So, } \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} = 1$$

$$32.(A) \quad ab + bc + ca = 0$$

$$\Rightarrow ab + ca = -bc$$

$$\therefore a^2 - bc = a^2 + ab + ac = a(a+b+c)$$

Similarly,

$$b^2 - ac = b(a+b+c)$$

$$c^2 - ab = c(a+b+c)$$

$$\therefore \frac{1}{a^2-bc} + \frac{1}{b^2-ca} + \frac{1}{c^2-ab}$$

$$= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$$

$$= \frac{1}{(a+b+c)} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$= \frac{1}{a+b+c} \left(\frac{bc+ca+ab}{abc} \right)$$

$$= \frac{1}{a+b+c} \times \frac{0}{abc} = 0$$

$$33.(D) \quad \text{Given,}$$

$$bc + ab + ca = abc$$

$$\therefore bc + ab = abc - ca$$

$$bc + ca = abc - ab$$

$\therefore$ Expression

$$= \frac{b+c}{abc-bc} + \frac{a+c}{abc-ac} + \frac{a+b}{abc-ab}$$

$$= \frac{b+c}{ab+ca} + \frac{a+c}{bc+ab} + \frac{a+b}{bc+ca}$$

$$= \frac{b+c}{a(b+c)} + \frac{a+c}{b(c+a)} + \frac{a+b}{c(a+b)}$$

$$= \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

$$= \frac{bc+ac+ab}{abc} = \frac{abc}{abc} = 1$$

34.(D) $\frac{a^2-bc}{a^2+bc} + \frac{b^2-ca}{b^2+ca} + \frac{c^2-ab}{c^2+ab} = 1$

$$\Rightarrow \left(\frac{a^2-bc}{a^2+bc} + 1 \right) + \left(\frac{b^2-ca}{b^2+ca} + 1 \right) + \left(\frac{c^2-ab}{c^2+ab} + 1 \right) = 4$$

$$\Rightarrow \frac{a^2-bc+a^2+bc}{a^2+bc} + \frac{b^2-ca+b^2+ca}{b^2+ca} + \frac{c^2-ab+c^2+ab}{c^2+ab}$$

$$+ \frac{c^2-ab+c^2+ab}{c^2+ab}$$

$$\Rightarrow \frac{2a^2}{a^2+bc} + \frac{2b^2}{b^2+ca} + \frac{2c^2}{c^2+ab} = 4$$

$$\Rightarrow \frac{a^2}{a^2+bc} + \frac{b^2}{b^2+ca} + \frac{c^2}{c^2+ab} = \frac{4}{2} = 2$$

35.(C) Given that,

$$x + \frac{1}{x} = 1$$

Expression,

Dividing numerator and denominator by x.

$$= \frac{x^2+3x+1}{x^2+7x+1} = \frac{x+\frac{1}{x}+3}{x+\frac{1}{x}+7}$$

Putting the value of $x + \frac{1}{x} = 1$

$$= \frac{1+3}{1+7} = \frac{4}{8} = \frac{1}{2}$$

36.(C) Expression,

$$= \frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$$

$$= \frac{(s^2 - 2sa + a^2 + s^2 + b^2 - 2sb + s^2 - 2sc + c^2 + s^2)}{a^2 + b^2 + c^2}$$

$$= \frac{4s^2 + a^2 + b^2 + c^2 - 2s(a+b+c)}{a^2 + b^2 + c^2}$$

$$= \frac{4s^2 + a^2 + b^2 + c^2 - 4s^2}{a^2 + b^2 + c^2} = 1$$

37.(D) $\left(a + \frac{1}{a}\right)^2 = 3 = (\sqrt{3})^2$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

Cubing both sides,

$$\left(a + \frac{1}{a}\right)^3 = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 0$$

38.(B) $a + b + c = 3; a^2 + b^2 + c^2 = 6$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\Rightarrow 3^2 = 6 + 2(ab + bc + ca)$$

$$\Rightarrow 9 - 6 = 2(ab + bc + ca)$$

$$\Rightarrow ab + bc + ca = \frac{3}{2}$$

$$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

$$\Rightarrow \frac{bc+ac+ab}{abc} = 1$$

$$\Rightarrow abc = ab + bc + ca = \frac{3}{2}$$

39.(D) $x = a(b - c)$

$$\Rightarrow \frac{x}{a} = b - c$$

Similarly, $y = b(c - a)$

$$\Rightarrow \frac{y}{b} = c - a \text{ and } \frac{z}{c} = a - b$$

$$\therefore \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = b - c + c - a + a - b = 0$$

$$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$$

$$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c} = \frac{3xyz}{abc}$$

$$(If a + b + c = 0, a^3 + b^3 + c^3 = 3abc)$$

40.(B) $\sqrt{\frac{x-a}{x-b}} - \sqrt{\frac{x-b}{x-a}} = \frac{b-a}{x}$

$$\Rightarrow \sqrt{\frac{(x-a)(x-a)}{(x-b)(x-a)}} - \sqrt{\frac{(x-b)(x-b)}{(x-a)(x-b)}}$$

$$= \frac{b-a}{x}$$

$$\Rightarrow \frac{x-a}{\sqrt{(x-b)(x-a)}} - \frac{x-b}{\sqrt{(x-a)(x-b)}}$$

$$= \frac{b-a}{x}$$

$$\Rightarrow \frac{x-a-x+b}{\sqrt{(x-a)(x-a)}} = \frac{b-a}{x}$$

$$\Rightarrow \frac{b-a}{\sqrt{(x-b)(x-a)}} = \frac{b-a}{x}$$

$$\Rightarrow x = \sqrt{(x-b)(x-a)}$$

On squaring,

$$x^2 = (x-b)(x-a)$$

$$\Rightarrow x^2 = x^2 - ax - bx + ab$$

$$\Rightarrow ax + bx = ab$$

$$\Rightarrow x(a+b) = ab$$

$$\Rightarrow x = \frac{ab}{a+b}$$

41.(A) $x = (\sqrt{2}-1)^{\frac{1}{2}}$

$$x^{-2} = \sqrt{2}-1$$

$$\therefore x^2 = \frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \sqrt{2}+1$$

$$\therefore \frac{1}{x^2} = \sqrt{2}-1$$

$$\therefore x^2 - \frac{1}{x^2} = \sqrt{2}+1 - (\sqrt{2}-1) = 2$$

42.(A) $x + \frac{1}{x} = 5$

On squaring both sides,

$$x^2 + \frac{1}{x^2} + 2 = 25$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2 = 23 \quad \dots (1)$$

Expression,

$$= \frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

$$= \frac{x^4 + 1 + 3x^3 + 3x + 5x^2}{x^4 + 1}$$

$$= \frac{x^2 \left(x^2 + \frac{1}{x^2} \right) + 3x^2 \left(x + \frac{1}{x} \right) + 5x^2}{x^2 \left(x^2 + \frac{1}{x^2} \right)}$$

$$= \frac{\left(x^2 + \frac{1}{x^2} \right) + 3 \left(x + \frac{1}{x} \right) + 5}{x^2 + \frac{1}{x^2}}$$

$$= \frac{23 + 3 \times 5 + 5}{23} = \frac{43}{23}$$

43.(D) Check through options.

If $x = y = z$, then

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{3}{x^2}$$

$$\text{and } \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$$

$$\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{3}{x^2}$$

44.(C) $a + b + c = 0$

$$\Rightarrow a + b = -c; b + c = -a, c + a = -b$$

$$\therefore \frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}$$

$$= \left(\frac{-c}{c} \right) + \left(\frac{-a}{a} \right) + \left(\frac{-b}{b} \right)$$

$$= -1 - 1 - 1 = -3$$

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{a}{-a-b} + \frac{b}{-a-b} + \frac{c}{-a-b}$$

$$= -1 - 1 - 1 = -3$$

$\therefore$ Value of expression,

$$= (-3) \times (-3) = 9$$

45.(D) $(p^{-1-3} q^{2-(-2)})^{1/4} \div (p^{6-(-2)} q^{-3-3})$

$$= (p^{-4} q^4)^{1/4} \div (p^8 q^{-6})$$

$$= (p^{-1} q) \div (p^8 q^{-6}) = p^{-1-8} q^{1-(-6)}$$

$$= p^{-9} q^7$$

$$p^a q^b = p^{-9} q^7$$

By comparing $a = -9, b = 7$

$$a + b = -9 + 7 = -2$$

46.(A)

$$x = a^{\frac{1}{2}} + a^{\frac{-1}{2}}$$

$$y = a^{\frac{1}{2}} - a^{\frac{-1}{2}}$$

$$\therefore x^2 - y^2 = 4a^{\frac{1}{2}} \cdot a^{\frac{-1}{2}} = 4$$

$$[\ominus (a+b)^2 - (a-b)^2 = 4ab]$$

$$y^2 - x^2 = -4, a^{\frac{1}{2}} \cdot a^{\frac{-1}{2}} = -4$$

Expression,

$$= (x^4 - x^2 y^2 - 1) + (y^4 - x^2 y^2 + 1)$$

$$= x^2 (x^2 - y^2) - 1 + y^2 (y^2 - x^2) + 1$$

$$= (x^2 - y^2)^2 = 4^2 = 16$$

47.(C)

$$\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$$

$$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$$

$$\Rightarrow \frac{3}{x} + \frac{3}{y} + \frac{3}{z} = 4 + 4 + 4 = 12$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{12}{3} = 4$$

48.(D)

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$$

Using componendo and dividendo,

$$\left(\frac{a}{b} = \frac{a+b}{a-b} \right)$$

$$\therefore \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{x}{2a} + \frac{x}{2b}$$

$$= \frac{4ab}{2a(a+b)} + \frac{4ab}{2b(a+b)}$$

$$= \frac{2b}{a+b} + \frac{2a}{a+b}$$

$$= \frac{2(b+a)}{a+b} = 2$$

49.(D)

$$x^2 - yz = x^2 + xy + zx$$

$$= x(x + y + z)$$

$$[\therefore xy + yz + zx = 0 \Rightarrow yz = -xy - zx]$$

Similarly,

$$y^2 - zx = y(x + y + z)$$

$$z^2 - xy = x(x + y + z)$$

 $\therefore$ Expression,

$$= \frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)}$$

$$+ \frac{1}{z(x+y+z)}$$

$$= \frac{yz + zx + xy}{xyz(x+y+z)} = 0$$

50.(B)

$$x = \frac{4\sqrt{15}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$x = \frac{4\sqrt{15}(\sqrt{5} - \sqrt{3})}{2}$$

$$= 2\sqrt{15}(\sqrt{5} - \sqrt{3})$$

$$x = 10\sqrt{3} - 6\sqrt{5}$$

$$\frac{x + \sqrt{20}}{x - \sqrt{20}} + \frac{x + \sqrt{12}}{x - \sqrt{12}}$$

$$= \frac{10\sqrt{3} - 6\sqrt{5} + 2\sqrt{5}}{10\sqrt{3} - 6\sqrt{5} - 2\sqrt{5}} + \frac{10\sqrt{3} - 6\sqrt{5} + 2\sqrt{3}}{10\sqrt{3} - 6\sqrt{5} - 2\sqrt{3}}$$

$$= \frac{5\sqrt{3} - 2\sqrt{5}}{5\sqrt{3} - 4\sqrt{5}} + \frac{6\sqrt{3} - 3\sqrt{5}}{4\sqrt{3} - 3\sqrt{5}}$$

$$= 1 + \frac{2\sqrt{5}}{5\sqrt{3} - 4\sqrt{5}} + 1 + \frac{2\sqrt{3}}{4\sqrt{3} - 3\sqrt{5}}$$

$$= 2 + \left[\frac{2\sqrt{5}(5\sqrt{3} + 4\sqrt{5})}{-5} + \frac{2\sqrt{3}(4\sqrt{3} + 3\sqrt{5})}{3} \right]$$

$$= 2 + [-2\sqrt{15} - 8 + 8 + 2\sqrt{15}]$$

$$= 2$$

51.(B)

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)}$$

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$[\ominus (a-c)^2 = (c-a)^2]$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

$$[\text{If } a + b + c = 0, a^3 + b^3 + c^3 = 3abc,$$

$$\text{Here, } a - b + b - c + c - a = 0]$$

52.(D)

$$\sqrt[3]{a} + \sqrt[3]{b} - \sqrt[3]{c} = 0$$

$$\therefore a + b - c = -3(abc)^{\frac{1}{3}}$$

On cubing both sides,

$$(a + b + c)^3 = -27abc$$

$$\therefore (a + b - c)^3 + 27abc = 0$$

53.(D)

$$\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = \frac{2}{1}$$

By componendo and dividendo,

$$\left[\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \right]$$

$$\frac{2\sqrt{x+4}}{2\sqrt{x-4}} = \frac{3}{1}$$

On squaring,

$$\frac{x+4}{x-4} = \frac{9}{1}$$

$$\Rightarrow 9x - 36 = x + 4$$

$$\Rightarrow 9x - x = 36 + 4$$

$$\Rightarrow 8x = 40, x = 5$$

54.(A)

$$\left(x + \frac{1}{x} \right)^2 = 3$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{3}$$

On cubing both sides,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

$$\Rightarrow x^6 + 1 = 0$$

$$\begin{aligned} x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1 \\ = x^{200}(x^6 + 1) + x^{84}(x^6 + 1) + x^{12}(x^6 + 1) + (x^6 + 1) = 0 \end{aligned}$$

$$55.(C) \quad \frac{1}{x^{99}} = \frac{1}{(-1)^{99}} = -1$$

$$\frac{1}{x^{98}} = \frac{1}{(-1)^{98}} = 1$$

$\therefore$ Expression,

$$= -1 + 1 - 1 - 1 + 1 - 1 = -2$$

$$56.(D) \quad x = 3 + 2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8} = 3 - 2\sqrt{2}$$

$$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$$

$$= 6$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times \left(\frac{1}{x}\right)$$

$$x^2 + \frac{1}{x^2} = 36 - 2 = 34$$

$\therefore$ Expression,

$$= \frac{x^6 + x^4 + x^2 + 1}{x^3}$$

$$= \frac{x^4(x^2 + 1) + 1(x^2 + 1)}{x^3}$$

$$= \frac{(x^4 + 1)(x^2 + 1)}{x^3}$$

$$= \frac{x^3 \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right)}{x^3}$$

$$= \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right)$$

$$= 34 \times 6 = 204$$

$$57.(B) \quad \frac{(a+b)^2}{(a-b)^2} = \frac{25}{4}$$

By componendo and dividendo,

$$\frac{2(a^2 + b^2)}{4ab} = \frac{29}{21}$$

$$\frac{a^2 + b^2}{2ab} = \frac{29}{21}$$

$$a^2 + b^2 = 58 \quad [\ominus ab = 21]$$

$$\therefore a^2 + b^2 + 3ab = 58 + 3(21) = 121$$

$$58.(B) \quad \frac{p^2 - p}{2p^3 + 6p^2} \div \frac{p^2 - 1}{p^2 + 3p} \div \frac{p^2}{p + 1}$$

$$= \frac{p(p-1)}{2p^2(p+3)} \div \frac{(p+1)(p-1)}{p(p+3)} \div \frac{p^2}{p+1}$$

$$= \frac{p(p-1)}{2p^2(p+3)} \times \frac{p(p+3)}{(p+1)(p-1)}$$

$$\times \frac{p+1}{p^2} = \frac{1}{2p^2}$$

If $p = 2$ then all option are different

$\Rightarrow$ Expression

$$\frac{2^2 - 2}{2 \times 2^3 + 6 \times 2^2} \times \frac{2^2 + 3 \times 2}{2^2 - 1} \times \frac{2 + 1}{2^2}$$

$$= \frac{2}{40} \times \frac{10}{3} \times \frac{3}{4} = \frac{1}{8}$$

$$= \frac{1}{2 \times 2^2} \times \frac{1}{2p^2}$$

$$59.(B) \quad \frac{(q-p) + (p-n) + (n-q)}{(p-n)(n-q)(q-p)}$$

$$\Rightarrow \frac{0}{(p-n)(n-q)(q-p)} = 0$$

$$60.(D) \quad u_1 = 1 - \frac{1}{2}$$

$$u_2 = \frac{1}{2} - \frac{1}{3}, u_3 = \frac{1}{3} - \frac{1}{4}$$

$$u_4 = \frac{1}{4} - \frac{1}{5}, u_5 = \frac{1}{5} - \frac{1}{6}$$

$$u_1 + u_2 + u_3 + u_4 + u_5$$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} = \frac{5}{6}$$

61.(D) It can be seen that for the series, the average of two terms is 2, for 3 terms the average is 3 and so on.

Thus, the sum to 2 terms is 2^2 , for 3 terms it is 3^2 and so on. For 11111 terms it would be $11111^2 = 123454321$.

62.(A) $\text{Area} = 36 + \frac{36}{2} + \frac{36}{4} + \frac{36}{8} \dots \infty$

$a = 36$

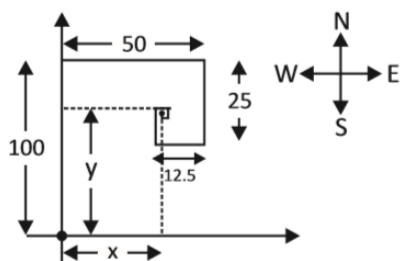
$r = \frac{1}{2}$

$\text{Sum of } \infty = \frac{a}{1-r}$

$= \frac{36}{1 - \frac{1}{2}} = 36 \times 2 = 72$

$= 72 \text{ cm}^2$

63.(B)



For x-co-ordinate,

$x = 50 - \frac{50}{4} + \frac{50}{16} - \frac{50}{64} + \dots \infty$

$x = 50 \left(1 - \frac{1}{2^2} + \frac{1}{2^4} - \frac{1}{2^6} + \dots \right)$

Here, $a = 50$ and $r = \frac{-1}{2^2}$

$x = \frac{50}{1 - (-1/4)} = \frac{50}{5} \times 4 = 40 \text{ m}$

For y-co-ordinate,

$y = 100 - \frac{100}{4} + \frac{100}{16} - \frac{100}{64} + \dots \infty$

$y = \frac{100}{1 - (-1/4)} = \frac{100}{5} \times 4 = 80 \text{ m}$

Distance between his final position and his initial position

$= \sqrt{40^2 + 80^2} = \sqrt{8000}$

$= 40\sqrt{5} \text{ m}$

64.(B)

Let, $S = 1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27}$

$+ \frac{8}{81} + \frac{10}{243} + \dots + \infty \dots (1)$

$3S = 3 + 2 + \frac{4}{3} + \frac{6}{9} + \frac{8}{27}$

$+ \frac{10}{81} + \dots + \infty \dots (2)$

Subtracting (1) from (2),

$2(S) = 4 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \dots + \infty$

$2(S) = 4 + 2 + \left[\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots + \infty \right]$

$2(S) = 4 + 2 \left[\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right]$

$2(S) = 4 + 2 \left[\frac{1}{2} \right]$

$2(S) = 5$

$(S) = \left[\frac{5}{2} \right]$

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